OCR
Oxford Cambridge and RSA

Monday 18 October 2021 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours

8980673136

You must have:
- the Printed Answer Booklet
- the Insert
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h\to 0}\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta,\ \cos\theta \approx 1 - \frac{1}{2}\theta^2,\ \tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

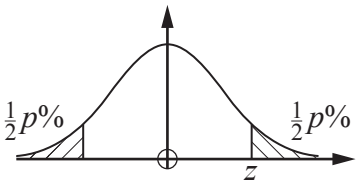
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Answer **all** the questions.

Section A (60 marks)

1 **(a)** Express x^2+8x+2 in the form $(x+a)^2+b$. **[2]**

(b) Write down the coordinates of the turning point of the curve $y=x^2+8x+2$. **[1]**

(c) State the transformation(s) which map(s) the curve $y=x^2$ onto the curve $y=x^2+8x+2$. **[2]**

2 Solve the equation $\sin 2x = 0.3$ for $0° \leqslant x \leqslant 180°$. Give your answer(s) correct to **1** decimal place. **[2]**

3 **(a)** Determine, in terms of k, the coordinates of the point where the lines with the following equations intersect.

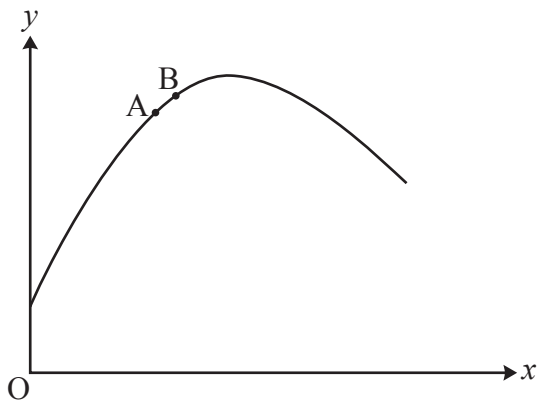
$$x+y=k$$
$$2x-y=1$$

[3]

(b) Determine, in terms of k, the coordinates of the points where the line $x+y=k$ crosses the curve $y=x^2+k$. **[4]**

4 The diagram shows points A and B on the curve $y=\left(\frac{x}{4}\right)^{-x}$.

The x-coordinate of A is 1 and the x-coordinate of B is 1.1.



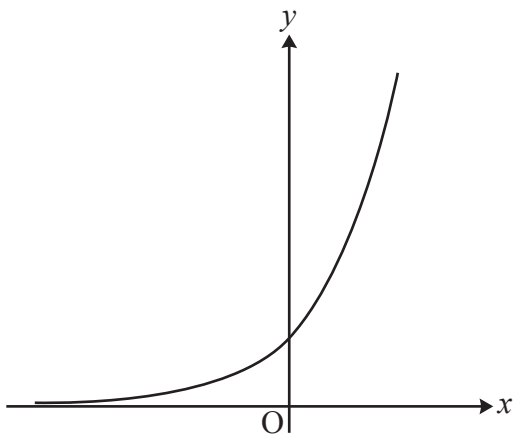
(a) Find the gradient of chord AB. Give your answer correct to **2** decimal places. **[2]**

(b) Give the x-coordinate of a point C on the curve such that the gradient of chord AC is a better approximation to the gradient of the tangent to the curve at A. **[1]**

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5 **(a)** The diagram shows the curve $y = e^x$.



On the axes in the Printed Answer Booklet, sketch graphs of

(i) $\frac{dy}{dx}$ against x, **[1]**

(ii) $\frac{dy}{dx}$ against y. **[2]**

(b) Wolves were introduced to Yellowstone National Park in 1995.

The population of wolves, y, is modelled by the equation

$y = Ae^{kt}$,

where A and k are constants and t is the number of years after 1995.

(i) Give a reason why this model might be suitable for the population of wolves. **[1]**

(ii) When $t = 0$, $y = 21$ and when $t = 1$, $y = 51$.

Find values of A and k consistent with the data. **[3]**

(iii) Give a reason why the model will not be a good predictor of wolf populations many years after 1995. **[1]**

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6 **In this question you must show detailed reasoning.**

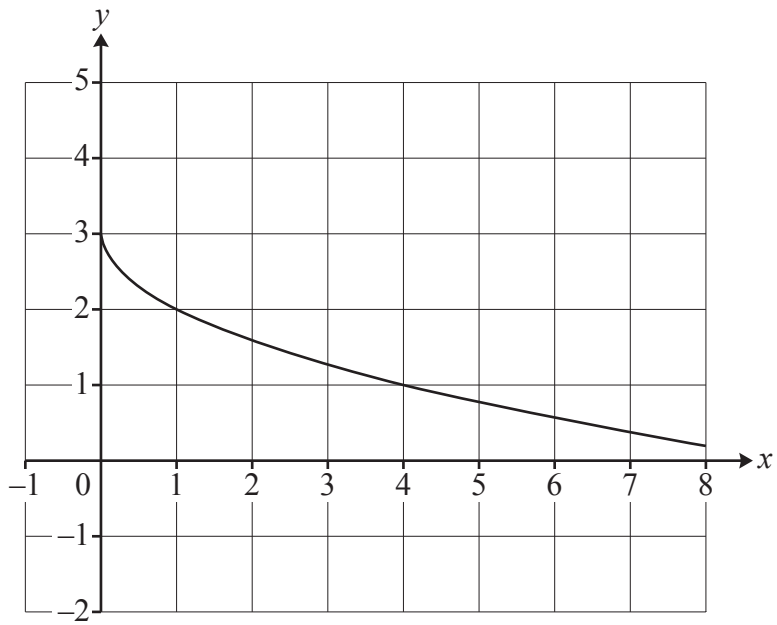
Show that $\sum_{r=1}^{3} \frac{1}{\sqrt{r+1}+\sqrt{r}} = 1$. **[4]**

7 Determine $\int x \cos 2x \, dx$. **[3]**

8 For a particular value of a, the curve $y = \frac{a}{x^2}$ passes through the point (3, 1).

Find the coordinates of all the other points on the curve where both the x-coordinate and the y-coordinate are integers. **[3]**

9 The diagram shows the curve $y = 3 - \sqrt{x}$.



(a) Draw the line $y = 5x - 1$ on the copy of the diagram in the Printed Answer Booklet. **[1]**

(b) **In this question you must show detailed reasoning.**

Determine the exact area of the region bounded by the curve $y = 3 - \sqrt{x}$, the lines $y = 5x - 1$ and $x = 4$ and the x-axis. **[10]**

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10 **(a)** Express $\frac{1}{(4x+1)(x+1)}$ in partial fractions. **[3]**

(b) A curve passes through the point (0, 2) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{y}{(4x+1)(x+1)},$$

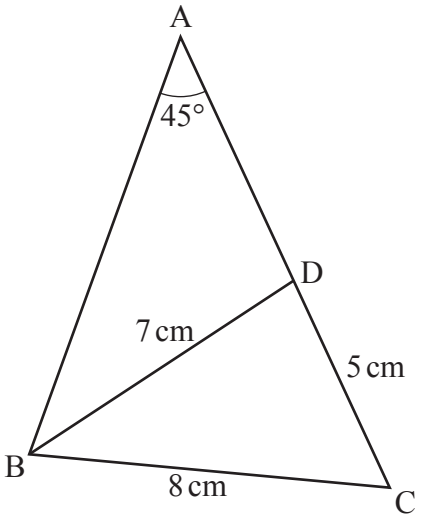
for $x > -\frac{1}{4}$.

Show by integration that $y = A\left(\frac{4x+1}{x+1}\right)^B$ where A and B are constants to be determined. **[6]**

11 **In this question you must show detailed reasoning.**

The diagram shows triangle ABC, with BC = 8 cm and angle BAC = 45°.

The point D on AC is such that DC = 5 cm and BD = 7 cm.



Determine the exact length of AB. **[5]**

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Answer **all** the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

12 Show that $\beta = \arctan\left(\frac{1}{3}\right)$, as given in line 15. **[3]**

13 **(a)** Use triangle ABE in **Fig. C2** to show that $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$, as given in line 29. **[1]**

(b) Sketch the graph of $y = \arctan x$. **[1]**

(c) What property of the arctan function ensures that $y > \frac{1}{x} \Rightarrow \arctan y > \arctan\left(\frac{1}{x}\right)$, as given in line 30? **[1]**

14 **(a)** Show that

$$\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right) \Rightarrow \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan 1. \quad \textbf{[1]}$$

(b) Use the arctan addition formula in line 23 to show that

$$\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right), \text{ as given in line 39.} \quad \textbf{[4]}$$

15 Prove that $\arctan 1 + \arctan 2 + \arctan 3 = \pi$, as given in line 41. **[4]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$x^2 + 8x + 2 = (x + 4)^2 - 16 + 2$

$= (x + 4)^2 - 14$, so $a = 4$, $b = -14$

1 (b)

Turning point $(-4, -14)$

1 (c)

Translation by vector $\begin{pmatrix} -4 \\ -14 \end{pmatrix}$

2

$0° \leq x \leq 180° \Rightarrow 0° \leq 2x \leq 360°$

$\sin 2x = 0.3 \Rightarrow 2x = 17.4576...° \text{ or } 2x = 180° - 17.4576...° = 162.5423...°$

$x = 8.7288...° \text{ or } x = 81.2711...°$

$x = 8.7°, \quad x = 81.3°$

3 (a)

Adding the equations: $3x = k + 1 \Rightarrow x = \frac{k + 1}{3}$

$y = k - x = k - \frac{k + 1}{3} = \frac{2k - 1}{3}$

Lines intersect at $\left(\frac{k + 1}{3}, \frac{2k - 1}{3}\right)$

3 (b)

$y = k - x$, so $k - x = x^2 + k$

$x^2 + x = 0 \Rightarrow x(x + 1) = 0 \Rightarrow x = 0 \text{ or } x = -1$

$x = 0$: $y = k - 0 = k$; $x = -1$: $y = k - (-1) = k + 1$

Points are $(0, k)$ and $(-1, k + 1)$

4 (a)

A: $x = 1$, $y = \left(\frac{1}{4}\right)^{-1} = 4$

B: $x = 1.1$, $y = \left(\frac{1.1}{4}\right)^{-1.1} = 4.13746...$

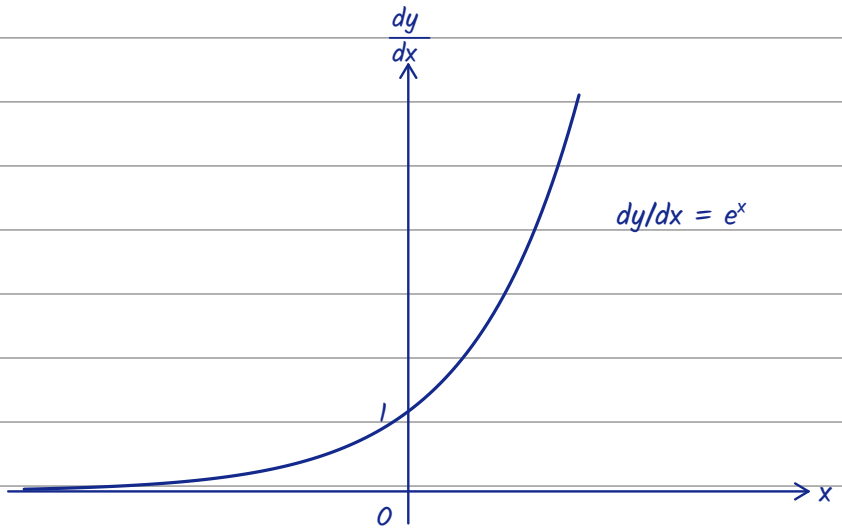
Gradient AB $= \frac{4.13746... - 4}{1.1 - 1} = 1.3746...$

Gradient of chord AB $= 1.37$ (2 d.p.)

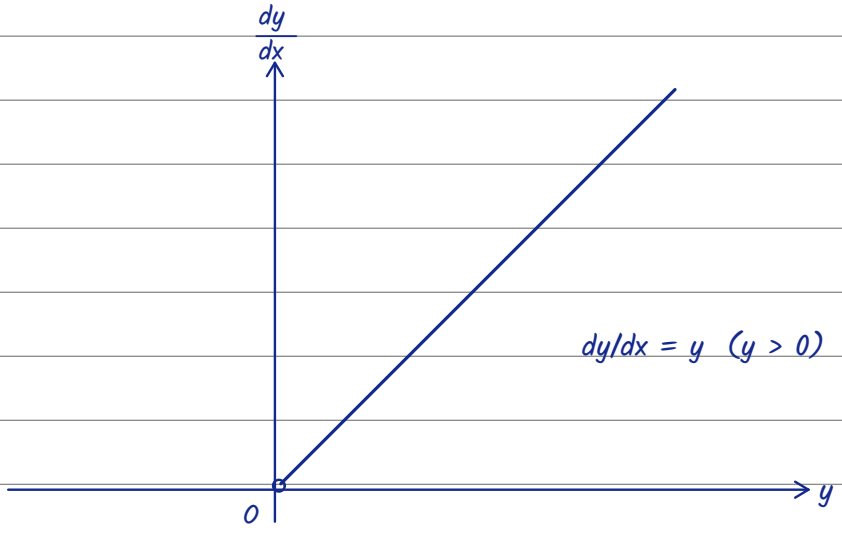
Question number

4 (b) $x = 1.01$ (any value between 1 and 1.1, closer to 1 than B is)

5 (a)(i) $y = e^x \Rightarrow \frac{dy}{dx} = e^x$



5 (a)(ii) $\frac{dy}{dx} = e^x = y$, so the graph is the line $\frac{dy}{dx} = y$, for $y > 0$ only

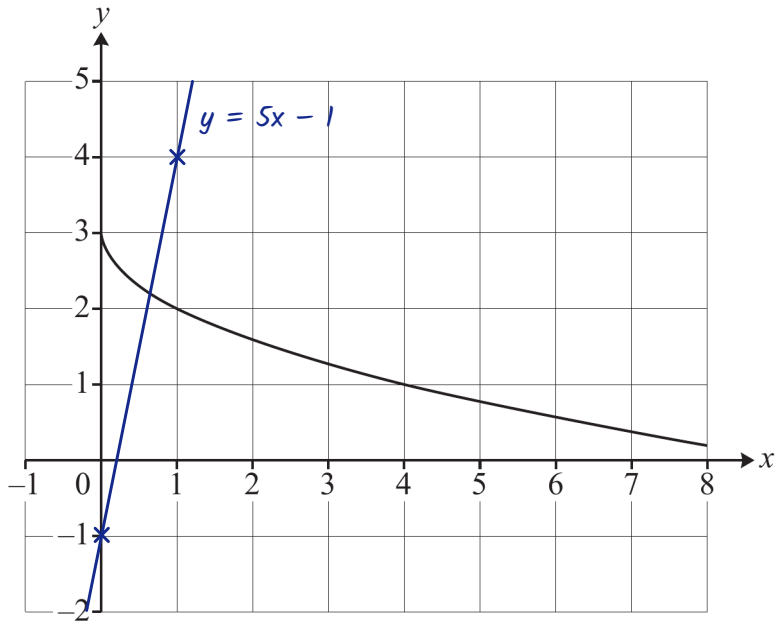


5 (b)(i) The rate of growth of the population is likely to be proportional to the population (more wolves means more births), which gives exponential growth.

Question number	
5 (b)(ii)	$t = 0,\ y = 21$: $21 = Ae^0 \Rightarrow A = 21$
	$t = 1,\ y = 51$: $51 = 21e^k \Rightarrow e^k = \frac{51}{21} = \frac{17}{7}$
	$k = \ln(\frac{17}{7}) = 0.88730...$
	$A = 21,\ k = 0.887$ (3 s.f.)
5 (b)(iii)	The model predicts the population keeps growing without limit, but the wolves will run out of food and space, so the population cannot keep growing.
6	Σ for $r = 1, 2, 3$: $\frac{1}{\sqrt{2} + \sqrt{1}} + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{4} + \sqrt{3}}$
	$\frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$
	$\frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{3} - \sqrt{2}}{3 - 2} = \sqrt{3} - \sqrt{2}$
	$\frac{1}{\sqrt{4} + \sqrt{3}} \times \frac{\sqrt{4} - \sqrt{3}}{\sqrt{4} - \sqrt{3}} = \frac{\sqrt{4} - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$
	Sum $= (\sqrt{2} - 1) + (\sqrt{3} - \sqrt{2}) + (2 - \sqrt{3})$
	$= -1 + 2 = 1$, so $\Sigma \frac{1}{\sqrt{(r+1)} + \sqrt{r}} = 1$ ($r = 1$ to 3) as required
7	Integrate by parts: $u = x,\ \frac{dv}{dx} = \cos 2x \Rightarrow \frac{du}{dx} = 1,\ v = \frac{1}{2}\sin 2x$
	$\int x \cos 2x\, dx = \frac{1}{2}x \sin 2x - \int \frac{1}{2} \sin 2x\, dx$
	$= \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + c$
8	$(3, 1)$: $1 = \frac{a}{3^2} \Rightarrow a = 9$, so $y = \frac{9}{x^2}$
	y is an integer only when x^2 is a factor of 9: $x^2 = 1$ or $x^2 = 9$
	$x = \pm 1 \Rightarrow y = 9$; $x = \pm 3 \Rightarrow y = 1$
	Other points: $(-3, 1)$, $(1, 9)$, $(-1, 9)$

Question number

9 (a) $y = 5x - 1$ passes through $(0, -1)$ and $(1, 4)$



9 (b) Intersection of curve and line: $3 - \sqrt{x} = 5x - 1$

$4 - 5x = \sqrt{x} \Rightarrow (4 - 5x)^2 = x$

$16 - 40x + 25x^2 = x \Rightarrow 25x^2 - 41x + 16 = 0$

$(x - 1)(25x - 16) = 0 \Rightarrow x = 1$ or $x = \frac{16}{25} = 0.64$

$x = 1$: $3 - 1 = 2$ but $5 - 1 = 4$, so reject. $\therefore x = 0.64$

$y = 5 \times 0.64 - 1 = 2.2$ (check: $3 - \sqrt{0.64} = 3 - 0.8 = 2.2$)

Line meets the x-axis when $5x - 1 = 0$, $x = 0.2$

Area = triangle (x from 0.2 to 0.64) + area under curve (x from 0.64 to 4)

Triangle: $\frac{1}{2} \times (0.64 - 0.2) \times 2.2 = 0.484 = \frac{121}{250}$

$\int_{0.64}^{4} (3 - \sqrt{x})\,dx = \left[3x - \frac{2}{3}x^{3/2}\right]_{0.64}^{4}$

$= (12 - \frac{2}{3} \times 8) - (1.92 - \frac{2}{3} \times 0.512)$

$= (12 - \frac{16}{3}) - (1.92 - \frac{128}{375}) = \frac{20}{3} - \frac{592}{375} = \frac{636}{125} = 5.088$

Total area = $0.484 + 5.088 = 5.572$

Exact area = $\frac{1393}{250}$

Question number

10 (a)

$\frac{1}{(4x+1)(x+1)} \equiv \frac{P}{4x+1} + \frac{Q}{x+1} \quad \Rightarrow \quad 1 \equiv P(x+1) + Q(4x+1)$

$x = -1$: $\quad 1 = -3Q \quad \Rightarrow \quad Q = -\frac{1}{3}$

$x = -\frac{1}{4}$: $\quad 1 = \frac{3}{4}P \quad \Rightarrow \quad P = \frac{4}{3}$

$\frac{1}{(4x+1)(x+1)} = \frac{4}{3(4x+1)} - \frac{1}{3(x+1)}$

10 (b)

Separate variables: $\int \frac{1}{y}\,dy = \int \frac{1}{(4x+1)(x+1)}\,dx$

$\int \frac{1}{y}\,dy = \int \left(\frac{4}{3(4x+1)} - \frac{1}{3(x+1)} \right) dx$

$\ln|y| = \frac{1}{3}\ln|4x+1| - \frac{1}{3}\ln|x+1| + c$

$x = 0,\ y = 2$: $\quad \ln 2 = \frac{1}{3}\ln 1 - \frac{1}{3}\ln 1 + c \quad \Rightarrow \quad c = \ln 2$

$\ln|y| = \frac{1}{3}\ln\left|\frac{4x+1}{x+1}\right| + \ln 2 = \ln\left(2\left(\frac{4x+1}{x+1}\right)^{1/3} \right)$

$x > -\frac{1}{4}$ so $4x + 1 > 0$ and $x + 1 > 0$; $y > 0$ since $y = 2$ when $x = 0$

$y = 2\left(\frac{4x+1}{x+1}\right)^{1/3}$, $\quad$ so $A = 2,\ B = \frac{1}{3}$

11

Cosine rule in triangle BDC: $\cos BDC = \frac{7^2 + 5^2 - 8^2}{2 \times 7 \times 5} = \frac{10}{70} = \frac{1}{7}$

$\sin BDC = \sqrt{\left(1 - \frac{1}{49}\right)} = \frac{\sqrt{48}}{7} = \frac{4\sqrt{3}}{7}$

$ADB = 180° - BDC$, $\quad$ so $\quad \sin ADB = \sin BDC = \frac{4\sqrt{3}}{7}$

Sine rule in triangle ABD: $\frac{AB}{\sin ADB} = \frac{BD}{\sin 45°}$

$AB = \frac{7 \sin ADB}{\sin 45°} = 7 \times \frac{4\sqrt{3}}{7} \div \frac{\sqrt{2}}{2} = 4\sqrt{3} \times \frac{2}{\sqrt{2}} = 4\sqrt{3} \times \sqrt{2}$

$AB = 4\sqrt{6}$ cm

12

$\tan(\alpha + \beta) = 1$ and $\tan\alpha = \frac{1}{2}$:

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1 \quad \Rightarrow \quad \frac{\frac{1}{2} + \tan\beta}{1 - \frac{1}{2}\tan\beta} = 1$

$\frac{1}{2} + \tan\beta = 1 - \frac{1}{2}\tan\beta$

$\frac{3}{2}\tan\beta = \frac{1}{2} \quad \Rightarrow \quad \tan\beta = \frac{1}{3}$

β is an acute angle in the triangle, so $\beta = \arctan\left(\frac{1}{3}\right)$

Question number

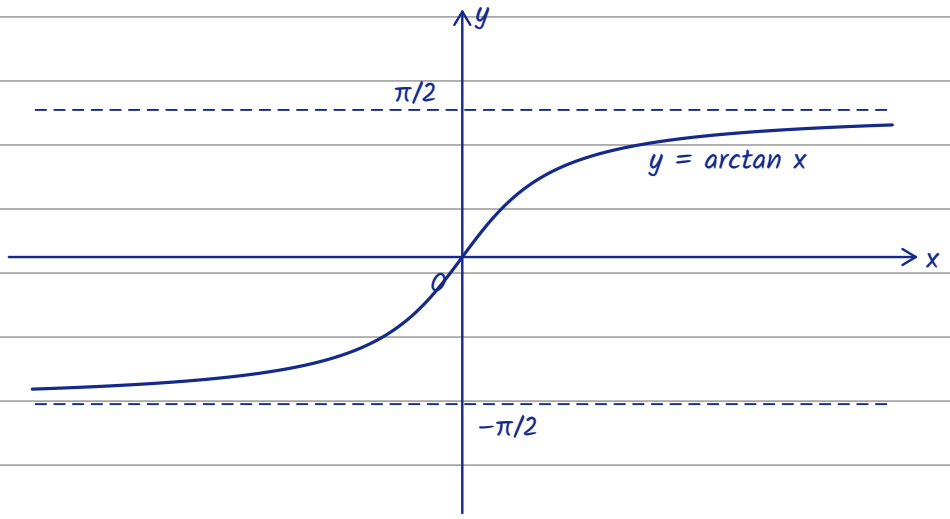
13 (a) Triangle ABE has a right angle at B: $\tan(\text{angle } BEA) = \frac{AB}{EB} = \frac{1}{x}$

so angle $BEA = \arctan\left(\frac{1}{x}\right)$

Angles in triangle ABE: $\theta + \text{angle } BEA = \frac{\pi}{2}$, and $\theta = \arctan x$

So $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$

13 (b)



13 (c) arctan is an increasing function (it is strictly increasing for all x)

14 (a) Put $n = 1$: $n + 1 = 2$, $n^2 + n + 1 = 3$ and $\frac{1}{n} = 1$,

so the result gives $\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan 1$

14 (b) $x = \frac{1}{n+1}$, $y = \frac{1}{n^2+n+1}$: for n a positive integer $xy = \frac{1}{(n+1)(n^2+n+1)} < 1$

so $\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{x+y}{1-xy}\right)$

$x + y = \frac{(n^2+n+1)+(n+1)}{(n+1)(n^2+n+1)} = \frac{n^2+2n+2}{(n+1)(n^2+n+1)}$

$1 - xy = \frac{(n+1)(n^2+n+1)-1}{(n+1)(n^2+n+1)} = \frac{n^3+2n^2+2n+1-1}{(n+1)(n^2+n+1)}$

$\frac{x+y}{1-xy} = \frac{n^2+2n+2}{n^3+2n^2+2n} = \frac{n^2+2n+2}{n(n^2+2n+2)} = \frac{1}{n}$

$\therefore \arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right)$

Question number

15

Using $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$ with $x = 2$ and $x = 3$:

$\arctan 2 + \arctan\left(\frac{1}{2}\right) = \frac{\pi}{2}$ and $\arctan 3 + \arctan\left(\frac{1}{3}\right) = \frac{\pi}{2}$

Adding: $\arctan 2 + \arctan 3 + \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \pi$

But $\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan 1$ (line 16)

$\therefore$ $\arctan 1 + \arctan 2 + \arctan 3 = \pi$ as required