



Oxford Cambridge and RSA

# Monday 18 October 2021 – Afternoon

## A Level Mathematics B (MEI)

### H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours



**You must have:**

- the Printed Answer Booklet
- the Insert
- a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

#### INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

#### ADVICE

- Read each question carefully before you start your answer.

## Formulae A Level Mathematics B (MEI) (H640)

### Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

### Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

### Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

### Differentiation

| $f(x)$                   | $f'(x)$                          |
|--------------------------|----------------------------------|
| $\tan kx$                | $k \sec^2 kx$                    |
| $\sec x$                 | $\sec x \tan x$                  |
| $\cot x$                 | $-\operatorname{cosec}^2 x$      |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

$$\text{Quotient Rule } y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

### Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

### Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Sample variance**

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation,  $s = \sqrt{\text{variance}}$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = r) = {}^nC_r p^r q^{n-r}$  where  $q = 1 - p$

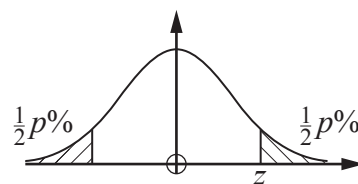
Mean of  $X$  is  $np$

**Hypothesis testing for the mean of a Normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the Normal distribution**

|     |       |       |       |       |
|-----|-------|-------|-------|-------|
| $p$ | 10    | 5     | 2     | 1     |
| $z$ | 1.645 | 1.960 | 2.326 | 2.576 |

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

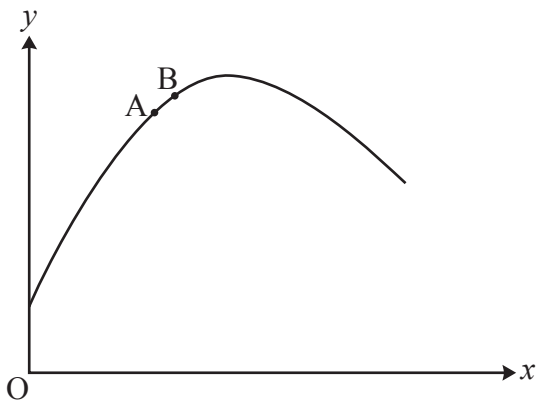
$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

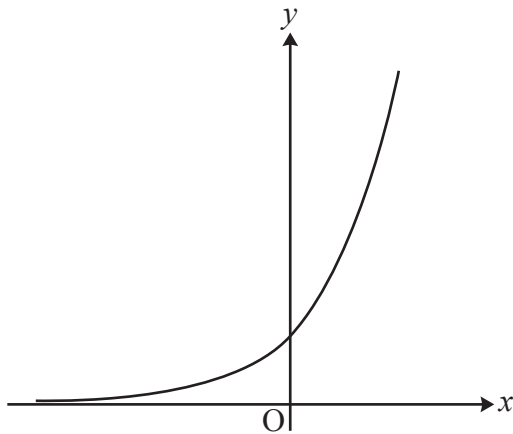
**Section A** (60 marks)

- 1 (a) Express  $x^2 + 8x + 2$  in the form  $(x + a)^2 + b$ . [2]
- (b) Write down the coordinates of the turning point of the curve  $y = x^2 + 8x + 2$ . [1]
- (c) State the transformation(s) which map(s) the curve  $y = x^2$  onto the curve  $y = x^2 + 8x + 2$ . [2]
- 2 Solve the equation  $\sin 2x = 0.3$  for  $0^\circ \leq x \leq 180^\circ$ . Give your answer(s) correct to **1** decimal place. [2]
- 3 (a) Determine, in terms of  $k$ , the coordinates of the point where the lines with the following equations intersect. [3]
- $$\begin{aligned} x + y &= k \\ 2x - y &= 1 \end{aligned}$$
- (b) Determine, in terms of  $k$ , the coordinates of the points where the line  $x + y = k$  crosses the curve  $y = x^2 + k$ . [4]
- 4 The diagram shows points A and B on the curve  $y = \left(\frac{x}{4}\right)^{-x}$ . [2]
- The  $x$ -coordinate of A is 1 and the  $x$ -coordinate of B is 1.1.



- (a) Find the gradient of chord AB. Give your answer correct to **2** decimal places. [2]
- (b) Give the  $x$ -coordinate of a point C on the curve such that the gradient of chord AC is a better approximation to the gradient of the tangent to the curve at A. [1]

- 5 (a) The diagram shows the curve  $y = e^x$ .



On the axes in the Printed Answer Booklet, sketch graphs of

(i)  $\frac{dy}{dx}$  against  $x$ , [1]

(ii)  $\frac{dy}{dx}$  against  $y$ . [2]

- (b) Wolves were introduced to Yellowstone National Park in 1995.

The population of wolves,  $y$ , is modelled by the equation

$$y = Ae^{kt},$$

where  $A$  and  $k$  are constants and  $t$  is the number of years after 1995.

- (i) Give a reason why this model might be suitable for the population of wolves. [1]

- (ii) When  $t = 0$ ,  $y = 21$  and when  $t = 1$ ,  $y = 51$ .

Find values of  $A$  and  $k$  consistent with the data. [3]

- (iii) Give a reason why the model will not be a good predictor of wolf populations many years after 1995. [1]

**6 In this question you must show detailed reasoning.**

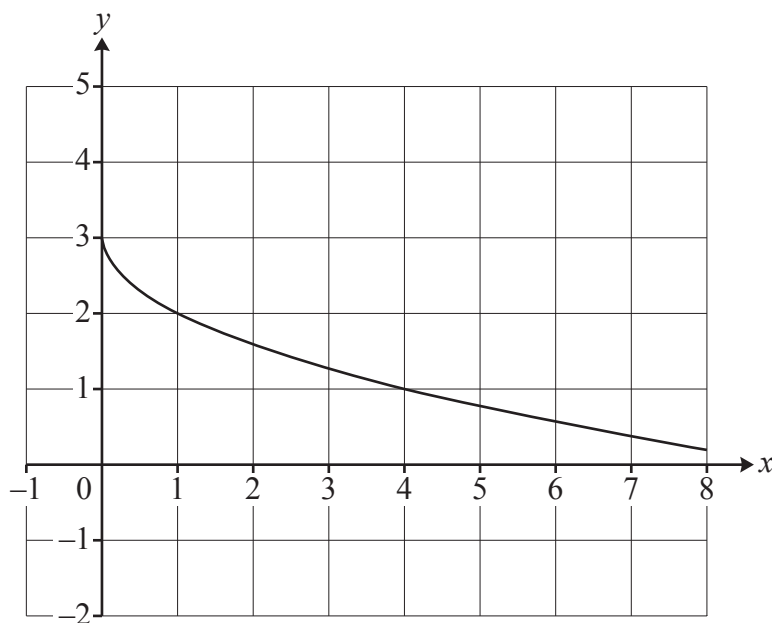
Show that  $\sum_{r=1}^3 \frac{1}{\sqrt{r+1} + \sqrt{r}} = 1.$  [4]

**7** Determine  $\int x \cos 2x \, dx.$  [3]

**8** For a particular value of  $a$ , the curve  $y = \frac{a}{x^2}$  passes through the point  $(3, 1).$

Find the coordinates of all the other points on the curve where both the  $x$ -coordinate and the  $y$ -coordinate are integers. [3]

**9** The diagram shows the curve  $y = 3 - \sqrt{x}.$



**(a)** Draw the line  $y = 5x - 1$  on the copy of the diagram in the Printed Answer Booklet. [1]

**(b) In this question you must show detailed reasoning.**

Determine the exact area of the region bounded by the curve  $y = 3 - \sqrt{x}$ , the lines  $y = 5x - 1$  and  $x = 4$  and the  $x$ -axis. [10]

10 (a) Express  $\frac{1}{(4x+1)(x+1)}$  in partial fractions. [3]

(b) A curve passes through the point (0, 2) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{y}{(4x+1)(x+1)},$$

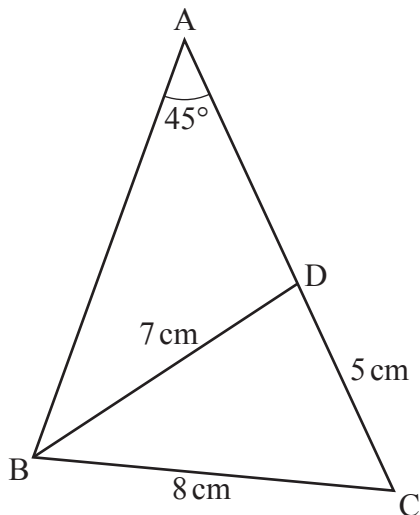
for  $x > -\frac{1}{4}$ .

Show by integration that  $y = A\left(\frac{4x+1}{x+1}\right)^B$  where  $A$  and  $B$  are constants to be determined. [6]

11 In this question you must show detailed reasoning.

The diagram shows triangle ABC, with  $BC = 8$  cm and angle  $BAC = 45^\circ$ .

The point D on AC is such that  $DC = 5$  cm and  $BD = 7$  cm.



Determine the exact length of AB. [5]

Answer **all** the questions.

**Section B** (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

12 Show that  $\beta = \arctan\left(\frac{1}{3}\right)$ , as given in line 15. [3]

13 (a) Use triangle ABE in **Fig. C2** to show that  $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$ , as given in line 29. [1]

(b) Sketch the graph of  $y = \arctan x$ . [1]

(c) What property of the arctan function ensures that  $y > \frac{1}{x} \Rightarrow \arctan y > \arctan\left(\frac{1}{x}\right)$ , as given in line 30? [1]

14 (a) Show that

$$\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right) \Rightarrow \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan 1. \quad [1]$$

(b) Use the arctan addition formula in line 23 to show that

$$\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right), \text{ as given in line 39.} \quad [4]$$

15 Prove that  $\arctan 1 + \arctan 2 + \arctan 3 = \pi$ , as given in line 41. [4]

**END OF QUESTION PAPER**

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