OCR
Oxford Cambridge and RSA

Thursday 12 June 2025 – Afternoon

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Time allowed: 2 hours

185844237 5

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \ (n \in \mathbb{N})$,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \ (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h\to 0}\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y \, \mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

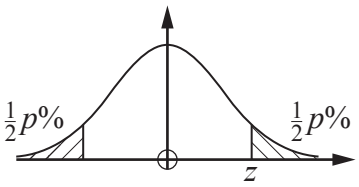
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A (23 marks)

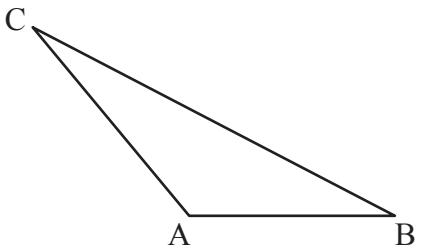
1 The equation of a circle is $(x-4)^2+(y+5)^2-64=0$.

(a) State the coordinates of the centre of the circle. **[1]**

(b) State the radius of the circle. **[1]**

2 Determine the coefficient of x^3 in the expansion of $(1+2x)^{12}$. **[2]**

3 The diagram shows triangle ABC.



Not to scale

AB = 4.7 cm, AC = 6.9 cm and BC = 11.2 cm.

Find the size of angle CAB. Give your answer correct to **3** significant figures. **[2]**

4 **(a)** Use the factor theorem to show that $(x-3)$ is a factor of $4x^3-8x^2-11x-3$. **[1]**

(b) Hence show that $4x^3-8x^2-11x-3=(x-3)(ax+b)^2$, where a and b are integers to be determined. **[2]**

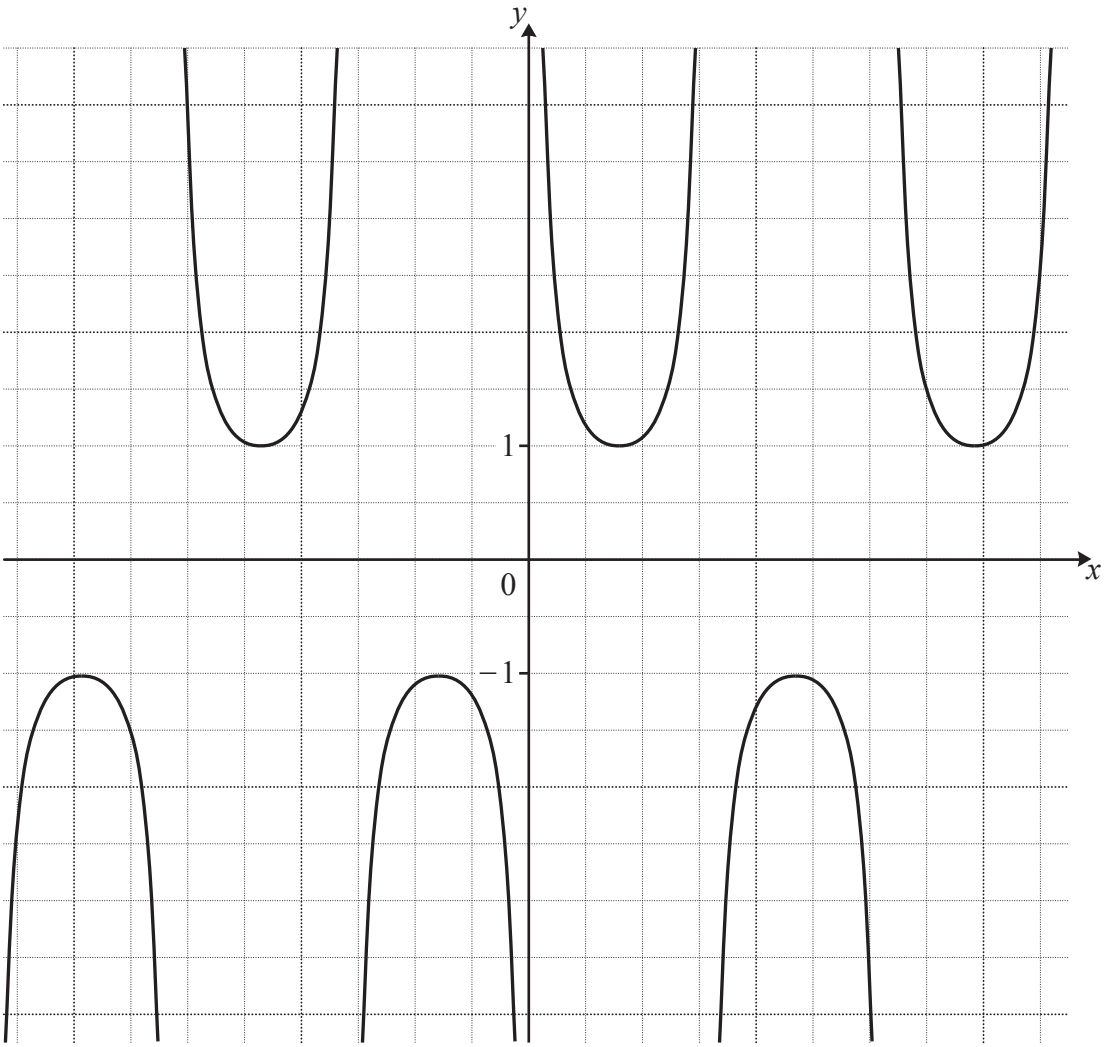
(c) Sketch the graph of $y=4x^3-8x^2-11x-3$ on the axes in the Printed Answer Booklet. **[2]**

5 Prove that the sum of the first n positive odd numbers is a square number. **[3]**

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6 The diagram shows part of the graph of $f(x) = \operatorname{cosec} 2x$.



(a) State which of the following is the equation of the curve $y = \dfrac{1}{f(x)}$.

$y = \cos 2x$ $y = \operatorname{cosec}(-2x)$ $y = -\operatorname{cosec} 2x$ $y = \sin 2x$ **[1]**

(b) State the exact equations of the asymptotes of $y = \operatorname{cosec} 2x$ for $0 \leqslant x \leqslant \pi$. **[1]**

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7 Some people have blood type A and some people have brown hair. A person may have both, just one, or neither of these characteristics.

A person is selected at random.

A is the event 'the person has blood type A'.
B is the event 'the person has brown hair'.

You are given the following probabilities.

$P(A \cap B') = 0.3382$, $P(A' \cap B) = 0.0682$ and $P((A \cup B)') = 0.5518$.

(a) Show this information on the Venn diagram in the Printed Answer Booklet. **[2]**

(b) Determine whether A and B are independent events. **[4]**

(c) Explain whether A and B are mutually exclusive events. **[1]**

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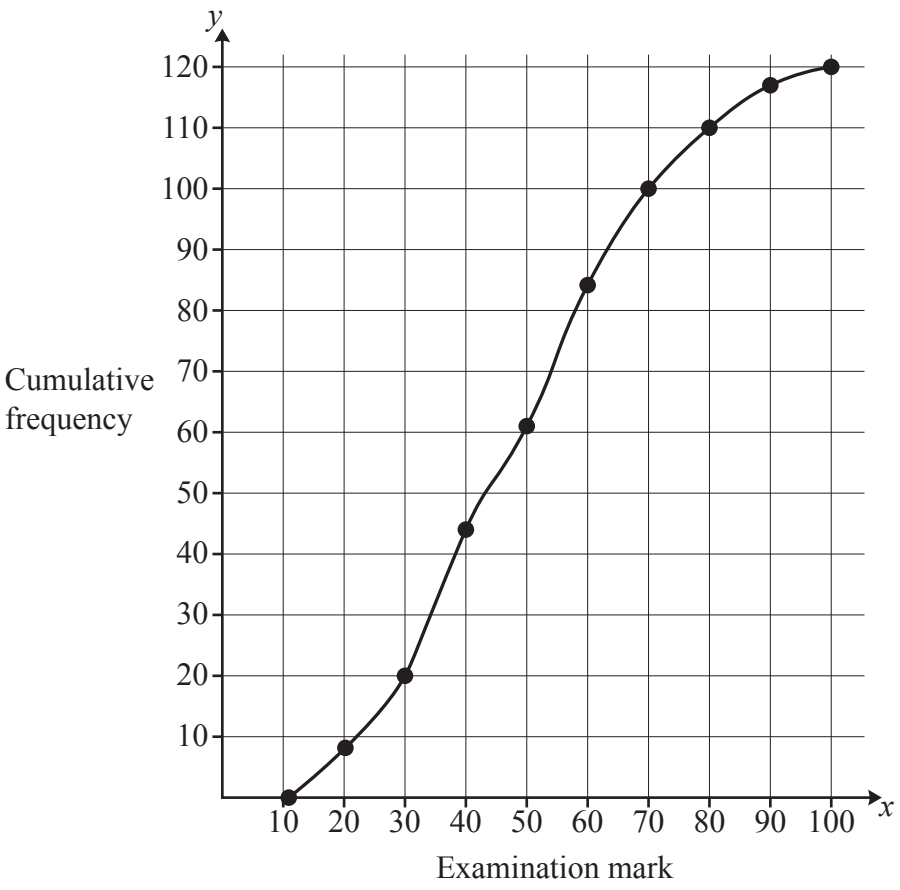
Section B (77 marks)

8 At the end of the summer term all 120 Year 8 pupils at Oakmount College sit the same mathematics examination.

The marks obtained by the pupils in the 2024 examination are summarised in the table.

Mark	11–20	21–30	31–40	41–50	51–60	61–70	71–80	81–90	91–100
Frequency	8	12	24	17	23	16	10	7	3

Software is used to produce the cumulative frequency curve shown below.



(a) It is proposed that the 30 pupils with the best marks in the examination are placed in the top set in Year 9.

Use the copy of the cumulative frequency curve in the **Printed Answer Booklet** to determine an estimate of the lowest mark needed for a pupil to be placed in the top set. **[2]**

(b) The person who will teach the top set states that pupils will **not** manage in this class unless they obtain at least 75 marks in the examination.

Use the copy of the cumulative frequency curve in the **Printed Answer Booklet** to determine an estimate of how many pupils will be in the top set if the lowest mark is set at 75. **[2]**

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9 The position vectors of the points A and B are $\overrightarrow{OA} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ respectively.

(a) Write down the vector $\overrightarrow{AB}$. **[1]**

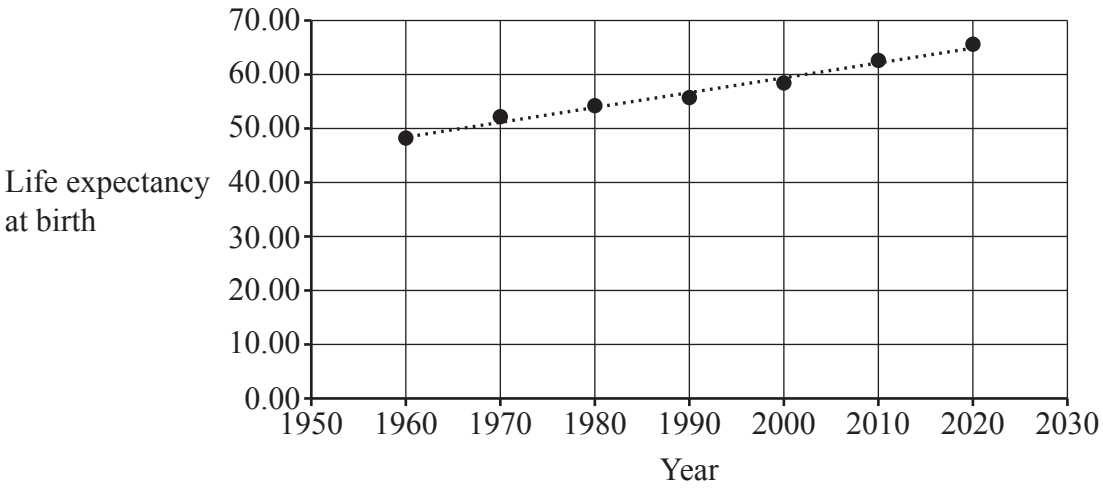
The point C lies on AB such that $2\overrightarrow{AC} = \overrightarrow{CB}$.

(b) Find the vector $\overrightarrow{OC}$. **[2]**

10 The pre-release material contains information about life expectancy at birth for countries of the world at 10-year intervals from 1960 until 2020.

The life expectancy at birth of the population of Sudan for this period, together with a line of best fit, is shown in **Fig. 10.1**.

Fig. 10.1



The equation of the line of best fit is $L = 0.275X - 490$, where X is the year and L is the life expectancy at birth, measured in years.

(a) Use the equation of the line of best fit to estimate the life expectancy at birth in Sudan in 1975. **[1]**

(b) Explain whether this estimate is likely to be close to the true value of life expectancy at birth in Sudan in 1975. **[1]**

(c) Use your knowledge of the pre-release material to explain whether your answer to part **(a)** is likely to be a close approximation to the life expectancy at birth in the United Kingdom in 1975. **[1]**

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The pre-release material also gives the median age of the population in countries of the world.

The table shows the median age of the population and the life expectancy at birth in 2020 for some countries in Africa.

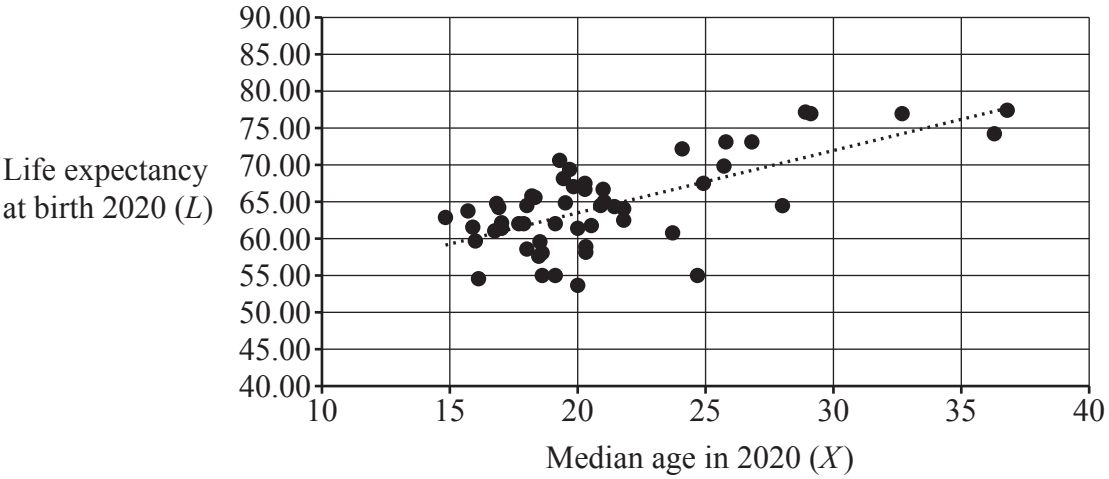
Country	Median age	Life expectancy at birth 2020
Nigeria	18.6	55.02
Rwanda	19.7	69.33
Saint Helena, Ascension and Tristan da Cunha	43.2	#N/A
Sao Tome and Principe	19.3	70.58
Senegal	19.4	68.21

(d) Explain how the data in the table should be cleaned before these data can be included in a scatter diagram for life expectancy at birth against median age for the countries in Africa. **[1]**

Fig. 10.2 shows a scatter diagram for the cleaned data of life expectancy at birth against median age in 2020 for the countries in Africa. It also shows a line of best fit.

The product moment correlation coefficient for these data is 0.680.

Fig. 10.2



(e) A student decides to use the line of best fit to estimate the life expectancy at birth in 2020 for Saint Helena, Ascension and Tristan da Cunha.

Explain whether this estimate is likely to be reliable. **[1]**

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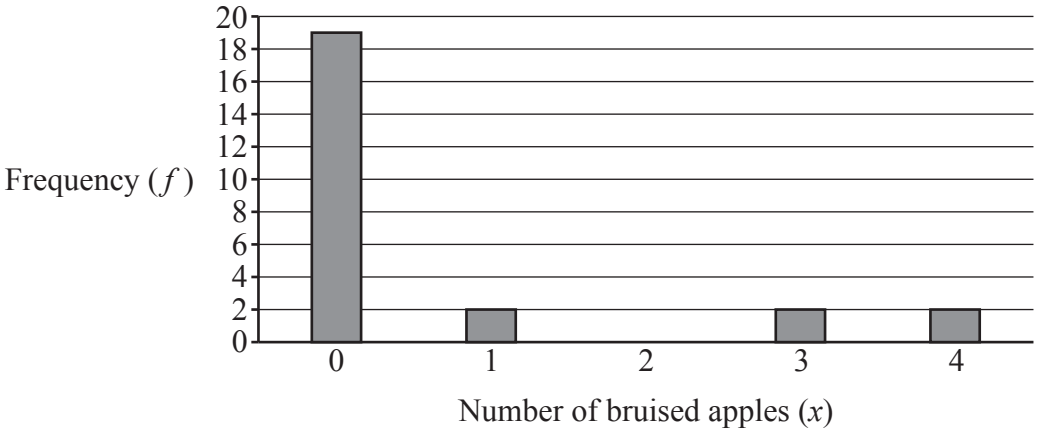
11 Apples are sold in packets of 4. Ling and Sam are investigating the frequency, f, of the number of bruised apples in a packet, x. They each collect a random sample of packets, note the number of bruised apples in each packet and draw a diagram to represent their data.

Ling's results are shown in **Table 11.1** and **Fig. 11.1**.

Table 11.1

x	0	1	2	3	4
f	19	2	0	2	2

Fig. 11.1



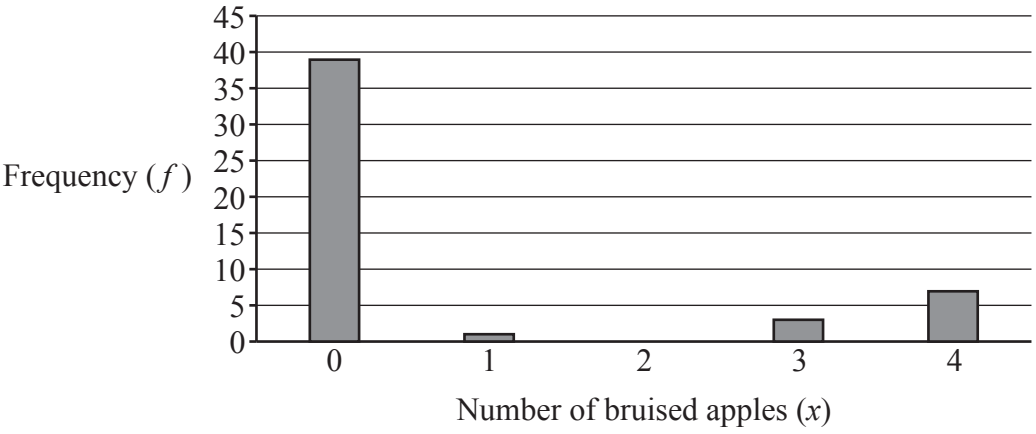
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Sam's results are shown in **Table 11.2** and **Fig. 11.2**.

Table 11.2

x	0	1	2	3	4
f	39	1	0	3	7

Fig. 11.2



(a) Explain why Sam's diagram is likely to be a better representation of the true distribution of the number of bruised apples in a packet than Ling's diagram. **[1]**

(b) Calculate the mean number of bruised apples per packet for Sam's data. **[1]**

Sam thinks that the distribution of bruised apples may be modelled by a binomial distribution.

(c) Use your answer to part **(b)** to calculate the value of p, the probability that an apple selected at random is bruised, for Sam's model. **[1]**

(d) Calculate the theoretical frequency distribution of the number of bruised apples per packet for Sam's model, giving your answers correct to **2** decimal places. **[3]**

(e) Comment on whether Sam's model appears to be a good fit for the data. **[1]**

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12 In this question you must show detailed reasoning

Solve the equation $3\sec^2\theta + 2\tan\theta - 4 = 0$ for $-180° < \theta < 180°$. **[6]**

13 According to the Office for National Statistics, in 2022 the proportion of people in England and Wales aged 16 or over who were married or in a civil partnership was 0.494.

In 2024 a researcher collected a random sample of 78 people aged 16 or over.

The researcher decided to conduct a hypothesis test at the 1% level to see if there was any evidence to suggest that the proportion of people aged 16 or over who are married or in a civil partnership was lower in 2024.

(a) State the hypotheses for the test. You must define the parameter used to carry out the test. **[2]**

(b) Determine the critical region for the test. **[3]**

The researcher found that 26 of the 78 people in the random sample were married or in a civil partnership.

(c) Use your answer to part **(b)** to carry out the hypothesis test. **[3]**

(d) State the probability that the null hypothesis is incorrectly rejected. **[1]**

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14 The equation of a curve is $y = \frac{16}{x^2} + \frac{3}{x}$.

(a) Determine the coordinates of the point where the curve cuts the x-axis. **[2]**

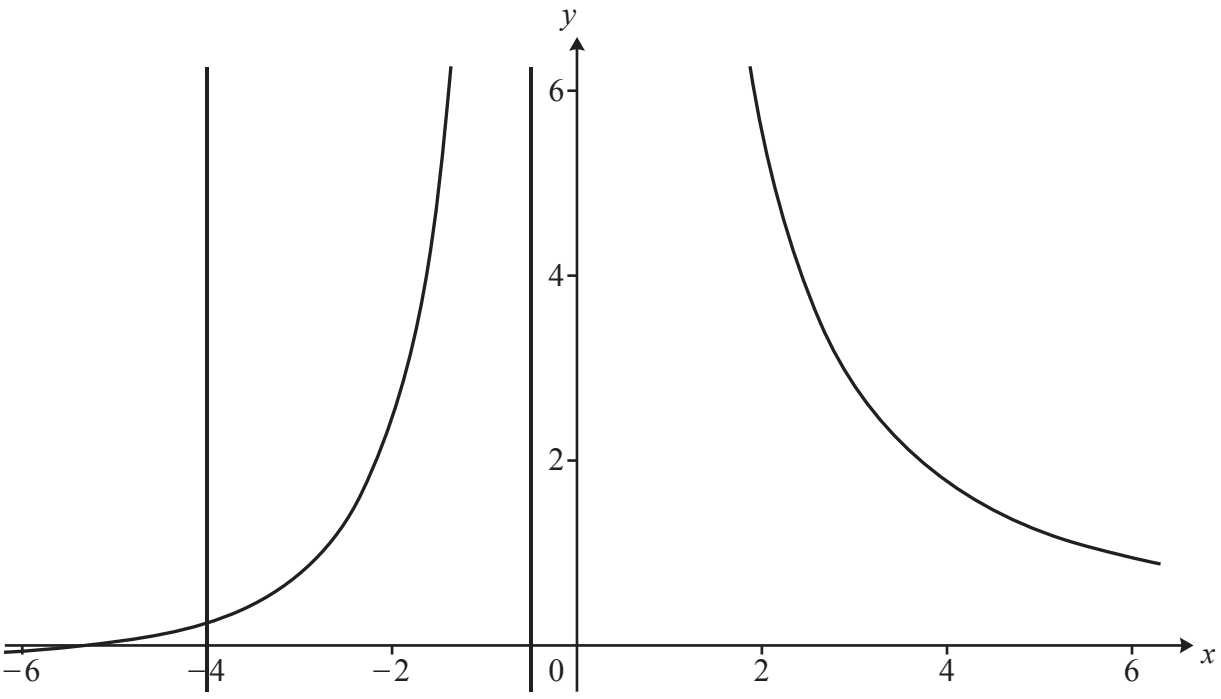
(b) (i) Find $\frac{dy}{dx}$. **[2]**

(ii) Hence determine the exact coordinates of the stationary point on the curve $y = \frac{16}{x^2} + \frac{3}{x}$. **[2]**

(c) (i) Find $\frac{d^2y}{dx^2}$. **[2]**

(ii) Hence determine the set of values of x for which the curve $y = \frac{16}{x^2} + \frac{3}{x}$ is concave downwards. **[2]**

The diagram shows parts of the curve $y = \frac{16}{x^2} + \frac{3}{x}$, the line $x = -4$ and the line $x = -\frac{1}{2}$.



(d) Determine the exact area bounded by the curve $y = \frac{16}{x^2} + \frac{3}{x}$, the x-axis and the lines $x = -4$ and $x = -\frac{1}{2}$. Give your answer in the form $a + b\ln 2$, where a and b are constants to be determined. **[5]**

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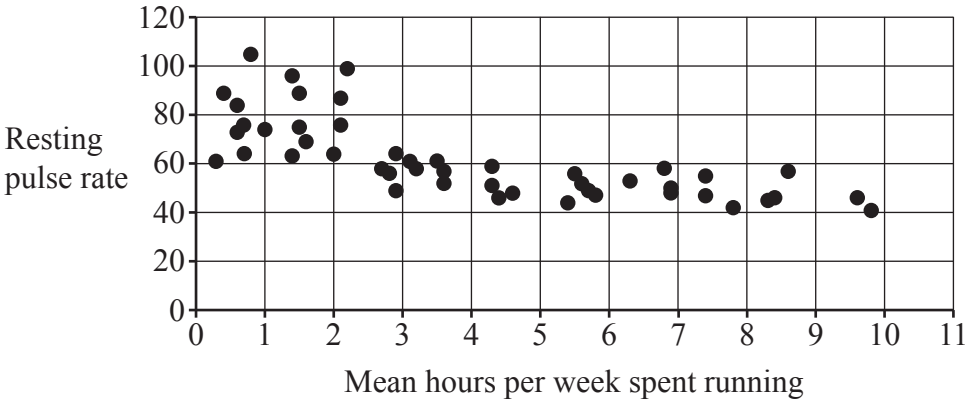
15 A personal trainer is investigating whether, in the general population, there is any association between resting pulse rate in beats per minute and mean hours per week spent running.

One week he collects a sample by asking 25 of his clients for the relevant data.

(a) (i) State the name of the sampling technique used by the personal trainer. **[1]**

(ii) Explain why this sampling technique might introduce bias. **[1]**

A biologist researching the same topic collects a random sample of size 47. She represents the data using the scatter diagram shown below.



(b) Describe the association between resting pulse rate and mean hours per week spent running. **[1]**

(c) The biologist identifies two distinct regions on the scatter diagram. Identify these regions on the copy of the scatter diagram in the **Printed Answer Booklet** by drawing an appropriate **vertical** line between them. **[1]**

According to medical research, the normal resting pulse rate for an adult is between 60 and 110 beats per minute.

The biologist separates the data into a group to the left of the vertical line on the scatter diagram and a group to the right of the vertical line on the scatter diagram. She calculates Spearman's rank correlation coefficient, r_s, and the associated p-value for each group.

The results are shown in the table.

Group	r_s	p-value
To the left of the vertical line	0.209 36	0.419 98
To the right of the vertical line	–0.613 49	0.000 31

(d) With reference to the two groups identified by the biologist and to the values in the table, explain what may be inferred about the association between resting pulse rate and mean number of hours per week spent running. **[6]**

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16 In this question you must show detailed reasoning

The population of puffins on a remote island is estimated to be $P = 58.5$, where P is measured in thousands of birds.

Just after this estimate is made there is a serious oil spill in the area. It is noted that there is a rapid decline in the population of puffins on the island.

The situation is modelled by the differential equation

$\frac{dP}{dt} = (72t - 108)e^{-0.8t}$, where t is the time in years after the pollution incident.

(a) Determine an expression for P in terms of t. **[7]**

(b) Determine whether, according to the model, P will recover to 58.5. You may assume that $\lim_{t \to \infty} te^{-0.8t} = 0$. **[2]**

17 In this question you must show detailed reasoning

Jars of honey contain a nominal mass of 340 grams of honey.

The jars are filled by an automated process so that the actual mass of honey in a jar, X grams, is Normally distributed with mean 350 grams. This is to minimize the chance of a jar which contains less than 340 grams of honey being sold.

During a quality control check, a random sample of 50 jars of honey are checked.

Summary statistics for the check are as follows.

$\Sigma x = 17367 \quad \Sigma x^2 = 6036597 \quad n = 50$

Carry out a hypothesis test at the 1% level to see if there is any evidence to suggest that the mean mass of honey in the jars is less than 350 grams. **[9]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) Centre $(4, -5)$

1 (b) Radius $= \sqrt{64} = 8$

2 Coefficient of $x^3 = {}^{12}C_3 \times 2^3$

$= 220 \times 8$

$= 1760$

3 $\cos CAB = \dfrac{4.7^2 + 6.9^2 - 11.2^2}{2 \times 4.7 \times 6.9} = \dfrac{-55.74}{64.86} = -0.859389...$

$CAB = 149.248...°$

Angle $CAB = 149°$ (3 s.f.)

4 (a) $f(x) = 4x^3 - 8x^2 - 11x - 3$

$f(3) = 4 \times 3^3 - 8 \times 3^2 - 11 \times 3 - 3$

$= 108 - 72 - 33 - 3 = 0$

$f(3) = 0$ so $(x - 3)$ is a factor of $4x^3 - 8x^2 - 11x - 3$

4 (b) $4x^3 - 8x^2 - 11x - 3 = (x - 3)(4x^2 + px + 1)$

x^2 terms: $p - 12 = -8 \Rightarrow p = 4$ (x terms: $1 - 3p = -11$ ✓)

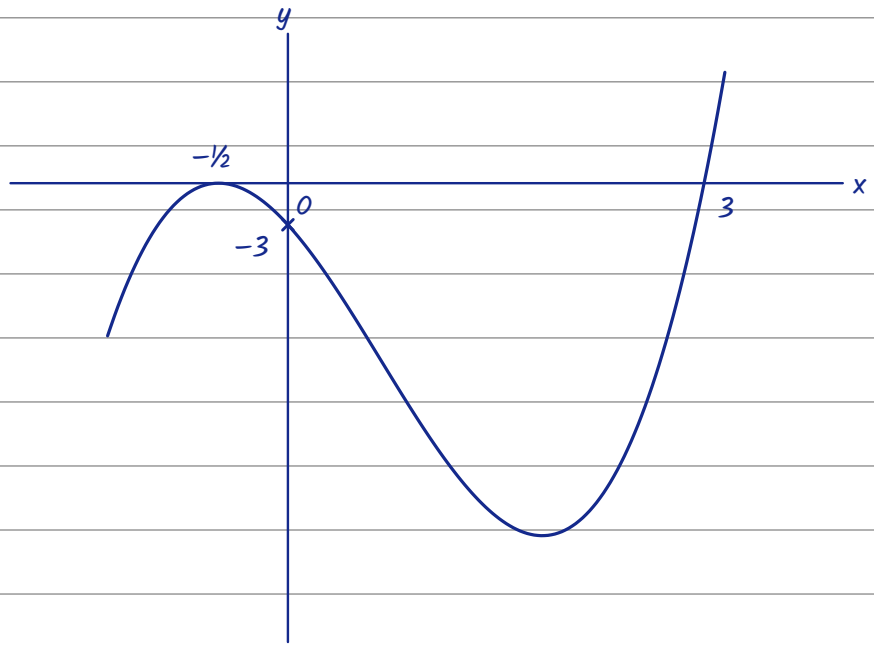
$4x^3 - 8x^2 - 11x - 3 = (x - 3)(4x^2 + 4x + 1)$

$= (x - 3)(2x + 1)^2$

$a = 2, b = 1$

Question number

4 (c)



5 Sum = 1 + 3 + 5 + ... + (2n − 1)

This is an arithmetic series with $a = 1$, $d = 2$ and n terms

$S_n = \frac{n}{2}(2 \times 1 + (n - 1) \times 2)$

$= \frac{n}{2} \times 2n$

$= n^2$

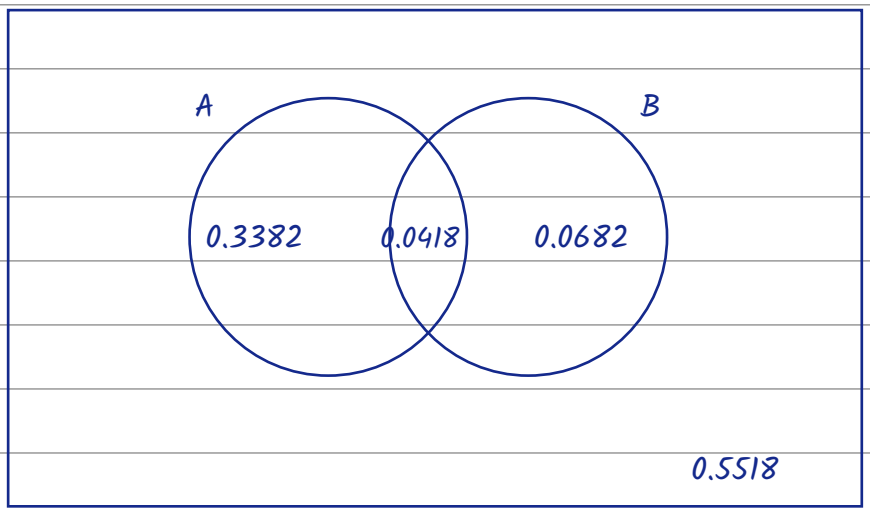
n^2 is a square number, so the sum of the first n positive odd numbers is a square number.

6 (a) $y = \sin 2x$

6 (b) $x = 0, \quad x = \frac{\pi}{2}, \quad x = \pi$

Question number

7 (a)



7 (b) $P(A \cap B) = 1 - 0.5518 - 0.3382 - 0.0682 = 0.0418$

$P(A) = 0.3382 + 0.0418 = 0.38$

$P(B) = 0.0682 + 0.0418 = 0.11$

$P(A) \times P(B) = 0.38 \times 0.11 = 0.0418 = P(A \cap B)$

so A and B are independent events.

7 (c) $P(A \cap B) = 0.0418 \neq 0$, so A and B are not mutually exclusive.

8 (a) Top 30 pupils: cf = 120 − 30 = 90

From the graph, cf = 90 at a mark of about 63

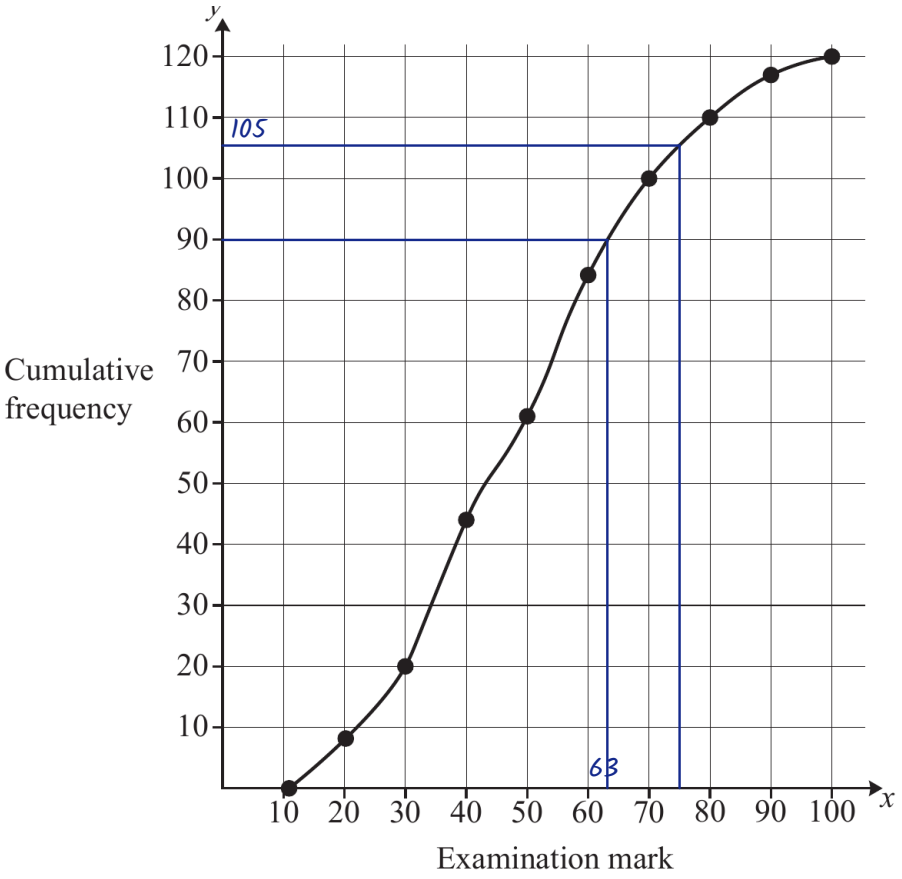
Lowest mark needed ≈ 63

8 (b) At a mark of 75, cf ≈ 105

Number in top set = 120 − 105 = 15 pupils

Question number

8 cont.



9 (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}$

9 (b) $2\overrightarrow{AC} = \overrightarrow{CB} \Rightarrow \overrightarrow{AC} = \frac{1}{3}\overrightarrow{AB}$

$\overrightarrow{OC} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$

10 (a) $L = 0.275 \times 1975 - 490 = 53.125$

Life expectancy ≈ 53.1 years

10 (b) Likely to be close: 1975 lies within the range of the data, so this is interpolation, and the points lie close to a straight line, so the linear model

Question number	
10 cont.	seems appropriate.
10 (c)	Not likely to be a close approximation: life expectancy at birth in the UK (Europe) is generally much higher than in Sudan (Africa).
10 (d)	Remove the row for Saint Helena, Ascension and Tristan da Cunha (the row with #N/A), as it has no life expectancy value.
10 (e)	Not reliable: a median age of 43.2 is outside the range of the median ages in the data (about 15 to 37), so this would be extrapolation.
11 (a)	Sam used a larger sample (50 packets compared with 25), so his data are likely to represent the true distribution better.
11 (b)	Mean $= \frac{0\times39 + 1\times1 + 2\times0 + 3\times3 + 4\times7}{50} = \frac{38}{50} = 0.76$
11 (c)	$np = 0.76 \Rightarrow p = \frac{0.76}{4} = 0.19$
11 (d)	$X \sim B(4, 0.19)$; expected frequency $= 50 \times P(X = x)$ $P(X = 0) = 0.81^4 = 0.430467\ldots \rightarrow 50 \times 0.430467 = 21.52$ $P(X = 1) = 4 \times 0.19 \times 0.81^3 = 0.403895\ldots \rightarrow 20.19$ $P(X = 2) = 6 \times 0.19^2 \times 0.81^2 = 0.142111\ldots \rightarrow 7.11$ $P(X = 3) = 4 \times 0.19^3 \times 0.81 = 0.022223\ldots \rightarrow 1.11$ $P(X = 4) = 0.19^4 = 0.001303\ldots \rightarrow 0.07$

Question number

11 cont.

x	0	1	2	3	4
P(X = x)	0.4305	0.4039	0.1421	0.0222	0.0013
Frequency	21.52	20.19	7.11	1.11	0.07

11 (e) Not a good fit: e.g. observed 39 packets with 0 bruised apples but expected 21.52, and observed 7 with 4 bruised but expected only 0.07. The model is a very poor fit.

12 $3\sec^2\theta + 2\tan\theta - 4 = 0$

$3(1 + \tan^2\theta) + 2\tan\theta - 4 = 0$

$3\tan^2\theta + 2\tan\theta - 1 = 0$

$(3\tan\theta - 1)(\tan\theta + 1) = 0$

$\tan\theta = \frac{1}{3}$ or $\tan\theta = -1$

$\tan\theta = \frac{1}{3}$: $\theta = 18.434...°$ or $18.434...° - 180° = -161.565...°$

$\tan\theta = -1$: $\theta = -45°$ or $-45° + 180° = 135°$

$\theta = -161.6°, -45°, 18.4°, 135°$

13 (a) H_0: $p = 0.494$ H_1: $p < 0.494$

where p is the proportion of people aged 16 or over in England and Wales who are married or in a civil partnership.

13 (b) Under H_0, $X \sim B(78, 0.494)$, where X is the number in the sample who are married or in a civil partnership

$P(X \leq 27) = 0.005946... < 0.01$

$P(X \leq 28) = 0.011178... > 0.01$

Critical region: $X \leq 27$

Question number	
13 (c)	$26 \leqslant 27$, so 26 lies in the critical region Reject H_0 (the result is significant) There is sufficient evidence at the 1% level to suggest that the proportion of people aged 16 or over who are married or in a civil partnership is lower in 2024 than in 2022.
13 (d)	$P(\text{reject } H_0 \text{ when } H_0 \text{ true}) = P(X \leqslant 27) = 0.005946\ldots$ $= 0.00595$ (3 s.f.)
14 (a)	$\frac{16}{x^2} + \frac{3}{x} = 0 \Rightarrow 16 + 3x = 0 \Rightarrow x = -\frac{16}{3}$ Curve cuts the x-axis at $\left(-\frac{16}{3}, 0\right)$
14 (b)(i)	$y = 16x^{-2} + 3x^{-1}$ $dy/dx = -2 \times 16x^{-3} - 3x^{-2} = -32x^{-3} - 3x^{-2} = -\frac{32}{x^3} - \frac{3}{x^2}$
14 (b)(ii)	$-\frac{32}{x^3} - \frac{3}{x^2} = 0 \Rightarrow -32 - 3x = 0 \Rightarrow x = -\frac{32}{3}$ $y = \frac{16}{(-32/3)^2} + \frac{3}{(-32/3)} = \frac{9}{64} - \frac{18}{64} = -\frac{9}{64}$ Stationary point is $\left(-\frac{32}{3}, -\frac{9}{64}\right)$
14 (c)(i)	$d^2y/dx^2 = 96x^{-4} + 6x^{-3} = \frac{96}{x^4} + \frac{6}{x^3}$
14 (c)(ii)	Concave downwards when $d^2y/dx^2 < 0$

Question number

14 cont. $\frac{96}{x^4} + \frac{6}{x^3} < 0 \quad \Rightarrow \quad (x\,x^4 > 0) \quad 96 + 6x < 0 \quad \Rightarrow \quad x < -16$

$\{x : x < -16\}$

14 (d) $\text{Area} = \int_{-4}^{-\frac{1}{2}} \left(\frac{16}{x^2} + \frac{3}{x} \right) dx = \left[-\frac{16}{x} + 3\ln|x| \right]_{-4}^{-\frac{1}{2}}$

$= \left(-\frac{16}{-\frac{1}{2}} + 3\ln\left|-\tfrac{1}{2}\right| \right) - \left(-\frac{16}{-4} + 3\ln|-4| \right)$

$= (32 + 3\ln \tfrac{1}{2}) - (4 + 3\ln 4)$

$= 32 - 3\ln 2 - 4 - 6\ln 2$

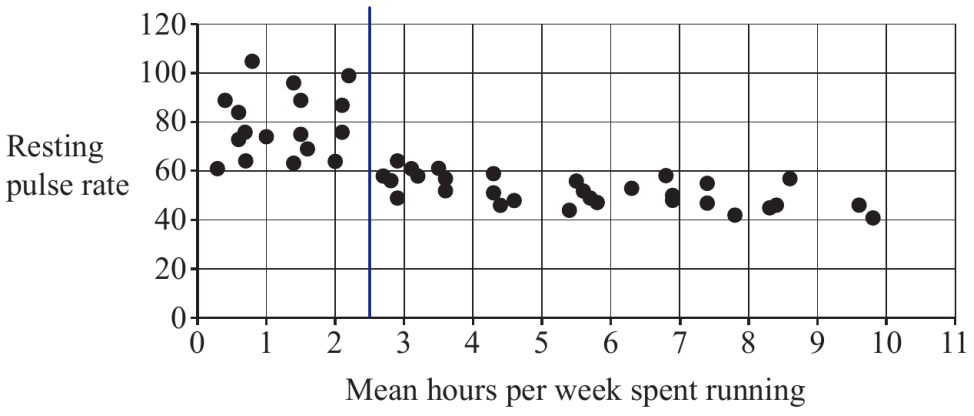
$= 28 - 9\ln 2$

15 (a)(i) Opportunity sampling

15 (a)(ii) Only his own clients are sampled; they are likely to be fitter than typical members of the general population, so the sample may not be representative.

15 (b) Negative association: as mean hours per week spent running increases, resting pulse rate tends to decrease.

15 (c)



Question number	
15 (d)	Left of the line (less than about 2.5 hours running per week): $r_s = 0.20936$ is small (close to 0) and the p-value 0.41998 is not close to 0 (it is greater than 0.05), so there is insufficient evidence to suggest any association between mean time spent running and resting pulse rate for this group. Most of these pulse rates are in the normal range 60 to 110. Right of the line (more than about 2.5 hours per week): $r_s = -0.61349$ is closer to -1 than to 0 and the p-value 0.00031 is very close to 0 (less than 0.01), so there is sufficient evidence to suggest a negative association between mean time spent running and resting pulse rate: those who run more tend to have lower resting pulse rates, many below the normal range.
16 (a)	$\int 1\, dP = \int (72t - 108)e^{-0.8t}\, dt$ $u = 72t - 108,\ dv/dt = e^{-0.8t} \Rightarrow du/dt = 72,\ v = \frac{e^{-0.8t}}{-0.8} = -1.25e^{-0.8t}$ $P = -1.25(72t - 108)e^{-0.8t} - \int 72 \times (-1.25e^{-0.8t})\, dt$ $= -90te^{-0.8t} + 135e^{-0.8t} + 90 \int e^{-0.8t}\, dt$ $= -90te^{-0.8t} + 135e^{-0.8t} + 90 \times \frac{e^{-0.8t}}{-0.8} + c$ $= -90te^{-0.8t} + 135e^{-0.8t} - 112.5e^{-0.8t} + c$ $P = -90te^{-0.8t} + 22.5e^{-0.8t} + c$ $t = 0,\ P = 58.5$: $58.5 = -90 \times 0 \times e^0 + 22.5e^0 + c$ $58.5 = 22.5 + c \Rightarrow c = 36$ $P = 36 + 22.5e^{-0.8t} - 90te^{-0.8t}$
16 (b)	As $t \to \infty$, $e^{-0.8t} \to 0$ and $te^{-0.8t} \to 0$ so $P \to 36 + 0 - 0 = 36$ $36 < 58.5$, so according to the model the population of puffins does not

Question number

16 cont. recover to 58.5 thousand; it tends to 36 thousand.

17 $\bar{x} = \frac{17367}{50} = 347.34$

$s^2 = \frac{6036597 - 50 \times 347.34^2}{49} = \frac{4343.22}{49} = 88.637142...$

$s = 9.414730...$

H_0: $\mu = 350$ H_1: $\mu < 350$

where μ is the population mean mass of honey in a jar (grams)

Under H_0: $\bar{X} \sim N(350, \frac{88.637...}{50}) = N(350, 1.772742...)$

$P(\bar{X} < 347.34) = P(Z < \frac{347.34 - 350}{\sqrt{1.772742...}}) = P(Z < -1.99783...)$

$= 0.022867... = 0.0229$ (3 s.f.)

$0.0229 > 0.01$ so the result is not significant: do not reject H_0

There is insufficient evidence at the 1% level to suggest that the mean mass of honey in a jar is less than 350 grams.