OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Wednesday 13 June 2018 – Morning

Time allowed: 2 hours

7009605404

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

© OCR 2018 [603/1038/8]
DC (SC/SW) 165432/3
OCR is an exempt Charity

stemup.co.uk

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \quad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \frac{u}{v}, \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Integration

$\int \frac{f'(x)}{f(x)}\mathrm{d}x = \ln|f(x)| + c$

$\int f'(x)(f(x))^n \mathrm{d}x = \frac{1}{n+1}(f(x))^{n+1} + c$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1 - \frac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

© OCR 2018

stemup.co.uk

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k+\frac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B|A) = \mathrm{P}(B)\mathrm{P}(A|B)$ **or** $\mathrm{P}(A|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \overline{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u+v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

© OCR 2018

stemup.co.uk

Section A: Pure Mathematics

Answer **all** the questions.

1 **(i)** Express $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$. **[4]**

(ii) Use your result to show that the equation $2x^2 - 12x + 23 = 0$ has no real roots. **[1]**

(iii) Given that the equation $2x^2 - 12x + k = 0$ has repeated roots, find the value of the constant k. **[2]**

2 The points A and B have position vectors $\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$ respectively.

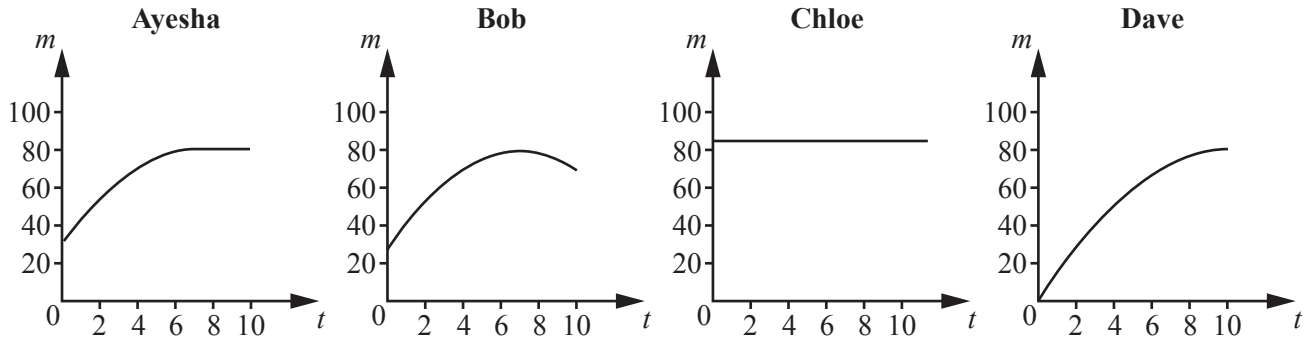
(i) Find the exact length of AB. **[2]**

(ii) Find the position vector of the midpoint of AB. **[1]**

The points P and Q have position vectors $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ respectively.

(iii) Show that $ABPQ$ is a parallelogram. **[3]**

3 Ayesha, Bob, Chloe and Dave are discussing the relationship between the time, t hours, they might spend revising for an examination, and the mark, m, they would expect to gain. Each of them draws a graph to model this relationship for himself or herself.



(i) Assuming Ayesha's model is correct, how long would you recommend that she spends revising? **[1]**

(ii) State one feature of Dave's model that is likely to be unrealistic. **[1]**

(iii) Suggest a reason for the shape of Bob's graph as compared with Ayesha's graph. **[1]**

(iv) What does Chloe's model suggest about her attitude to revision? **[1]**

4 Prove that $\sin^2(\theta + 45)° - \cos^2(\theta + 45)° \equiv \sin 2\theta°$. **[4]**

© OCR 2018

stemup.co.uk

5 Charlie claims to have proved the following statement.

"The sum of a square number and a prime number cannot be a square number."

(i) Give an example to show that Charlie's statement is not true. **[1]**

Charlie's attempt at a proof is below.

Assume that the statement is not true.
⇒ There exist integers n and m and a prime p such that $n^2 + p = m^2$.
⇒ $p = m^2 - n^2$
⇒ $p = (m - n)(m + n)$
⇒ p is the product of two integers.
⇒ p is not prime, which is a contradiction.
⇒ Charlie's statement is true.

(ii) Explain the error that Charlie has made. **[1]**

(iii) Given that 853 is a prime number, find the square number S such that $S + 853$ is also a square number. **[4]**

6 **In this question you must show detailed reasoning.**

A curve has equation $y = \frac{\ln x}{x}$.

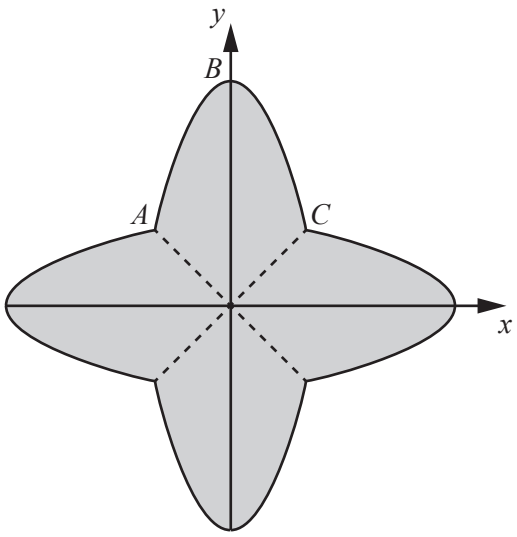
(i) Find the x-coordinate of the point where the curve crosses the x axis. **[2]**

(ii) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x-axis. **[2]**

(iii) Find the coordinates of the turning point on the curve. **[4]**

(iv) Determine whether this turning point is a maximum or a minimum. **[5]**

7 The diagram shows a part ABC of the curve $y = 3 - 2x^2$, together with its reflections in the lines $y = x$, $y = -x$ and $y = 0$.



Find the area of the shaded region. **[7]**

stemup.co.uk

Section B: Statistics

Answer **all** the questions.

8 **(i)** The variable X has the distribution N(20, 9).

(a) Find $P(X > 25)$. **[1]**

(b) Given that $P(X > a) = 0.2$, find a. **[1]**

(c) Find b such that $P(20 - b < X < 20 + b) = 0.5$. **[3]**

(ii) The variable Y has the distribution $N(\mu, \frac{\mu^2}{9})$. Find $P(Y > 1.5\mu)$. **[3]**

9 Briony suspects that a particular 6-sided dice is biased in favour of 2. She plans to throw the dice 35 times and note the number of times that it shows a 2. She will then carry out a test at the 4% significance level. Find the rejection region for the test. **[7]**

10 A certain forest contains only trees of a particular species. Dipak wished to take a random sample of 5 trees from the forest. He numbered the trees from 1 to 784. Then, using his calculator, he generated the random digits 14 781 049. Using these digits, Dipak formed 5 three-digit numbers. He took the first, second and third digits, followed by the second, third and fourth digits and so on. In this way he obtained the following list of numbers for his sample.

147 478 781 104 49

(i) Explain why Dipak omitted the number 810 from his list. **[1]**

(ii) Explain why Dipak's sample is not random. **[1]**

The mean height of all trees of this species is known to be 4.2 m. Dipak wishes to test whether the mean height of trees in the forest is less than 4.2 m. He now uses a correct method to choose a random sample of 50 trees and finds that their mean height is 4.0 m. It is given that the standard deviation of trees in the forest is 0.8 m.

(iii) Carry out the test at the 2% significance level. **[7]**

© OCR 2018

stemup.co.uk

11 Christa used Pearson's product-moment correlation coefficient, r, to compare the use of public transport with the use of private vehicles for travel to work in the UK.

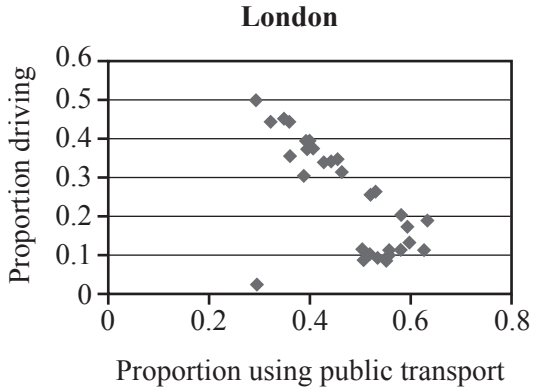
(i) Using the pre-release data set for all 348 UK Local Authorities, she considered the following four variables.

Number of employees using public transport	x
Number of employees using private vehicles	y
Proportion of employees using public transport	a
Proportion of employees using private vehicles	b

(a) Explain, in context, why you would expect strong, positive correlation between x and y. **[1]**

(b) Explain, in context, what kind of correlation you would expect between a and b. **[2]**

(ii) Christa also considered the data for the 33 London boroughs alone and she generated the following scatter diagram.



One London Borough is represented by an outlier in the diagram.

(a) Suggest what effect this outlier is likely to have on the value of r for the 32 London Boroughs. **[1]**

(b) Suggest what effect this outlier is likely to have on the value of r for the whole country. **[1]**

(c) What can you deduce about the area of the London Borough represented by the outlier? Explain your answer. **[1]**

stemup.co.uk

12 The discrete random variable X takes values 1, 2, 3, 4 and 5, and its probability distribution is defined as follows.

$$\mathrm{P}(X = x) = \begin{cases} a & x = 1, \\ \frac{1}{2}\mathrm{P}(X = x - 1) & x = 2,3,4,5, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a = \frac{16}{31}$. **[2]**

The discrete probability distribution for X is given in the table.

x	1	2	3	4	5
$\mathrm{P}(X = x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

(ii) Find the probability that X is odd. **[1]**

Two independent values of X are chosen, and their sum S is found.

(iii) Find the probability that S is odd. **[2]**

(iv) Find the probability that S is greater than 8, given that S is odd. **[3]**

Sheila sometimes needs several attempts to start her car in the morning. She models the number of attempts she needs by the discrete random variable Y defined as follows.

$$\mathrm{P}(Y = y + 1) = \frac{1}{2}\mathrm{P}(Y = y) \quad \text{for all positive integers } y.$$

(v) Find $\mathrm{P}(Y = 1)$. **[2]**

(vi) Give a reason why one of the variables, X or Y, might be more appropriate as a model for the number of attempts that Sheila needs to start her car. **[1]**

© OCR 2018

stemup.co.uk

13 In this question you must show detailed reasoning.

The probability that Paul's train to work is late on any day is 0.15, independently of other days.

(i) The number of days on which Paul's train to work is late during a 450-day period is denoted by the random variable Y. Find a value of a such that $\mathrm{P}(Y > a) \approx \frac{1}{6}$. **[3]**

In the expansion of $(0.15 + 0.85)^{50}$, the terms involving 0.15^r and 0.15^{r+1} are denoted by T_r and T_{r+1} respectively.

(ii) Show that $\frac{T_r}{T_{r+1}} = \frac{17(r+1)}{3(50-r)}$. **[3]**

(iii) The number of days on which Paul's train to work is late during a 50-day period is modelled by the random variable X.

(a) Find the values of r for which $\mathrm{P}(X = r) \leqslant \mathrm{P}(X = r+1)$. **[4]**

(b) Hence find the most likely number of days on which the train will be late during a 50-day period. **[2]**

END OF QUESTION PAPER

stemup.co.uk

BLANK PAGE

© OCR 2018

stemup.co.uk

BLANK PAGE

© OCR 2018

stemup.co.uk

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

stemup.co.uk

Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (i)

$2x^2 - 12x + 23 = 2(x^2 - 6x) + 23$

$= 2[(x - 3)^2 - 9] + 23$

$= 2(x - 3)^2 - 18 + 23$

$= 2(x - 3)^2 + 5$ $\quad (a = 2,\ b = -3,\ c = 5)$

1 (ii)

$(x - 3)^2 \geq 0$ for all x, so $2(x - 3)^2 + 5 \geq 5 > 0$ for all x.

So $2x^2 - 12x + 23$ is never 0: the equation has no real roots.

1 (iii)

$2x^2 - 12x + k = 2(x - 3)^2 + k - 18$

Repeated roots $\Rightarrow$ the minimum value is $0 \Rightarrow k - 18 = 0$

$k = 18$

2 (i)

$\overrightarrow{AB} = b - a = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ -3 \end{bmatrix}$

$|AB| = \sqrt{((-4)^2 + 1^2 + (-3)^2)} = \sqrt{(16 + 1 + 9)} = \sqrt{26}$

2 (ii)

Midpoint $= \frac{1}{2}(a + b) = \frac{1}{2}\begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix} = \begin{bmatrix} -1 \\ -1.5 \\ 3.5 \end{bmatrix}$

2 (iii)

$\overrightarrow{BA} = a - b = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} - \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$

Question number	
2 cont.	$\overrightarrow{PQ} = q - p = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$
	$\overrightarrow{BA} = \overrightarrow{PQ}$, so BA is parallel to PQ and $\lvert BA \rvert = \lvert PQ \rvert$.
	One pair of opposite sides is parallel and equal in length,
	hence ABPQ is a parallelogram.
3 (i)	About 7 hours (her mark stops increasing after about 7 hours).
3 (ii)	Dave would get 0 marks if he did no revision at all ($m = 0$ when
	$t = 0$), which is unrealistic.
3 (iii)	Bob believes that too much revision makes him tired or stressed
	(less sleep), so his mark goes down after about 7 hours, whereas
	Ayesha believes extra revision just stops helping.
3 (iv)	Chloe thinks that however much (or little) she revises, it will make
	no difference to her mark: revision will not increase her mark.
4	$\text{LHS} = \sin^2(\theta + 45)^\circ - \cos^2(\theta + 45)^\circ$
	$= -[\cos^2(\theta + 45)^\circ - \sin^2(\theta + 45)^\circ]$
	$= -\cos(2\theta + 90)^\circ$ (using $\cos 2A \equiv \cos^2 A - \sin^2 A$)
	$= -[\cos 2\theta^\circ \cos 90^\circ - \sin 2\theta^\circ \sin 90^\circ]$
	$= -[\cos 2\theta^\circ \times 0 - \sin 2\theta^\circ \times 1]$
	$= \sin 2\theta^\circ = \text{RHS}$ $\therefore \sin^2(\theta + 45)^\circ - \cos^2(\theta + 45)^\circ \equiv \sin 2\theta^\circ$
5 (i)	$1 + 3 = 4$: 1 is a square number, 3 is prime and $4 = 2^2$ is square.

Question number

5 (ii) If $m - n = 1$, then $p = 1 \times (m + n)$, and p could be prime: being the product of two integers does not mean p is not prime if one factor is 1.

5 (iii) Let $S = n^2$ and $S + 853 = m^2$, so $853 = m^2 - n^2 = (m - n)(m + n)$

853 is prime, so $m - n = 1$ and $m + n = 853$

i.e. the other square number is $(n + 1)^2$:

$853 = (n + 1)^2 - n^2 = 2n + 1 \Rightarrow n = 426$

$S = 426^2 = 181476$

6 (i) $\frac{\ln x}{x} = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$

6 (ii) $x = 2$: $y = \frac{\ln 2}{2}$ $x = 4$: $y = \frac{\ln 4}{4} = \frac{2 \ln 2}{4} = \frac{\ln 2}{2}$

A and B have the same y-coordinate, so AB is parallel to the x-axis.

6 (iii) $\frac{dy}{dx} = \frac{x \times (1/x) - \ln x \times 1}{x^2} = \frac{1 - \ln x}{x^2}$

Turning point: $\frac{dy}{dx} = 0 \Rightarrow 1 - \ln x = 0 \Rightarrow \ln x = 1 \Rightarrow x = e$

$y = \frac{\ln e}{e} = \frac{1}{e}$ Turning point is $(e, \frac{1}{e})$

6 (iv) $\frac{d^2y}{dx^2} = \frac{x^2 \times (-1/x) - (1 - \ln x) \times 2x}{x^4} = \frac{-3x + 2x \ln x}{x^4} = \frac{2 \ln x - 3}{x^3}$

At $x = e$: $\frac{d^2y}{dx^2} = \frac{2 \ln e - 3}{e^3} = -\frac{1}{e^3} = -0.04978...$

$\frac{d^2y}{dx^2} < 0$ at $x = e$, so the turning point is a maximum.

Question number	
7	C is where $y = 3 - 2x^2$ meets $y = x$: $3 - 2x^2 = x$
	$2x^2 + x - 3 = 0 \Rightarrow (2x + 3)(x - 1) = 0 \Rightarrow x = 1$ $(C = (1, 1))$
	$\int_0^1 (3 - 2x^2)\,dx = \left[3x - \frac{2x^3}{3}\right]_0^1 = 3 - \frac{2}{3} = \frac{7}{3}$
	Area between the curve and $y = 1$ for $0 \leqslant x \leqslant 1$: $\frac{7}{3} - 1 \times 1 = \frac{4}{3}$
	By symmetry the shaded region is the square with vertices
	$(\pm 1, \pm 1)$, of area $2 \times 2 = 4$, plus 8 of these pieces.
	Shaded area $= 8 \times \frac{4}{3} + 4 = \frac{32}{3} + \frac{12}{3} = \frac{44}{3}$
8 (i)(a)	$X \sim N(20, 3^2)$: $P(X > 25) = 0.04779\ldots = 0.0478$
8 (i)(b)	$P(X > a) = 0.2 \Rightarrow a = 22.524\ldots = 22.5$
8 (i)(c)	By symmetry, $P(X < 20 + b) = 0.5 + 0.25 = 0.75$
	$20 + b = 22.0234\ldots$ $(= 20 + 0.6745 \times 3)$
	$b = 2.0234\ldots = 2.02$
8 (ii)	$\sigma = \frac{\mu}{3}$ $P(Y > 1.5\mu) = P\left(Z > \frac{1.5\mu - \mu}{\mu/3}\right) = P\left(Z > \frac{3}{2}\right)$
	$= P(Z > 1.5) = 0.06680\ldots = 0.0668$
9	Let $p = P$(dice shows 2 on one throw). H_0: $p = \frac{1}{6}$ H_1: $p > \frac{1}{6}$
	Let X = number of 2s in 35 throws. Under H_0, $X \sim B\left(35, \frac{1}{6}\right)$
	$P(X \geqslant 10) = 1 - P(X \leqslant 9) = 1 - 0.9450 = 0.0550 > 0.04$
	$P(X \geqslant 11) = 1 - P(X \leqslant 10) = 1 - 0.9768 = 0.0232 < 0.04$

Question number	
9 cont.	0.04 lies between these, so the rejection region is $X \geq 11$
10 (i)	There are only 784 trees and 810 > 784, so there is no tree 810.
10 (ii)	Each number is not independent of the previous one: consecutive numbers share two digits (the 2nd digit of each is the 1st of the next).
10 (iii)	H_0: $\mu = 4.2$ H_1: $\mu < 4.2$, where μ is the mean height (m) of all trees in the forest.
	Under H_0: $\bar{X} \sim N(4.2, \frac{0.8^2}{50})$
	$P(\bar{X} < 4.0) = 0.03854... = 0.0385$
	0.0385 > 0.02, so do not reject H_0.
	There is insufficient evidence at the 2% level to suggest that the mean height of trees in the forest is less than 4.2 m.
11 (i)(a)	Both x and y depend on the number of employees (the population) of each Local Authority: LAs with large populations have large numbers of people using public transport and large numbers using vehicles.
11 (i)(b)	Negative correlation: if a larger proportion of employees use public transport, then a smaller proportion use private vehicles (and vice versa).
11 (ii)(a)	It will make r less negative (closer to 0), i.e. decrease the size of r.
11 (ii)(b)	Little effect: the population of this borough is very small compared with the population of the whole country (1 of 348 LAs).

Question number	
11 (ii)(c)	Very few people there drive or use public transport, so most must walk or cycle: people live close to their jobs (do not travel far to work).
12 (i)	$P(X = 1) = a$, $P(X = 2) = \frac{a}{2}$, $P(X = 3) = \frac{a}{4}$, $P(X = 4) = \frac{a}{8}$, $P(X = 5) = \frac{a}{16}$ $a(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}) = 1 \Rightarrow \frac{31}{16}a = 1 \Rightarrow a = \frac{16}{31}$
12 (ii)	$P(X \text{ odd}) = P(X = 1, 3 \text{ or } 5) = \frac{16 + 4 + 1}{31} = \frac{21}{31}$
12 (iii)	S is odd when one value is odd and the other is even: $P(S \text{ odd}) = 2 \times \frac{21}{31} \times \frac{10}{31} = \frac{420}{961}$ $(= 0.437)$
12 (iv)	$S > 8$ and odd $\Rightarrow$ $S = 9$: (4, 5) or (5, 4) $P(S > 8 \text{ and } S \text{ odd}) = \frac{2}{31} \times \frac{1}{31} + \frac{1}{31} \times \frac{2}{31} = \frac{4}{961}$ $P(S > 8 \mid S \text{ odd}) = \frac{P(S > 8 \text{ and } S \text{ odd})}{P(S \text{ odd})} = \frac{4}{961} \div \frac{420}{961} = \frac{1}{105}$ $(= 0.00952)$
12 (v)	Let $P(Y = 1) = p$. Probabilities form a geometric series, $r = \frac{1}{2}$: $S_\infty = \frac{p}{1 - 0.5} = 1 \Rightarrow 2p = 1 \Rightarrow P(Y = 1) = 0.5$
12 (vi)	Y is more appropriate: X cannot be more than 5, but Sheila might need more than 5 attempts; Y allows any number of attempts.

Question number

13 (i)

$Y \sim B(450, 0.15)$: $np = 67.5$ and $np(1-p) = 57.375$ are large,

so $Y \approx N(67.5, 57.375)$, $\sigma = \sqrt{57.375} = 7.5746...$ $\quad P(Y < a) \approx \frac{5}{6}$

$$\frac{a - 67.5}{7.5746...} = \Phi^{-1}\left(\frac{5}{6}\right) = 0.9674...$$

$$a = 67.5 + 0.9674... \times 7.5746... = 74.83...$$

$$a = 75$$

13 (ii)

$$T_r = \frac{50!}{r!(50-r)!}\, 0.15^r\, 0.85^{50-r}, \quad T_{r+1} = \frac{50!}{(r+1)!(49-r)!}\, 0.15^{r+1}\, 0.85^{49-r}$$

$$\frac{T_r}{T_{r+1}} = \frac{(r+1)!(49-r)!}{r!(50-r)!} \times \frac{0.15^r\, 0.85^{50-r}}{0.15^{r+1}\, 0.85^{49-r}} = \frac{r+1}{50-r} \times \frac{0.85}{0.15}$$

$$= \frac{r+1}{50-r} \times \frac{17}{3} = \frac{17(r+1)}{3(50-r)} \quad \text{as required}$$

13 (iii)(a)

$X \sim B(50, 0.15)$, so $P(X = r) = T_r$

$$P(X = r) \leqslant P(X = r+1) \iff \frac{T_r}{T_{r+1}} \leqslant 1 \iff \frac{17(r+1)}{3(50-r)} \leqslant 1$$

$17r + 17 \leqslant 150 - 3r \qquad (50 - r > 0)$

$20r \leqslant 133 \implies r \leqslant 6.65$

r is an integer, so $r \leqslant 6 \quad (r = 0, 1, 2, ..., 6)$

13 (iii)(b)

From (a), $P(X = r) \leqslant P(X = r+1)$ for $r \leqslant 6$ and $P(X = r) > P(X = r+1)$ for $r \geqslant 7$, so the most likely value is 6 or 7. Check:

$$\frac{P(X=6)}{P(X=7)} = \frac{17(6+1)}{3(50-6)} = \frac{119}{132} = 0.902 < 1, \text{ so } P(X=6) < P(X=7)$$

Most likely number of late days is 7.