OCR
Oxford Cambridge and RSA

Wednesday 14 October 2020 – Afternoon

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Time allowed: 2 hours



You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}, \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

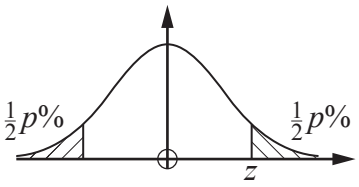
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Answer **all** the questions.

Section A (23 marks)

1 Fig. 1 shows triangle ABC.

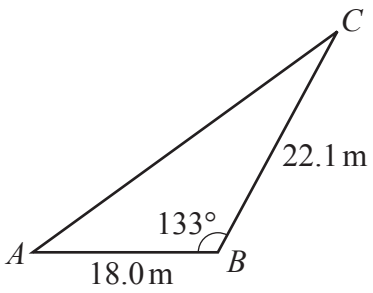


Fig. 1

Calculate the area of triangle ABC, giving your answer correct to 3 significant figures. **[2]**

2 Fig. 2 shows a sector of a circle of radius 8 cm.

The angle of the sector is 2.1 radians.

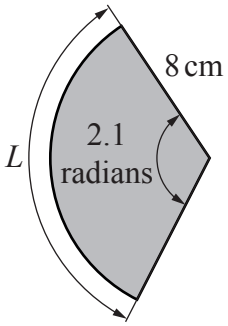


Fig. 2

(a) Calculate the length of the arc L. **[1]**

(b) Calculate the area of the sector. **[2]**

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3 You are given that $y = 4x + \sin 8x$.

(a) Find $\frac{dy}{dx}$. **[2]**

(b) Find the smallest positive value of x for which $\frac{dy}{dx} = 0$, giving your answer in an exact form. **[2]**

4 Fig. 4 shows a cumulative frequency diagram for the time spent revising mathematics by year 11 students at a certain school during a week in the summer term.

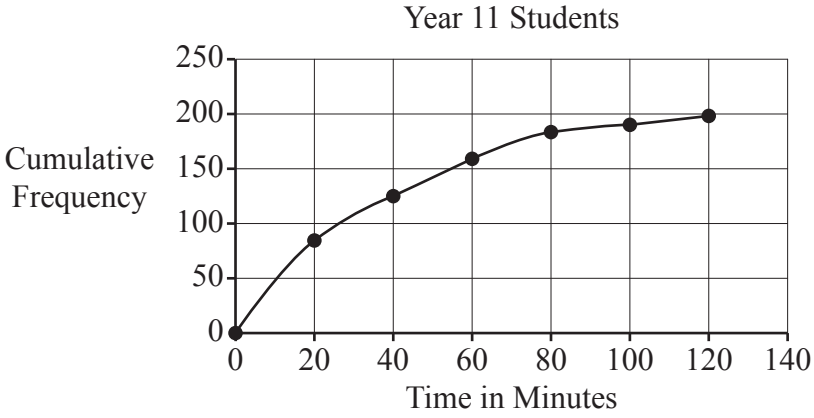


Fig. 4

(a) Use the diagram to estimate the median time spent revising mathematics by these students. **[1]**

A teacher comments that 90% of the students spent less than an hour revising mathematics during this week.

(b) Determine whether the information in the diagram supports this comment. **[1]**

5 The first n terms of an arithmetic series are

$17 + 28 + 39 + \ldots + 281 + 292$.

(a) Find the value of n. **[1]**

(b) Find the sum of these n terms. **[2]**

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6 **(a)** Find the first three terms in ascending powers of x of the binomial expansion of $(1+4x)^{\frac{1}{2}}$. **[3]**

(b) State the range of values of x for which this expansion is valid. **[1]**

7 You are given that $\mathrm{P}(A) = 0.6$, $\mathrm{P}(B) = 0.5$ and $\mathrm{P}(A \cup B)' = 0.2$.

(a) Find $\mathrm{P}(A \cap B)$. **[2]**

(b) Find $\mathrm{P}(A \mid B)$. **[2]**

(c) State, with a reason, whether A and B are independent. **[1]**

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Answer **all** the questions.

Section B (77 marks)

8 Rosella is carrying out an investigation into the age at which adults retire from work in the city where she lives. She collects a sample of size 50, ensuring this comprises of 25 randomly selected retired men and 25 randomly selected retired women.

(a) State the name of the sampling method she uses. **[1]**

Fig. 8.1 shows the data she obtains in a frequency table and Fig. 8.2 shows these data displayed in a histogram.

Age in years at retirement	45 –	50 –	55 –	60 –	65 –	70 –	75 – 80
Frequency density	0.4	1.8	2.4	2.2	1.8	1.2	0.2

Fig. 8.1

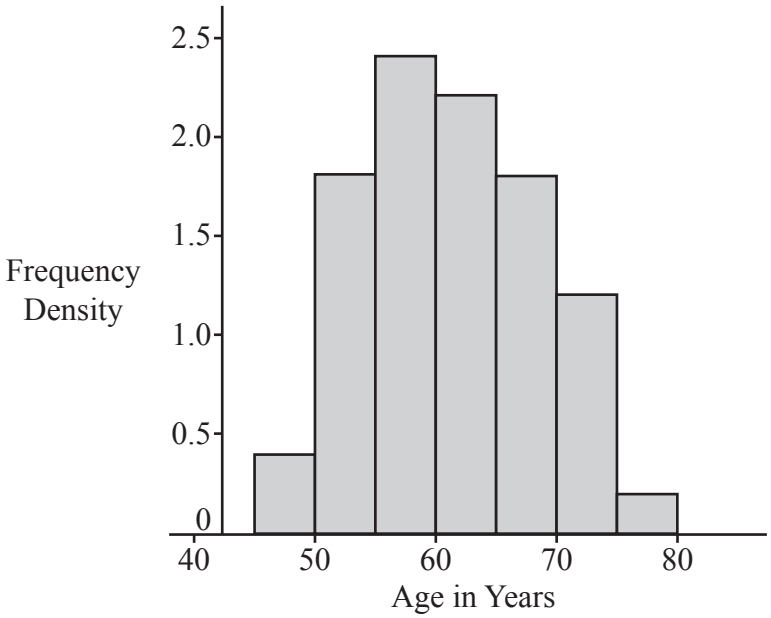


Fig. 8.2

(b) How many people in the sample are aged between 50 and 55? **[1]**

Rosella obtains a list of the names of all 4960 people who have retired in the city during the previous month.

(c) Describe how Rosella could collect a sample of size 200 from her list using

- systematic sampling such that every item on the list could be selected,
- simple random sampling. **[4]**

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Rosella collects two simple random samples, one of size 200 and one of size 500, from her list. The histograms in Fig. 8.3 show the data from the sample of size 200 on the left and the data from the sample of 500 on the right.

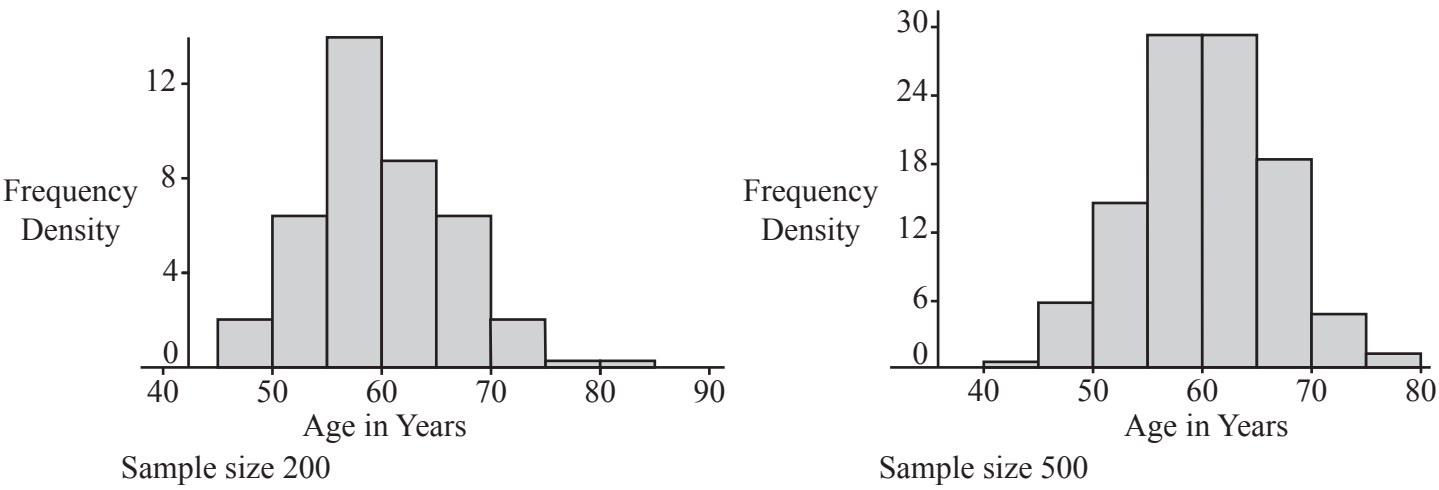


Fig. 8.3

(d) With reference to the histograms shown in Fig. 8.2 and Fig. 8.3, explain why it appears reasonable to model the age of retirement in this city using the Normal distribution. **[1]**

Summary statistics for the sample of 500 are shown in Fig. 8.4.

Statistics

n	500
Mean	60.0515
σ	6.5717
s	6.5783
Σx	30025.7601
Σx^2	1824686.322
Min	36.0793
Q1	55.2573
Median	59.9202
Q3	64.4239
Max	81.742

Fig. 8.4

(e) Use an appropriate Normal model based on the information in Fig. 8.4 to estimate the number of people aged over 65 who retired in the city in the previous month. **[4]**

(f) Identify a limitation in using this model to predict the number of people aged over 65 retiring in the **following** month. **[1]**

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9 A company supplies computers to businesses. In the past the company has found that computers are kept by businesses for a mean time of 5 years before being replaced. Claud, the manager of the company, thinks that the mean time before replacing computers is now different.

(a) Describe how Claud could obtain a cluster sample of 120 computers used by businesses the company supplies. **[1]**

Claud decides to conduct a hypothesis test at the 5% level to test whether there is evidence to suggest that the mean time that businesses keep computers is not 5 years. He takes a random sample of 120 computers. Summary statistics for the length of time computers in this sample are kept are shown in Fig. 9.

Statistics

n	120
Mean	4.8855
σ	2.6941
s	2.7054
Σx	586.2566
Σx^2	3735.1475
Min	0.1213
Q1	2.5472
Median	4.8692
Q3	7.0349
Max	9.9856

Fig. 9

(b) In this question you must show detailed reasoning.

- State the hypotheses for this test, explaining why the alternative hypothesis takes the form it does.
- Use a suitable distribution to carry out the test. **[8]**

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10 In this question you must show detailed reasoning.

The equation of a curve is

$$y = \frac{\sin 2x - x}{x \sin x}.$$

(a) Use the small angle approximation given in the list of formulae on pages 2–3 of this question paper to show that

$$\int_{0.01}^{0.05} y \, dx \approx \ln 5. \quad \textbf{[4]}$$

(b) Use the same small angle approximation to show that

$$\frac{dy}{dx} \approx -10000 \text{ at the point where } x = 0.01. \quad \textbf{[2]}$$

The equation $y = 0$ has a root near $x = 1$. Joan uses the Newton-Raphson method to find this root. The output from the spreadsheet she uses is shown in Fig. 10.1.

n	0	1	2	3	4	5	6	7
x_n	1	0.958509	0.950084	0.948261	0.94786	0.947772	0.947753	0.947748

Fig. 10.1

Joan carries out some analysis of this output. The results are shown in Fig. 10.2.

x	y
0.9477475	–7.79967E–07
0.9477485	–2.90821E–06
x	y
0.947745	4.54066E–06
0.947755	–1.67417E–05

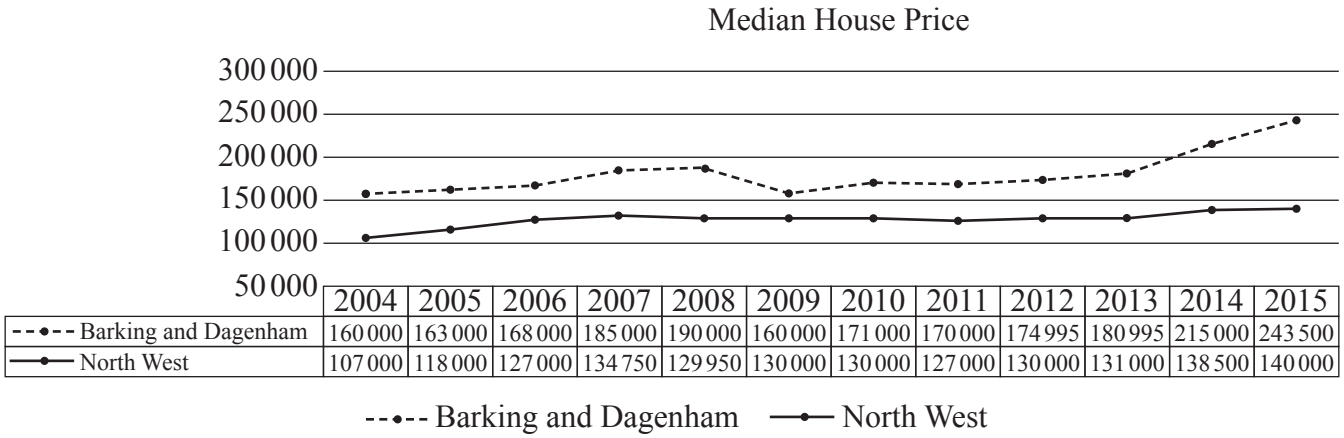
Fig. 10.2

(c) Consider the information in Fig. 10.1 and Fig. 10.2.

- Write 4.54066E–06 in standard mathematical notation.
- State the value of the root as accurately as you can, justifying your answer. **[3]**

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11 The pre-release material contains information concerning median house prices over the period 2004 – 2015. A spreadsheet has been used to generate a time series graph for two areas: the London borough of "Barking and Dagenham" and "North West". This is shown together with the raw data in Fig. 11.1.



	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Barking and Dagenham	160 000	163 000	168 000	185 000	190 000	160 000	171 000	170 000	174 995	180 995	215 000	243 500
North West	107 000	118 000	127 000	134 750	129 950	130 000	130 000	127 000	130 000	131 000	138 500	140 000

Fig. 11.1

Dr Procter suggests that it is unusual for median house prices in a London borough to be consistently higher than those in other parts of the country.

(a) Use your knowledge of the large data set to comment on Dr Procter's suggestion. **[1]**

Dr Procter wishes to predict the median house price in Barking and Dagenham in 2016. She uses the spreadsheet function LINEST to find the equation of the line of best fit for the given data. She obtains the equation

$P = 4897Y - 9\,657\,847$, where P is the median house price in pounds and Y is the calendar year, for example 2015.

(b) Use Dr Procter's equation to predict the median house price in Barking and Dagenham in

- 2016
- 2017. **[2]**

Professor Jackson uses a simpler model by using the data from 2014 and 2015 only to form a straight-line model.

(c) Find the equation Professor Jackson uses in her model. **[2]**

(d) Use Professor Jackson's equation to predict the median house price in Barking and Dagenham in

- 2016
- 2017. **[2]**

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Professor Jackson carries out some research online. She finds some information about median house prices in Barking and Dagenham, which is shown in Fig. 11.2.

2016	2017
£290 000	£300 000

Fig. 11.2

(e) Comment on how well

- Dr Procter's model fits the data,
- Professor Jackson's model fits the data. **[2]**

(f) Explain which, if any, of the models is likely to be more reliable for predicting median house prices in Barking and Dagenham in 2020. **[1]**

12 In this question you must show detailed reasoning.

A 5-sided spinner can give scores of 1, 2, 3, 4 or 5. After observing a large number of spins, Elaine models the probability distribution of X, the score on the spinner, as shown in Fig. 12.

x	1	2	3	4	5
$P(X = x)$	0.2	0.3	p	p	q

Fig. 12

When the spinner is spun twice, the probability of obtaining a total score of 9 is 0.06.

(a) Given that $q < 2p$, determine the values of p and q. **[6]**

(b) The spinner is spun 10 times. Calculate the probability that exactly one 5 is obtained. **[2]**

Elaine's teacher believes that the probability that the spinner shows a 1 is greater than 0.2. The spinner is spun 100 times and gives a score of 1 on 28 occasions.

(c) Conduct a hypothesis test at the 5% level to determine whether there is any evidence to suggest that the probability of obtaining a score of 1 is greater than 0.2. **[7]**

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13 The pre-release material contains information concerning median house prices, recycling rates and employment rates. Fig. 13.1 shows a scatter diagram of recycling rate against employment rate for a random sample of 33 regions.

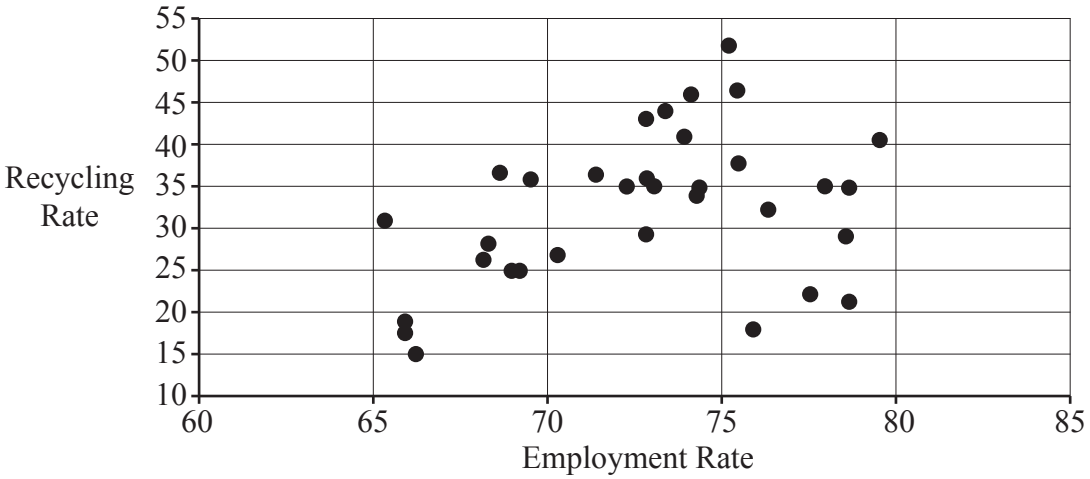


Fig. 13.1

The product moment correlation coefficient for this sample is 0.37154 and the associated p-value is 0.033.

Lee conducts a hypothesis test at the 5% level to test whether there is any evidence to suggest there is positive correlation between recycling rate and employment rate. He concludes that there is no evidence to suggest positive correlation because $0.033 \approx 0$ and $0.37154 > 0.05$.

(a) Explain whether Lee's reasoning is correct. **[2]**

Fig. 13.2 shows a scatter diagram of recycling rate against median house price for a random sample of 33 regions.

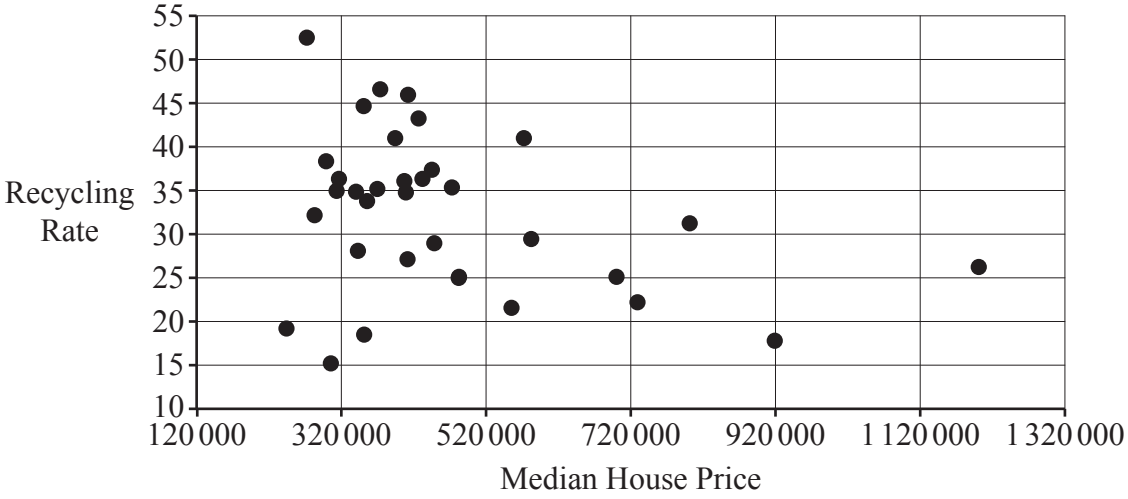


Fig. 13.2

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The product moment correlation coefficient for this sample is −0.33278 and the associated *p*-value is 0.058.

Fig. 13.3 shows summary statistics for the median house prices for the data in this sample.

Statistics	
n	33
Mean	465467.9697
σ	201236.1345
s	204356.2606
Σx	15360443
Σx^2	8486161617387
Min	243500
Q1	342500
Median	410000
Q3	521000
Max	1200000

Fig. 13.3

(b) Use the information in Fig. 13.3 and Fig. 13.2 to show that there are at least two outliers. **[2]**

(c) Describe the effect of removing the outliers on

- the product moment correlation coefficient between recycling rate and median house price,
- the *p*-value associated with this correlation coefficient,

in each case explaining your answer. **[2]**

All 33 items in the sample are areas in London. A student suggests that it is very unlikely that only areas in London would be selected in a random sample.

(d) Use your knowledge of the pre-release material to explain whether you think the student's suggestion is reasonable. **[1]**

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14 In this question you must show detailed reasoning.

Fig. 14 shows the graphs of $y = \sin x \cos 2x$ and $y = \frac{1}{2} - \sin 2x \cos x$.

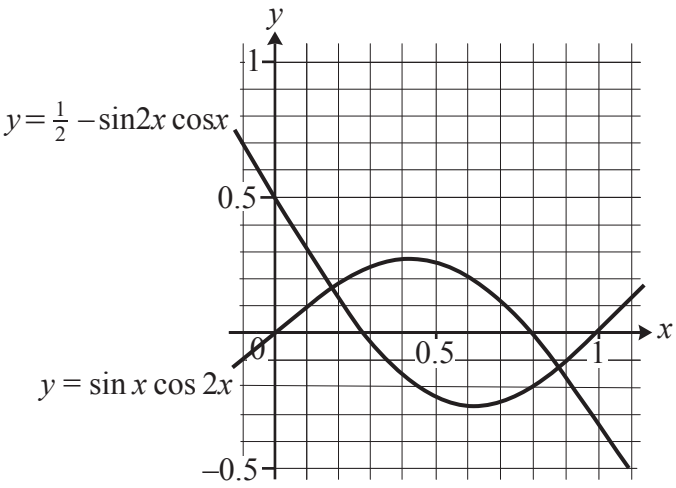


Fig. 14

Use integration to find the area between the two curves, giving your answer in an exact form. **[8]**

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15 Functions $f(x)$ and $g(x)$ are defined as follows.

$f(x) = \sqrt{x}$ for $x > 0$ and $g(x) = x^3 - x - 6$ for $x > 2$.

The function $h(x)$ is defined as

$h(x) = fg(x)$.

(a) Find $h(x)$ in terms of x and state its domain. **[2]**

(b) Find $h(3)$. **[1]**

Fig. 15 shows $h(x)$ and $h^{-1}(x)$, together with the straight line $y = x$.

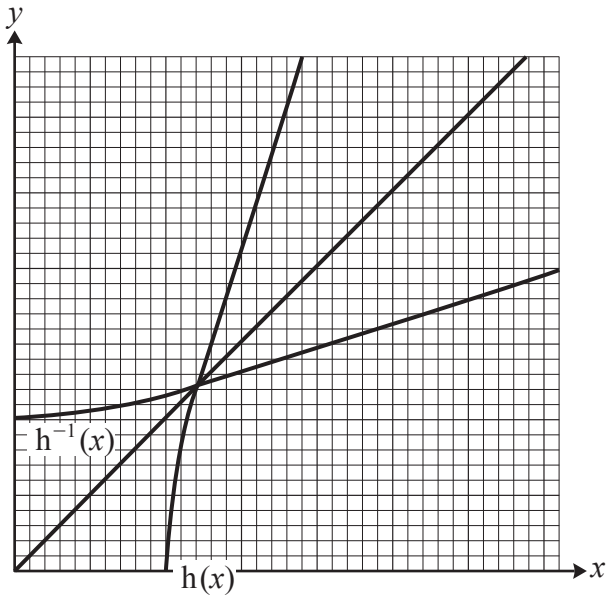


Fig. 15

(c) Determine the gradient of $y = h^{-1}(x)$ at the point where $y = 3$. **[4]**

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 Area = ½ × 18.0 × 22.1 × sin 133°

= 145.466... = 145 m^2 (3 s.f.)

2 (a) $L = r\theta = 8 \times 2.1 = 16.8$ cm

2 (b) Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times 2.1$

= 67.2 cm^2

3 (a)

$\frac{dy}{dx} = 4 + 8\cos 8x$

3 (b) $4 + 8\cos 8x = 0 \Rightarrow \cos 8x = -\frac{1}{2}$

$8x = 2\pi/3$ (smallest positive solution)

$x = \frac{\pi}{12}$

4 (a) Median at cumulative frequency ½ × 200 = 100

Median ≈ 27 minutes

4 (b) At 60 minutes the cumulative frequency is about 160

160 ÷ 200 = 80%, so only about 80% (not 90%) spent less than an hour.

No, the diagram does not support the comment.

5 (a) $a = 17$, $d = 11$: $17 + (n - 1) \times 11 = 292$

$(n - 1) \times 11 = 275 \Rightarrow n - 1 = 25 \Rightarrow n = 26$

Question number	
5 (b)	$S_{26} = \frac{26}{2}(17 + 292) = 13 \times 309 = 4017$
6 (a)	$(1 + 4x)^{1/2} = 1 + \frac{1}{2}(4x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(4x)^2 + \ldots$ $= 1 + 2x - \frac{1}{8} \times 16x^2 + \ldots$ $= 1 + 2x - 2x^2$
6 (b)	$\lvert 4x \rvert < 1 \Rightarrow \lvert x \rvert < \frac{1}{4}$, i.e. $-\frac{1}{4} < x < \frac{1}{4}$
7 (a)	$P(A \cup B) = 1 - 0.2 = 0.8$ $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.5 - 0.8$ $= 0.3$
7 (b)	$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.5} = 0.6$
7 (c)	$P(A \mid B) = 0.6 = P(A)$ (equivalently $0.6 \times 0.5 = 0.3 = P(A \cap B)$), so A and B are independent.
8 (a)	Quota sampling
8 (b)	Frequency = frequency density × class width = $1.8 \times 5 = 9$ people
8 (c)	Systematic: number the list 1 to 4960; $4960 \div 200 = 24.8$, so take every 24th name. Choose the starting name at random from the whole list (1 to 4960), then select every 24th name after it, cycling back to the start of the list when the end is reached, until 200 are selected.

Question number	
8 cont.	Simple random: assign each name a unique number from 1 to 4960. Use a random number generator to generate numbers in this range, ignoring repeats, until 200 different names have been selected.
8 (d)	As the sample size increases (50, 200, 500) the histograms become more symmetrical and bell-shaped, i.e. closer to a Normal shape.
8 (e)	X = age at retirement, $X \sim N(60.0515, 6.5783^2)$ $P(X > 65) = 0.225951\ldots \approx 0.226$ Expected number = $4960 \times 0.225951\ldots = 1120.7\ldots$ $\approx$ 1121 people aged over 65
8 (f)	Retirement ages may vary seasonally, e.g. many teachers retire in August, so the next month's distribution may differ from this month's.
9 (a)	Randomly select 12 of the businesses the company supplies, then randomly select 10 computers from each of these businesses.
9 (b)	H_0: $\mu = 5$ $\quad$ H_1: $\mu \neq 5$ where μ is the population mean time (years) for which computers are kept before being replaced. H_1 is two-tailed because Claud is testing whether the mean time is different from 5 years (not specifically more or less). Under H_0: $\bar{X} \sim N(5, 2.7054^2/120)$ Standard deviation of $\bar{X} = \frac{2.7054}{\sqrt{120}} = 0.246968\ldots$ $P(\bar{X} < 4.8855) = 0.321459\ldots \approx 0.32$ Two-tailed test at 5%: compare with 0.025 $0.32 > 0.025$, so the result is not significant: do not reject H_0.

Question number	
9 cont.	There is insufficient evidence at the 5% level to suggest that the mean length of time computers are kept is not 5 years.
10 (a)	For small x: $\sin 2x \approx 2x$ and $\sin x \approx x$
	$y \approx \frac{2x - x}{x \times x} = \frac{x}{x^2} = \frac{1}{x}$
	$\int_{0.01}^{0.05} y\,dx \approx \int_{0.01}^{0.05} \frac{1}{x}\,dx = [\ln x]_{0.01}^{0.05}$
	$= \ln 0.05 - \ln 0.01 = \ln \frac{0.05}{0.01} = \ln 5$ as required
10 (b)	$y \approx x^{-1} \Rightarrow \frac{dy}{dx} \approx -x^{-2} = -\frac{1}{x^2}$
	At $x = 0.01$: $\frac{dy}{dx} \approx -\frac{1}{0.01^2} = -\frac{1}{0.0001} = -10000$ as required
10 (c)	$4.54066E{-}06 = 4.54066 \times 10^{-6}$
	There is a sign change in y between x = 0.947745 and x = 0.947755,
	so the root lies between them: root = 0.94775 to 5 d.p.
	There is no sign change between 0.9477475 and 0.9477485, so 0.947748
	is not confirmed to 6 d.p. Root = 0.94775 (to 5 d.p.)
11 (a)	In the large data set, median house prices in London boroughs are
	generally higher than elsewhere in the country, so Dr Procter's
	suggestion is probably wrong.
11 (b)	2016: $P = 4897 \times 2016 - 9\,657\,847 = £214\,505$
	2017: $P = 4897 \times 2017 - 9\,657\,847 = £219\,402$

Question number	
11 (c)	Gradient $= \frac{243\,500 - 215\,000}{2015 - 2014} = 28\,500$ $P - 215\,000 = 28\,500(Y - 2014)$ $P = 28\,500Y - 57\,184\,000$
11 (d)	2016: $P = 28\,500 \times 2016 - 57\,184\,000 = £272\,000$ 2017: $P = 28\,500 \times 2017 - 57\,184\,000 = £300\,500$
11 (e)	Dr Procter's model is a very poor fit: £214 505 and £219 402 are far below the actual £290 000 and £300 000. Professor Jackson's model is a good fit: £300 500 is very close for 2017, although £272 000 is an underestimate for 2016.
11 (f)	Neither: 2020 is well outside the range of the data, so predicting it would be extrapolation and is unreliable for both models.
12 (a)	Probabilities sum to 1: $0.2 + 0.3 + p + p + q = 1 \Rightarrow 2p + q = 0.5$ Total of 9 from (4, 5) or (5, 4): $2 \times p \times q = 0.06 \Rightarrow pq = 0.03$ $q = 0.5 - 2p \Rightarrow p(0.5 - 2p) = 0.03$ $2p^2 - 0.5p + 0.03 = 0$ $p = \frac{0.5 \pm \sqrt{(0.25 - 0.24)}}{4} = \frac{0.5 \pm 0.1}{4} \Rightarrow p = 0.15$ or $p = 0.1$ $p = 0.15 \Rightarrow q = 0.2$; $p = 0.1 \Rightarrow q = 0.3$ $q < 2p$: $0.2 < 0.3$ ✓, but $0.3 < 0.2$ ✗ So $p = 0.15$ and $q = 0.2$
12 (b)	Y = number of 5s, $Y \sim B(10, 0.2)$ $P(Y = 1) = 10 \times 0.2 \times 0.8^9 = 0.268435\ldots \approx 0.268$

Question number	
12 (c)	H_0: $p = 0.2$ H_1: $p > 0.2$ where p is the probability that the spinner shows a 1 on any spin. Under H_0: $X \sim B(100, 0.2)$, X = number of 1s in 100 spins $P(X \geqslant 28) = 1 - P(X \leqslant 27) = 1 - 0.965848... = 0.034151...$ $0.0342 < 0.05$, so the result is significant: reject H_0. There is sufficient evidence at the 5% level to suggest that the probability of obtaining a score of 1 is greater than 0.2.
13 (a)	Lee is wrong. He should compare the p-value 0.033 with 0.05: $0.033 < 0.05$, so the result is significant. He should compare $r = 0.37154$ with 0: $r > 0$ suggests positive correlation. So there is evidence at the 5% level to suggest positive correlation.
13 (b)	Mean + 2s = 465 467.97 + 2 × 204 356.26 = 874 180.5 (approx. £874 180) From Fig. 13.2 there are points at about £920 000 and £1 200 000, both above £874 180, so there are at least two outliers.
13 (c)	The pmcc would be closer to 0, because without the outliers the remaining points are less well modelled by a straight line (the outliers create much of the negative trend). The p-value would increase, because a value of r closer to 0 is more likely to occur if there is no correlation.
13 (d)	The suggestion is reasonable: the large data set contains many other regions outside London, so a random sample of 33 consisting only of London areas would be very unlikely.
14	Intersections: $\sin x \cos 2x = \frac{1}{2} - \sin 2x \cos x$ $\sin x \cos 2x + \cos x \sin 2x = \frac{1}{2} \Rightarrow \sin(x + 2x) = \frac{1}{2} \Rightarrow \sin 3x = \frac{1}{2}$

Question number

14 cont.

$3x = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow x = \frac{\pi}{18}$ and $x = \frac{5\pi}{18}$

Between these the curve $y = \sin x \cos 2x$ is above $y = \frac{1}{2} - \sin 2x \cos x$.

$\text{Area} = \int_{\pi/18}^{5\pi/18} \left(\sin x \cos 2x - \left(\frac{1}{2} - \sin 2x \cos x \right) \right) dx$

$= \int_{\pi/18}^{5\pi/18} \left(\sin 3x - \frac{1}{2} \right) dx$

$= \left[-\frac{\cos 3x}{3} - \frac{x}{2} \right]_{\pi/18}^{5\pi/18}$

$= \left(-\frac{1}{3} \cos \frac{5\pi}{6} - \frac{5\pi}{36} \right) - \left(-\frac{1}{3} \cos \frac{\pi}{6} - \frac{\pi}{36} \right)$

$= \left(\frac{\sqrt{3}}{6} - \frac{5\pi}{36} \right) - \left(-\frac{\sqrt{3}}{6} - \frac{\pi}{36} \right)$

$= \frac{\sqrt{3}}{3} - \frac{\pi}{9} \quad \left(= \frac{3\sqrt{3} - \pi}{9} \right)$

15 (a) $h(x) = fg(x) = \sqrt{(x^3 - x - 6)}$, domain $x > 2$

15 (b) $h(3) = \sqrt{(27 - 3 - 6)} = \sqrt{18} \quad (= 3\sqrt{2} = 4.24\ldots)$

15 (c) $h(x) = (x^3 - x - 6)^{1/2}$

$h'(x) = \frac{1}{2}(x^3 - x - 6)^{-1/2} \times (3x^2 - 1) = \frac{3x^2 - 1}{2\sqrt{(x^3 - x - 6)}}$

On $y = h^{-1}(x)$, $y = 3$ corresponds to $x = h(3) = \sqrt{18}$

$h'(3) = \frac{27 - 1}{2\sqrt{18}} = \frac{26}{2\sqrt{18}} = \frac{13}{\sqrt{18}}$

Gradient of $h^{-1} = \frac{1}{h'(3)} = \frac{\sqrt{18}}{13} = \frac{3\sqrt{2}}{13} = 0.326$