OCR
Oxford Cambridge and RSA

Wednesday 5 June 2019 – Morning

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

7662337652

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \,|\, A) = \mathrm{P}(B)\mathrm{P}(A \,|\, B)$ **or** $\mathrm{P}(A \,|\, B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \tfrac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$
$s = \tfrac{1}{2}(u+v)t$	$\mathbf{s} = \tfrac{1}{2}(\mathbf{u}+\mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \tfrac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

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Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Solve the inequality $10x^2 + x - 2 > 0$. **[4]**

2 The point A is such that the magnitude of $\overrightarrow{OA}$ is 8 and the direction of $\overrightarrow{OA}$ is 240°.

(a) **(i)** Show the point A on the axes provided in the Printed Answer Booklet. **[1]**

(ii) Find the position vector of point A.
Give your answer in terms of **i** and **j**. **[3]**

The point B has position vector $6\mathbf{i}$.

(b) Find the exact area of triangle AOB. **[2]**

The point C is such that $OABC$ is a parallelogram.

(c) Find the position vector of C.
Give your answer in terms of **i** and **j**. **[2]**

3 The function f is defined by $\mathrm{f}(x) = (x-3)^2 - 17$ for $x \geqslant k$, where k is a constant.

(a) Given that $\mathrm{f}^{-1}(x)$ exists, state the least possible value of k. **[1]**

(b) Evaluate ff(5). **[2]**

(c) Solve the equation $\mathrm{f}(x) = x$. **[3]**

(d) Explain why your solution to part **(c)** is also the solution to the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. **[1]**

4 Sam starts a job with an annual salary of £16 000. It is promised that the salary will go up by the same amount every year. In the second year Sam is paid £17 200.

(a) Find Sam's salary in the tenth year. **[2]**

(b) Find the number of complete years needed for Sam's **total** salary to first exceed £500 000. **[4]**

(c) Comment on how realistic this model may be in the long term. **[1]**

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5 A curve has equation $x^3 - 3x^2y + y^2 + 1 = 0$.

(a) Show that $\frac{dy}{dx} = \frac{6xy - 3x^2}{2y - 3x^2}$. **[4]**

(b) Find the equation of the normal to the curve at the point (1, 2). **[4]**

6 Let $f(x) = 2x^3 + 3x$. Use differentiation from first principles to show that $f'(x) = 6x^2 + 3$. **[6]**

7 **In this question you must show detailed reasoning.**

A sequence $u_1, u_2, u_3 \ldots$ is defined by $u_n = 25 \times 0.6^n$.

Use an algebraic method to find the smallest value of N such that $\sum_{n=1}^{\infty} u_n - \sum_{n=1}^{N} u_n < 10^{-4}$. **[8]**

8 A cylindrical tank is initially full of water. There is a small hole at the base of the tank out of which the water leaks.

The height of water in the tank is x m at time t seconds. The rate of change of the height of water may be modelled by the assumption that it is proportional to the square root of the height of water.

When $t = 100$, $x = 0.64$ and, at this instant, the height is decreasing at a rate of $0.0032\,\text{ms}^{-1}$.

(a) Show that $\frac{dx}{dt} = -0.004\sqrt{x}$. **[2]**

(b) Find an expression for x in terms of t. **[4]**

(c) Hence determine at what time, according to this model, the tank will be empty. **[2]**

9 **(a)** Express $3\cos 3x + 7\sin 3x$ in the form $R\cos(3x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[3]**

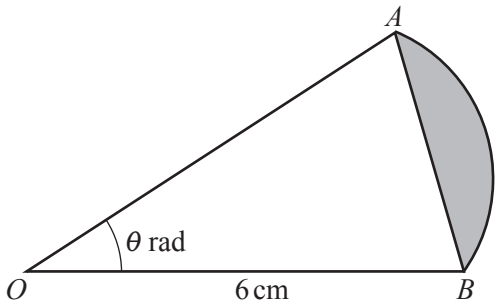
(b) Give full details of a sequence of three transformations needed to transform the curve $y = \cos x$ to the curve $y = 3\cos 3x + 7\sin 3x$. **[4]**

(c) Determine the **greatest** value of $3\cos 3x + 7\sin 3x$ as x varies and give the smallest positive value of x for which it occurs. **[2]**

(d) Determine the **least** value of $3\cos 3x + 7\sin 3x$ as x varies and give the smallest positive value of x for which it occurs. **[2]**

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The diagram shows a sector AOB of a circle with centre O and radius 6 cm.
The angle AOB is θ radians.
The area of the segment bounded by the chord AB and the arc AB is $7.2\,\text{cm}^2$.

(a) Show that $\theta = 0.4 + \sin\theta$. **[3]**

(b) Let $\text{F}(\theta) = 0.4 + \sin\theta$.

By considering the value of $\text{F}'(\theta)$ where $\theta = 1.2$, explain why using an iterative method based on the equation in part **(a)** will converge to the root, assuming that 1.2 is sufficiently close to the root. **[2]**

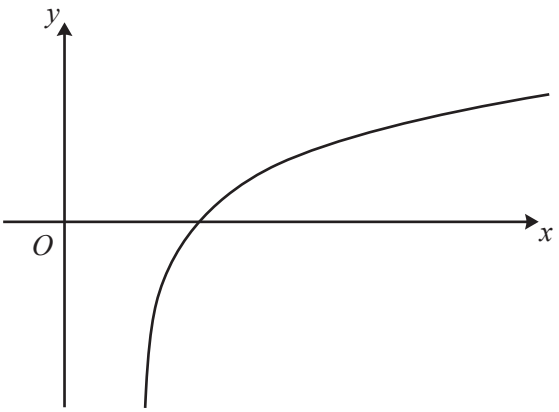
(c) Use the iterative formula $\theta_{n+1} = 0.4 + \sin\theta_n$ with a starting value of 1.2 to find the value of θ correct to 4 significant figures.
You should show the result of each iteration. **[3]**

(d) Use a change of sign method to show that the value of θ found in part **(c)** is correct to 4 significant figures. **[3]**

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11



The diagram shows part of the curve $y = \ln(x-4)$.

(a) Use integration by parts to show that $\int \ln(x-4)\,\mathrm{d}x = (x-4)\ln|x-4| - x + c$. **[5]**

(b) State the equation of the vertical asymptote to the curve $y = \ln(x-4)$. **[1]**

(c) Find the total area enclosed by the curve $y = \ln(x-4)$, the x-axis and the lines $x = 4.5$ and $x = 7$. Give your answer in the form $a\ln 3 + b\ln 2 + c$ where a, b and c are constants to be found. **[4]**

12 A curve has equation $y = a^{3x^2}$, where a is a constant greater than 1.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6xa^{3x^2}\ln a$. **[3]**

(b) The tangent at the point $(1, a^3)$ passes through the point $(\frac{1}{2}, 0)$.

Find the value of a, giving your answer in an exact form. **[4]**

(c) By considering $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ show that the curve is convex for all values of x. **[5]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 $10x^2 + x - 2 = 0$

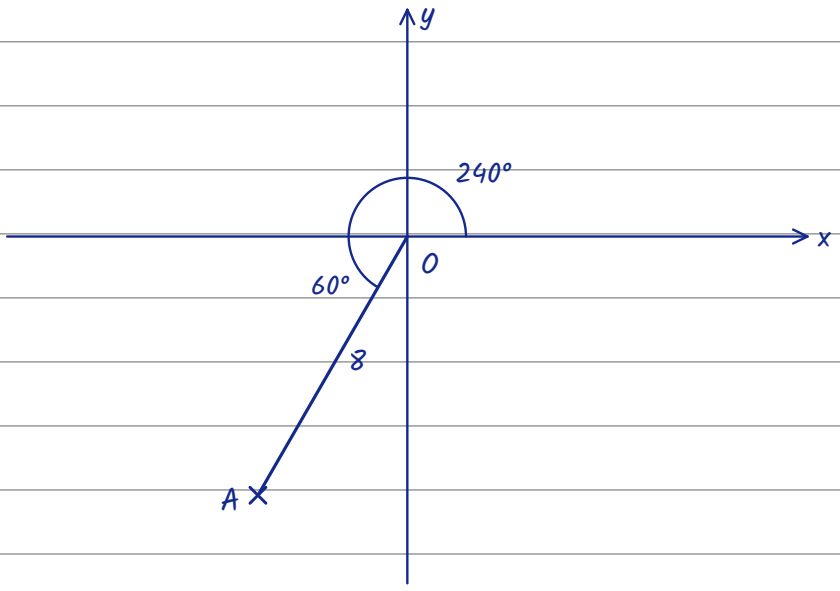
$(5x - 2)(2x + 1) = 0$

Critical values: $x = \frac{2}{5}$ or $x = -\frac{1}{2}$

$y = 10x^2 + x - 2$ is a ∪-shaped parabola, so $y > 0$ outside the roots

$x < -\frac{1}{2}$ or $x > \frac{2}{5}$

2 (a)(i)



2 (a)(ii) $x = 8\cos 240° = 8 \times (-½) = -4$

$y = 8\sin 240° = 8 \times (-\sqrt{3}/2) = -4\sqrt{3}$

$OA = -4i - 4\sqrt{3}\,j$

Question number	
2 (b)	Angle $AOB = 360° - 240° = 120°$
	Area $= \frac{1}{2} \times 8 \times 6 \times \sin 120°$
	$= 24 \times \sqrt{3}/2 = 12\sqrt{3}$
2 (c)	$OC = AB = OB - OA = 6i - (-4i - 4\sqrt{3}\,j)$
	$OC = 10i + 4\sqrt{3}\,j$
3 (a)	Vertex of the parabola is at $x = 3$, so least $k = 3$
3 (b)	$f(5) = (5 - 3)^2 - 17 = 4 - 17 = -13$
	-13 is not in the domain of f $(x \geq 3)$, so $f(-13)$ does not exist:
	$ff(5)$ is not defined
3 (c)	$(x - 3)^2 - 17 = x$
	$x^2 - 6x + 9 - 17 - x = 0 \quad \Rightarrow \quad x^2 - 7x - 8 = 0$
	$(x - 8)(x + 1) = 0 \quad \Rightarrow \quad x = 8$ or $x = -1$
	$x = -1$ is not valid as the domain is $x \geq 3$, so $x = 8$
3 (d)	The graph of f^{-1} is the reflection of the graph of f in the line $y = x$,
	so the curves meet on $y = x$, where $f(x) = x$. Hence $x = 8$ also solves
	$f(x) = f^{-1}(x)$.
4 (a)	Arithmetic sequence with $a = 16000$, $d = 17200 - 16000 = 1200$
	$u_{10} = 16000 + 9 \times 1200 = £26\,800$

Question number

4 (b)

$S_N = \frac{1}{2}N(2 \times 16000 + (N-1) \times 1200) = \frac{1}{2}N(32000 + 1200N - 1200)$

$S_N = 600N^2 + 15400N$

$600N^2 + 15400N > 500000 \quad \Rightarrow \quad 600N^2 + 15400N - 500000 > 0$

Roots: $N = \dfrac{-15400 \pm \sqrt{(15400^2 + 4 \times 600 \times 500000)}}{2 \times 600}$

$N = 18.758...$ (or $N = -44.42...$, rejected)

Check: $S_{18} = £471\,600 < £500\,000$, $S_{19} = £509\,200 > £500\,000$

So 19 complete years are needed

4 (c)

Not realistic in the long term: Sam is unlikely to stay in the same role for that long, and salaries are likely to level off (a pay cap) rather than keep rising by the same amount every year.

5 (a)

Differentiate $x^3 - 3x^2y + y^2 + 1 = 0$ implicitly with respect to x:

$3x^2 - 6xy - 3x^2\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} = 0$

$(2y - 3x^2)\dfrac{dy}{dx} = 6xy - 3x^2$

$\dfrac{dy}{dx} = \dfrac{6xy - 3x^2}{2y - 3x^2}$ as required

5 (b)

At (1, 2): $\dfrac{dy}{dx} = \dfrac{6(1)(2) - 3(1)^2}{2(2) - 3(1)^2} = \dfrac{9}{1} = 9$

Gradient of normal $= -\dfrac{1}{9}$

$y - 2 = -\dfrac{1}{9}(x - 1)$

$9y - 18 = -x + 1$

$x + 9y = 19$

Question number

6

$(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$f(x + h) - f(x) = 2(x + h)^3 + 3(x + h) - (2x^3 + 3x)$

$= 2x^3 + 6x^2h + 6xh^2 + 2h^3 + 3x + 3h - 2x^3 - 3x$

$= 6x^2h + 6xh^2 + 2h^3 + 3h$

$\frac{f(x + h) - f(x)}{h} = \frac{6x^2h + 6xh^2 + 2h^3 + 3h}{h} = 6x^2 + 6xh + 2h^2 + 3$

$f'(x) = \lim_{h \to 0} (6x^2 + 6xh + 2h^2 + 3)$

As $h \to 0$, $6xh \to 0$ and $2h^2 \to 0$, so $f'(x) = 6x^2 + 3$

7

$u_1 = 25 \times 0.6 = 15$, so geometric with $a = 15$, $r = 0.6$

$S_\infty = \frac{15}{1 - 0.6} = 37.5$

$S_N = \frac{15(1 - 0.6^N)}{1 - 0.6} = 37.5(1 - 0.6^N)$

$S_\infty - S_N < 10^{-4}$

$37.5 - 37.5(1 - 0.6^N) < 10^{-4}$

$37.5 \times 0.6^N < 10^{-4}$

$0.6^N < \frac{10^{-4}}{37.5} = 2.6667 \times 10^{-6}$

$N \ln 0.6 < \ln(2.6667 \times 10^{-6})$

$N > \frac{\ln(2.6667 \times 10^{-6})}{\ln 0.6}$ ($\ln 0.6 < 0$, so the inequality reverses)

$N > 25.125...$

Smallest $N = 26$

Question number

8 (a)

$\frac{dx}{dt} = k\sqrt{x}$

When $x = 0.64$, $dx/dt = -0.0032$: $-0.0032 = k\sqrt{0.64} = 0.8k$

$k = -0.004$

$\frac{dx}{dt} = -0.004\sqrt{x}$ as required

8 (b)

$\int x^{-\frac{1}{2}}\, dx = \int -0.004\, dt$

$2x^{\frac{1}{2}} = -0.004t + c$

$t = 100$, $x = 0.64$: $2\sqrt{0.64} = -0.4 + c \Rightarrow 1.6 = -0.4 + c \Rightarrow c = 2$

$2\sqrt{x} = 2 - 0.004t$

$\sqrt{x} = 1 - 0.002t$

$x = (1 - 0.002t)^2$

8 (c)

Tank empty when $x = 0$: $1 - 0.002t = 0$

$t = 500$, so the tank is empty after 500 seconds

9 (a)

$R\cos(3x - \alpha) = R\cos 3x \cos\alpha + R\sin 3x \sin\alpha$

$R\cos\alpha = 3$, $R\sin\alpha = 7$

$R^2 = 3^2 + 7^2 = 58 \Rightarrow R = \sqrt{58}$

$\tan\alpha = \frac{7}{3} \Rightarrow \alpha = 1.1659... = 1.17$ (3 s.f.)

$3\cos 3x + 7\sin 3x = \sqrt{58}\cos(3x - 1.17)$

Question number	
9 (b)	$y = \cos x \rightarrow y = \cos(x - 1.17)$:
	translation by 1.17 in the positive x-direction
	$y = \cos(x - 1.17) \rightarrow y = \cos(3x - 1.17)$:
	stretch parallel to the x-axis, scale factor $\frac{1}{3}$
	$y = \cos(3x - 1.17) \rightarrow y = \sqrt{58}\cos(3x - 1.17)$:
	stretch parallel to the y-axis, scale factor $\sqrt{58}$
9 (c)	Greatest value $= \sqrt{58}$, when $\cos(3x - 1.1659\ldots) = 1$
	$3x - 1.1659\ldots = 0 \Rightarrow x = 0.38863\ldots$
	Greatest value $\sqrt{58}$ when $x = 0.389$
9 (d)	Least value $= -\sqrt{58}$, when $\cos(3x - 1.1659\ldots) = -1$
	$3x - 1.1659\ldots = \pi \Rightarrow x = 1.43583\ldots$
	Least value $-\sqrt{58}$ when $x = 1.44$
10 (a)	Area of sector $= \frac{1}{2} \times 6^2 \times \theta = 18\theta$
	Area of triangle AOB $= \frac{1}{2} \times 6^2 \times \sin\theta = 18\sin\theta$
	Segment: $\frac{1}{2} \times 6^2 \times (\theta - \sin\theta) = 7.2$
	$\theta - \sin\theta = \frac{7.2}{18} = 0.4$
	$\theta = 0.4 + \sin\theta$ as required
10 (b)	$F'(\theta) = \cos\theta$, $F'(1.2) = \cos 1.2 = 0.3623\ldots$
	$\lvert F'(1.2)\rvert < 1$ $(-1 < 0.362 < 1)$, so the iteration will converge
10 (c)	$\theta_1 = 0.4 + \sin 1.2 = 1.33204$
	$\theta_2 = 1.37163$, $\theta_3 = 1.38023$, $\theta_4 = 1.38190$
	$\theta_5 = 1.38221$, $\theta_6 = 1.38227$, $\theta_7 = 1.38228$
	$\theta = 1.382$ (4 s.f.)

Question number

10 (d) Let $g(\theta) = \theta - 0.4 - \sin\theta$

$g(1.3815) = 1.3815 - 0.4 - \sin 1.3815 = -0.000637 < 0$

$g(1.3825) = 1.3825 - 0.4 - \sin 1.3825 = 0.000175 > 0$

Change of sign and g is continuous, so $1.3815 < \theta < 1.3825$

Hence $\theta = 1.382$ correct to 4 significant figures

11 (a) $\int 1 \times \ln(x-4)\,dx$: $u = \ln(x-4)$, $dv/dx = 1$

$du/dx = \frac{1}{x-4}$, $v = x$

$\int \ln(x-4)\,dx = x\ln|x-4| - \int \frac{x}{x-4}\,dx$

$\int \frac{x}{x-4}\,dx = \int \frac{(x-4)+4}{x-4}\,dx = \int 1 + \frac{4}{x-4}\,dx = x + 4\ln|x-4|$

$\int \ln(x-4)\,dx = x\ln|x-4| - x - 4\ln|x-4| + c$

$= (x-4)\ln|x-4| - x + c$ as required

11 (b) $x = 4$

11 (c) Curve crosses the x-axis at $x = 5$ (below for $4.5 < x < 5$, above for $x > 5$)

$\int_5^7 \ln(x-4)\,dx = [(x-4)\ln|x-4| - x]$ evaluated from 5 to 7

$= (3\ln 3 - 7) - (0 - 5) = 3\ln 3 - 2$

$\int_{4.5}^5 \ln(x-4)\,dx = (0-5) - (0.5\ln 0.5 - 4.5) = -0.5 - 0.5\ln 0.5$

$= -0.5 + 0.5\ln 2 = -0.153\ldots$ (below the axis)

Total area $= (3\ln 3 - 2) + (0.5 - 0.5\ln 2)$

$= 3\ln 3 - \frac{1}{2}\ln 2 - \frac{3}{2}$ $(a = 3, b = -\frac{1}{2}, c = -1.5)$

Question number

12 (a) Let $u = 3x^2$, so $y = a^u$

$du/dx = 6x$, $\frac{dy}{du} = a^u \ln a$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 6x\, a^u \ln a = 6xa^{3x^2} \ln a$ as required

12 (b) At $x = 1$: gradient $m = 6a^3 \ln a$

Tangent: $y - a^3 = 6a^3 \ln a\,(x - 1)$

Through $(\frac{1}{2}, 0)$: $0 - a^3 = 6a^3 \ln a\,(\frac{1}{2} - 1)$

$-a^3 = -3a^3 \ln a \Rightarrow a^3(3\ln a - 1) = 0$

$a^3 \neq 0$, so $\ln a = \frac{1}{3}$ $a = e^{1/3}$

12 (c) Product rule with $u = 6x \ln a$, $v = a^{3x^2}$:

$du/dx = 6\ln a$, $dv/dx = 6xa^{3x^2} \ln a$

$\frac{d^2y}{dx^2} = 6\ln a \times a^{3x^2} + 6x \ln a \times 6xa^{3x^2} \ln a$

$= 6\ln a \times a^{3x^2}(1 + 6x^2 \ln a)$

$a > 1$, so $\ln a > 0$; $a^{3x^2} > 0$ for all x;

$6x^2 \geq 0$ and $\ln a > 0$, so $1 + 6x^2 \ln a \geq 1 > 0$

Hence $\frac{d^2y}{dx^2} > 0$ for all x, so the curve is convex for all x