OCR
Oxford Cambridge and RSA

Wednesday 4 June 2025 – Afternoon

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours

185641861 2

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}, \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^{n}\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$

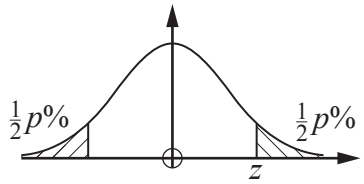
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A (20 marks)

1 **(a)** Sketch the function $y = |2x - 3|$. **[2]**

(b) **In this question you must show detailed reasoning.**

Solve the equation $|2x - 3| = 4 - x$. **[3]**

2 Express $\dfrac{7x - 25}{(x-1)(x-4)^2}$ in partial fractions. **[4]**

3 **(a)** Evaluate

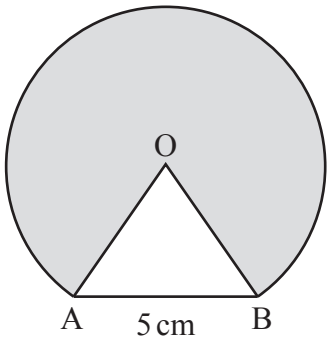
$$\sum_{r=1}^{4} \frac{1}{r}$$

giving your answer as a fraction in its lowest terms. **[1]**

(b) Write the sum $1 + 3 + 5 + 7 + 9$ in a similar way to the series in part **(a)**. **[2]**

(c) Explain why the sum to infinity of $1 + 3 + 5 + 7 + 9 + \ldots$ is **not** well defined. **[1]**

4 The diagram shows part of a circle with centre O and radius 5 cm. The circle passes through the points A and B. The length AB is 5 cm.



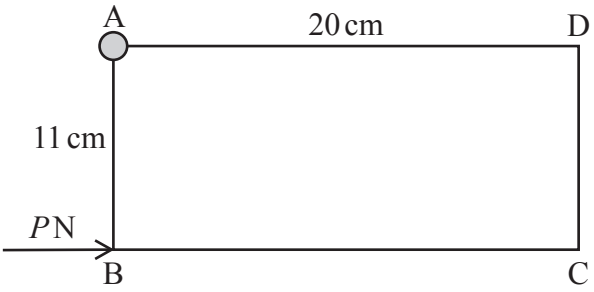
Calculate the area of the shaded region. **[4]**

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5 A uniform rectangular lamina ABCD has a mass of 0.12 kg. Length AB is 11 cm and length AD is 20 cm.

The lamina ABCD is held by a smooth hinge at A.

A horizontal force of magnitude P N is applied at B so that the lamina is in equilibrium in a vertical plane with AD horizontal as shown in the diagram.

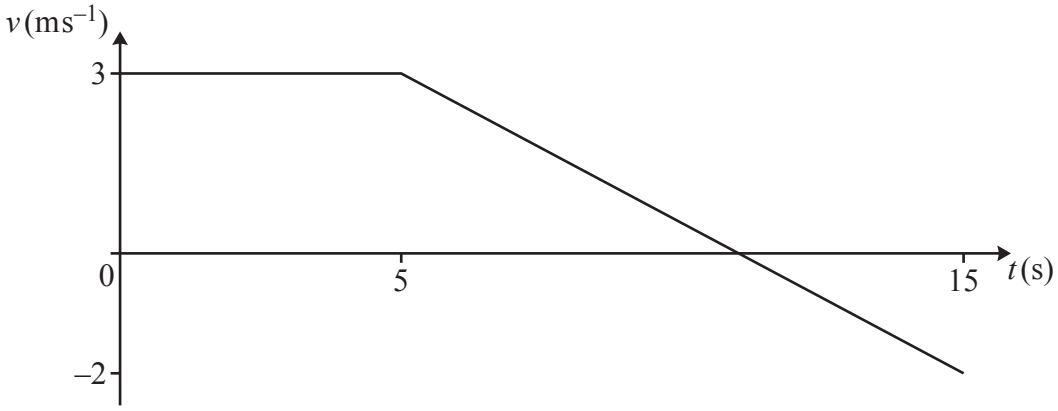


Show that P is 1.07 to **3** significant figures. **[3]**

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Section B (80 marks)

6 A car is travelling along a straight horizontal road. The car's velocity $v\,\text{m s}^{-1}$ at time $t\,\text{s}$ is shown in the velocity-time graph below. The points (0, 3) and (5, 3) are joined with a line segment, and the points (5, 3) and (15, −2) are joined with another line segment.



(a) Calculate the total distance travelled by the car in the first 15 s. **[3]**

The car is then attached to a caravan by means of a light inextensible horizontal tow bar and continues travelling along the road. You are given the following information.

- The car and caravan accelerate at $1.5\,\text{m s}^{-2}$.
- The mass of the car is 1400 kg and the mass of the caravan is 900 kg.
- The driving force acting on the car is D N and the tension in the tow bar is T N.
- The resistances to motion acting on the car and caravan are 400 N and 450 N respectively.

(b) Write down the equations of motion for the car and the caravan separately. **[2]**

(c) Calculate the values of D and T. **[3]**

7 **In this question you must show detailed reasoning.**

Show that the area of the finite region enclosed by the curves $y = x^2 - 7x + 2$ and $y = 14 - 9x - x^2$ is $\frac{125}{3}$. **[7]**

8 **(a)** Determine the **two** values of k for which the line $y = 3x + 5$ is a tangent to the curve $y = k - kx - x^2$. **[4]**

(b) These values of k are used to define two curves of the form $y = k - kx - x^2$.

Determine the coordinates of the point of intersection of these two curves. **[3]**

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9 The table below shows information about four of the moons of the planet Jupiter. The semi-major axis is the greatest distance that each moon reaches from the centre of its orbit around Jupiter. The orbital period is the time in Earth days that it takes to orbit the planet.

Moon	Semi-major axis (km)	Orbital period (Earth days)
Io	421 800	1.7627
Europa	671 100	3.5255
Ganymede	1 070 400	7.1556
Callisto	1 882 700	16.690

A student uses the equation $T = kd^n$ to model the orbital period of Jupiter's moons, where T is the orbital period in Earth days, d is the semi-major axis in km, and k and n are constants.

(a) Show that the equation $T = kd^n$ can be rewritten in the form $\log_{10} T = \log_{10} k + n \log_{10} d$. **[1]**

The student uses graph drawing software to plot $\log_{10} T$ against $\log_{10} d$. They find that the line of best fit for the data has gradient 1.504 and intercepts the $\log_{10} T$ axis at -8.215.

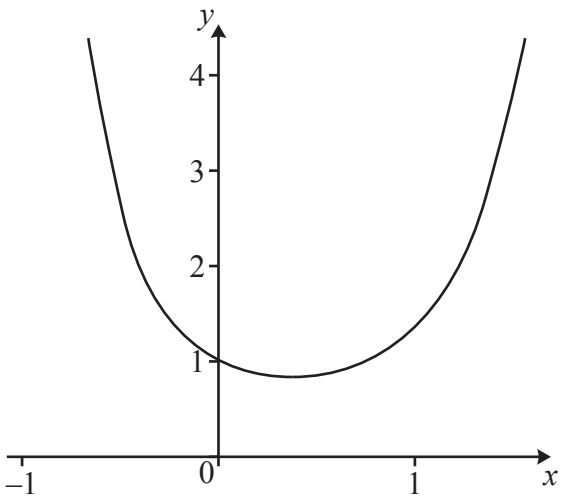
(b) Determine values of k and n that are consistent with this information. **[3]**

The student uses their equation to predict the orbital period for another moon of Jupiter called Thebe which has a semi-major axis of 221 900 km. They look up the value in an online encyclopedia and find it is 0.6761 Earth days.

(c) Comment on the suitability of the student's equation to model the orbital period of Thebe. **[2]**

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10 The diagram shows part of the graph of the function $y = \dfrac{e^{x^2}}{x+1}$ which is defined for $x > -1$.



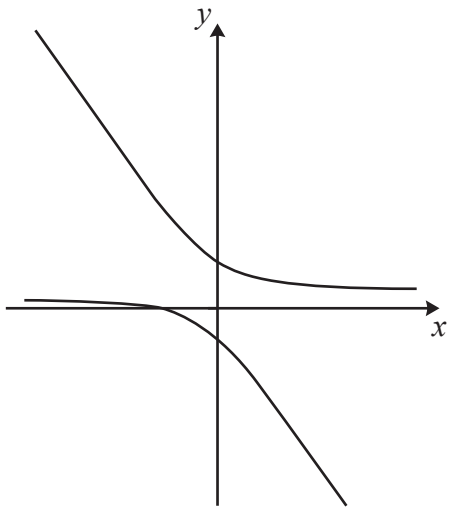
(a) Find an expression for $\dfrac{dy}{dx}$. **[3]**

(b) Determine the range of values of x where the gradient of the function is negative. **[3]**

(c) Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the axes and the line $x = 1$. **[3]**

(d) Determine whether the estimate in part **(c)** is an under- or over-estimate. Give a reason for your answer. **[1]**

11 The curve in the graph below is defined implicitly by $(2x+y)(y-1) = 6$.



(a) Show that $\dfrac{dy}{dx} = \dfrac{2(1-y)}{2y+2x-1}$. **[4]**

(b) Find the equation of the normal to the curve at the point (2, 2). **[3]**

(c) Show that there is **no** point on the curve at which the tangent is parallel to the x-axis. **[3]**

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12 In this question x and y are the horizontal and upwards vertical directions respectively.

An astronaut is standing on the surface of the moon exploring the motion of a ball.

(a) The astronaut drops a ball from rest from 1 m above the surface. It takes 1.1 s to hit the surface.

Calculate the value of the acceleration due to the moon's gravity. Give your answer correct to **3** significant figures. **[2]**

The astronaut stands in a crater of the moon and hits the ball with a golf club from the moon's surface. The initial velocity of the ball is $25\,\text{m}\,\text{s}^{-1}$ at an angle of 40° above the horizontal in the x-direction.

(b) Taking the origin to be the point of projection, determine the equation of the trajectory of the ball. Give your answer in the form $y = \text{f}(x)$, with each of the coefficients correct to **3** significant figures. **[4]**

(c) The edge of the crater is 40 m away from the point of projection and 15 m above it.

Determine whether the ball goes over the crater's edge. **[2]**

13 The displacement $\mathbf{r}$ m of a parachutist t s after opening their parachute is modelled by

$$\mathbf{r} = \begin{pmatrix} 50t \\ 280 + 5t - 280\text{e}^{-0.16t} \end{pmatrix}$$

where the x-direction is horizontal and the y-direction is vertically **downwards**.

(a) Calculate the **distance** from the parachutist's initial position that the model predicts after 10 s. **[3]**

(b) Find a vector expression for the velocity of the parachutist according to the model. **[3]**

(c) Determine what velocity the model predicts for large values of t. **[2]**

(d) Parachutists usually land travelling approximately vertically.

Explain a factor that should be included in the model to better reflect this. **[1]**

Turn over for question 14

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14 In this question the **i** and **j** vectors are horizontal and vertically upward respectively.

A particle of mass 5 kg is at rest on a rough horizontal shelf. The coefficient of friction between the particle and the shelf is μ.

(a) A force $\mathbf{P} = 9\mathbf{i} + 20\mathbf{j}$ N acts on the particle. The particle is on the point of sliding along the shelf.

Determine the value of μ. **[5]**

The force **P** is removed. One end of the shelf is lifted so that it is inclined at $\alpha°$ to the horizontal.

(b) The particle is on the point of sliding **down** the shelf.

Show that $\alpha = 17.2$ to **3** significant figures. **[4]**

(c) The particle is projected **up** the shelf with an initial speed of $5\,\text{m}\,\text{s}^{-1}$.

Given that the particle remains in contact with the shelf, determine the time after projection at which the particle first comes to rest. **[6]**

END OF QUESTION PAPER

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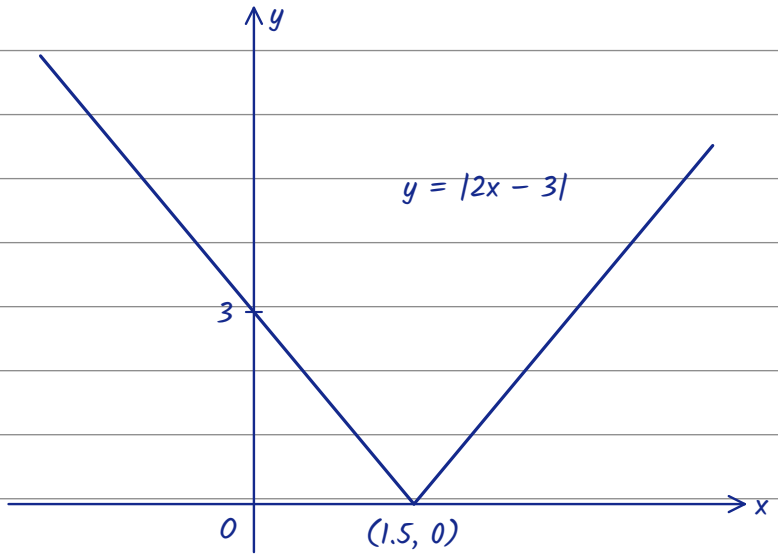
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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)



1 (b)

$2x - 3 = 4 - x$ or $2x - 3 = -(4 - x)$

$3x = 4 + 3$ or $2x - 3 = x - 4$

$x = \frac{7}{3}$ or $x = -1$

Both check: $4 - x > 0$ for each, so $x = 7/3$ or $x = -1$

2

$$\frac{7x - 25}{(x - 1)(x - 4)^2} \equiv \frac{A}{x - 1} + \frac{B}{x - 4} + \frac{C}{(x - 4)^2}$$

$7x - 25 \equiv A(x - 4)^2 + B(x - 1)(x - 4) + C(x - 1)$

$x = 4$: $28 - 25 = 3C \Rightarrow C = 1$

$x = 1$: $7 - 25 = 9A \Rightarrow -18 = 9A \Rightarrow A = -2$

Coefficients of x^2: $0 = A + B \Rightarrow B = 2$

$$\frac{7x - 25}{(x - 1)(x - 4)^2} = \frac{-2}{x - 1} + \frac{2}{x - 4} + \frac{1}{(x - 4)^2}$$

Question number

3 (a) $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{12 + 6 + 4 + 3}{12} = \frac{25}{12}$

3 (b) $1 + 3 + 5 + 7 + 9 = \sum_{r=1}^{5} (2r - 1)$

3 (c) The terms keep increasing, so the sum of n terms forms a divergent sequence: it does not converge to a finite value, so there is no well defined sum to infinity.

4 OA = OB = AB = 5 cm, so triangle AOB is equilateral

$\therefore$ angle AOB = 60° = π/3

Angle of major sector = 2π − π/3 = 5π/3

$\text{Area} = \frac{1}{2} r^2\theta = \frac{1}{2} \times 5^2 \times \frac{5\pi}{3} = \frac{125\pi}{6}$

$= 65.449... = 65.4 \text{ cm}^2$ (3 s.f.)

5 Weight 0.12g N acts at the centre, 10 cm = 0.1 m horizontally from A.

P acts horizontally at B, perpendicular distance AB = 0.11 m from A.

Moments about A: $0.12g \times 0.1 = P \times 0.11$

$0.1176 = 0.11P$

$P = 0.1176 \div 0.11 = 1.06909...$

$\therefore$ P = 1.07 (3 s.f.) as required

6 (a) Gradient of second segment = (−2 − 3)/(15 − 5) = −0.5

v = 0 when $3 - 0.5(t - 5) = 0 \Rightarrow t = 11$ s

$\text{Distance} = 5 \times 3 + \frac{1}{2} \times 6 \times 3 + \frac{1}{2} \times 4 \times 2$

$= 15 + 9 + 4 = 28$ m

Question number

6 (b)

Car: $D - T - 400 = 1400 \times 1.5$

Caravan: $T - 450 = 900 \times 1.5$

6 (c)

Caravan: $T - 450 = 1350 \Rightarrow T = 1800$

Adding: $D - 850 = 2300 \times 1.5 = 3450$

$D = 4300$

$D = 4300$ N, $T = 1800$ N

7

Intersect: $x^2 - 7x + 2 = 14 - 9x - x^2$

$2x^2 + 2x - 12 = 0 \Rightarrow x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0 \Rightarrow x = -3, \ x = 2$

Between these, $14 - 9x - x^2$ is above $x^2 - 7x + 2$ (e.g. $x = 0$: $14 > 2$)

Area $= \int_{-3}^{2} ((14 - 9x - x^2) - (x^2 - 7x + 2))\, dx$

$= \int_{-3}^{2} (12 - 2x - 2x^2)\, dx$

$= \left[12x - x^2 - \frac{2x^3}{3} \right]_{-3}^{2}$

$= \left(24 - 4 - \frac{16}{3} \right) - (-36 - 9 + 18)$

$= \frac{44}{3} - (-27) = \frac{44}{3} + \frac{81}{3} = \frac{125}{3}$ as required

8 (a)

$3x + 5 = k - kx - x^2$

$x^2 + (k + 3)x + (5 - k) = 0$

Tangent $\Rightarrow$ equal roots $\Rightarrow$ discriminant $b^2 - 4ac = 0$

$(k + 3)^2 - 4(5 - k) = 0$

$k^2 + 6k + 9 - 20 + 4k = 0 \Rightarrow k^2 + 10k - 11 = 0$

$(k + 11)(k - 1) = 0 \Rightarrow k = 1$ or $k = -11$

Question number

8 (b)

$k = 1$: $y = 1 - x - x^2$ $\qquad$ $k = -11$: $y = -11 + 11x - x^2$

$1 - x - x^2 = -11 + 11x - x^2$

$12 = 12x \quad \Rightarrow \quad x = 1$

$y = 1 - 1 - 1 = -1$

Point of intersection: $(1, -1)$

9 (a)

$T = kd^n \quad \Rightarrow \quad \log_{10} T = \log_{10}(kd^n) = \log_{10} k + \log_{10} d^n$

$\therefore \quad \log_{10} T = \log_{10} k + n \log_{10} d$

9 (b)

Comparing with $Y = c + mX$: $\quad n$ = gradient $= 1.504$

$\log_{10} k$ = intercept $= -8.215$

$k = 10^{-8.215} = 6.0953... \times 10^{-9}$

$k = 6.10 \times 10^{-9}$ (3 s.f.), $\quad n = 1.504$

9 (c)

Model: $T = 6.095 \times 10^{-9} \times 221900^{1.504} = 0.6692... = 0.669$ days

This is close to the actual value of 0.6761 days (about 1% out),

so the model is suitable for predicting the orbital period of Thebe.

10 (a)

$u = e^{x^2}$, $\quad du/dx = 2xe^{x^2}$; $\qquad v = x + 1$, $\quad dv/dx = 1$

$$\frac{dy}{dx} = \frac{(x+1) \times 2xe^{x^2} - e^{x^2} \times 1}{(x+1)^2} = \frac{e^{x^2}(2x^2 + 2x - 1)}{(x+1)^2}$$

10 (b)

$e^{x^2} > 0$ and $(x+1)^2 > 0$, so $dy/dx < 0$ when $2x^2 + 2x - 1 < 0$

$$2x^2 + 2x - 1 = 0 \quad \Rightarrow \quad x = \frac{-2 \pm \sqrt{12}}{4} = \frac{-1 \pm \sqrt{3}}{2} \quad (= 0.366 \text{ or } -1.37)$$

Quadratic is negative between its roots, and $x > -1$, so

$$-1 < x < \frac{-1 + \sqrt{3}}{2} \qquad (\text{i.e. } -1 < x < 0.366)$$

Question number	
10 (c)	$h = 0.25$: x: 0, 0.25, 0.5, 0.75, 1
	y: 1, 0.851596, 0.856017, 1.002888, 1.359141
	Area $\approx \frac{0.25}{2}\,[\,1 + 1.359141 + 2(0.851596 + 0.856017 + 1.002888)\,]$
	$= 0.125 \times 7.780143 = 0.97252\ldots \approx 0.973$
10 (d)	Over-estimate: the curve is concave upwards (gradient increasing) on $0 \leqslant x \leqslant 1$, so the tops of the trapeziums lie above the curve and the trapeziums include extra area that is not in the region.
11 (a)	Differentiate $(2x + y)(y - 1) = 6$ implicitly (product rule):
	$(2 + \frac{dy}{dx})(y - 1) + (2x + y)\frac{dy}{dx} = 0$
	$2y - 2 + (y - 1)\frac{dy}{dx} + (2x + y)\frac{dy}{dx} = 0$
	$(2y + 2x - 1)\frac{dy}{dx} = 2 - 2y$
	$\therefore \frac{dy}{dx} = \frac{2(1 - y)}{2y + 2x - 1}$ as required
11 (b)	At (2, 2): $\frac{dy}{dx} = \frac{2(1 - 2)}{4 + 4 - 1} = -\frac{2}{7}$
	Gradient of normal $= \frac{7}{2}$
	$y - 2 = \frac{7}{2}(x - 2) \quad \Rightarrow \quad y = 3.5x - 5$
11 (c)	Tangent parallel to x-axis $\Rightarrow \frac{dy}{dx} = \frac{2(1 - y)}{2y + 2x - 1} = 0$

Question number

$\Rightarrow \quad 2 - 2y = 0 \quad \Rightarrow \quad y = 1$

Substitute $y = 1$ into the curve: $(2x + 1)(1 - 1) = 6$

$\Rightarrow \quad (2x + 1) \times 0 = 6 \quad \Rightarrow \quad 0 = 6$, which is impossible.

$\therefore$ there is no point on the curve where the tangent is parallel to the x-axis.

12 (a) $s = ut + \frac{1}{2}at^2$ with $s = -1$, $u = 0$, $t = 1.1$

$-1 = \frac{1}{2}a(1.1)^2 \quad \Rightarrow \quad a = -\frac{2}{1.21} = -1.65289...$

Acceleration due to gravity = 1.65 m s^{-2} (3 s.f.) downwards $(a = -1.65)$

12 (b) Horizontal: $x = 25\cos 40° \, t = 19.151t \quad \Rightarrow \quad t = x / (25\cos 40°)$

Vertical: $y = 25\sin 40° \, t + \frac{1}{2}at^2$ with $a = -1.65289$

$$y = 25\sin 40° \frac{x}{25\cos 40°} - \frac{1}{2} \times 1.65289 \frac{x^2}{(25\cos 40°)^2}$$

$y = 0.83909... \, x - 0.0022533... \, x^2$

$y = 0.839x - 0.00225x^2$

12 (c) When $x = 40$: $y = 0.839 \times 40 - 0.00225 \times 40^2 = 33.56 - 3.6 = 29.96$

$y = 30.0$ m > 15 m

So the ball goes over the crater's edge.

13 (a) When $t = 10$: $x = 50 \times 10 = 500$

$y = 280 + 50 - 280e^{-1.6} = 330 - 56.531... = 273.469...$

Distance $= |r| = \sqrt{(500^2 + 273.469^2)}$

$= \sqrt{324785.3} = 569.899... = 570$ m (569.9 m)

Question number

13 (b) $v = \frac{dr}{dt} = (50,\ 5 + 44.8e^{-0.16t})$ $(280 \times 0.16 = 44.8)$

i.e. $v = 50i + (5 + 44.8e^{-0.16t})j$ m s^{-1}

13 (c) For large t, $e^{-0.16t} \to 0$

So $v \to 50i + 5j$ m s^{-1} (50 m s^{-1} horizontally, 5 m s^{-1} vertically downwards)

13 (d) The model should include a resistance force in the horizontal (x) direction, so that the horizontal component of velocity decreases towards zero.

14 (a) Forces: weight $-5g\,j$, reaction $R\,j$, friction F (opposing), $P = 9i + 20j$

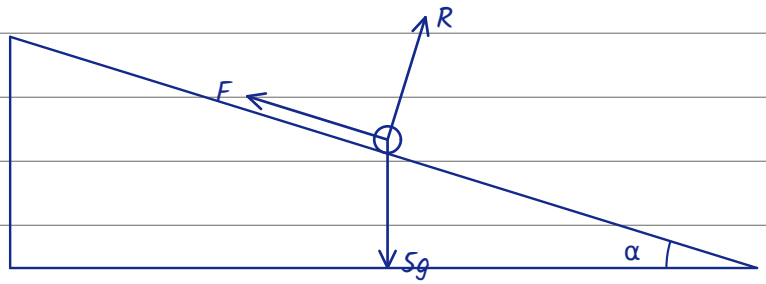
Horizontal: $F = 9$

Vertical: $R + 20 - 5g = 0 \Rightarrow R = 5g - 20 = 49 - 20 = 29$

On the point of sliding: $F = \mu R \Rightarrow 9 = 29\mu$

$\mu = \frac{9}{29} = 0.31034\ldots = 0.310$ (3 s.f.)

14 (b)



Resolve perpendicular to slope: $R = 5g\cos\alpha$

Resolve parallel to slope: $F = 5g\sin\alpha$ (friction acts up the slope)

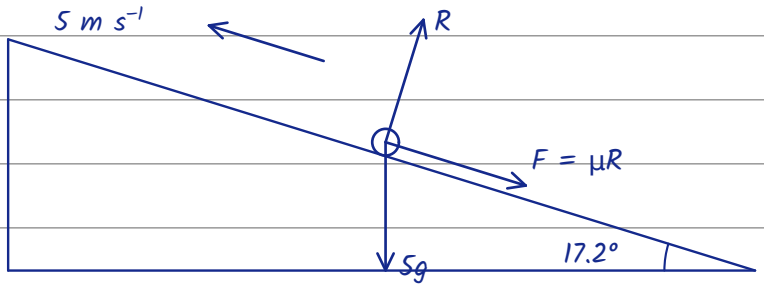
Limiting friction: $F = \mu R \Rightarrow 5g\sin\alpha = \mu \times 5g\cos\alpha$

$\tan\alpha = \mu = \frac{9}{29} \Rightarrow \alpha = \tan^{-1}\frac{9}{29} = 17.241\ldots$

$\therefore\ \alpha = 17.2°$ (3 s.f.) as required

Question number

14 (c)



Up the slope positive. $R = 5g\cos 17.2°$, friction $F = \mu R$ acts down the slope.

N2L up the slope: $-(5g\sin 17.2° + \mu \times 5g\cos 17.2°) = 5a$

$-(14.4897 + \frac{9}{29} \times 46.8086) = 5a \Rightarrow -(14.4897 + 14.5268) = 5a$

$a = -29.0165 \div 5 = -5.8033...\ m\ s^{-2}$

$v = u + at$: $0 = 5 - 5.8033t$

$t = 5 \div 5.8033 = 0.86157... = 0.862\ s$