



Oxford Cambridge and RSA

Tuesday 21 June 2022 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- the Insert
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

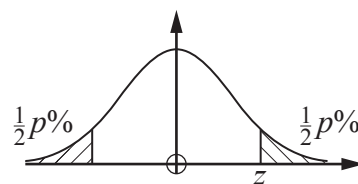
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

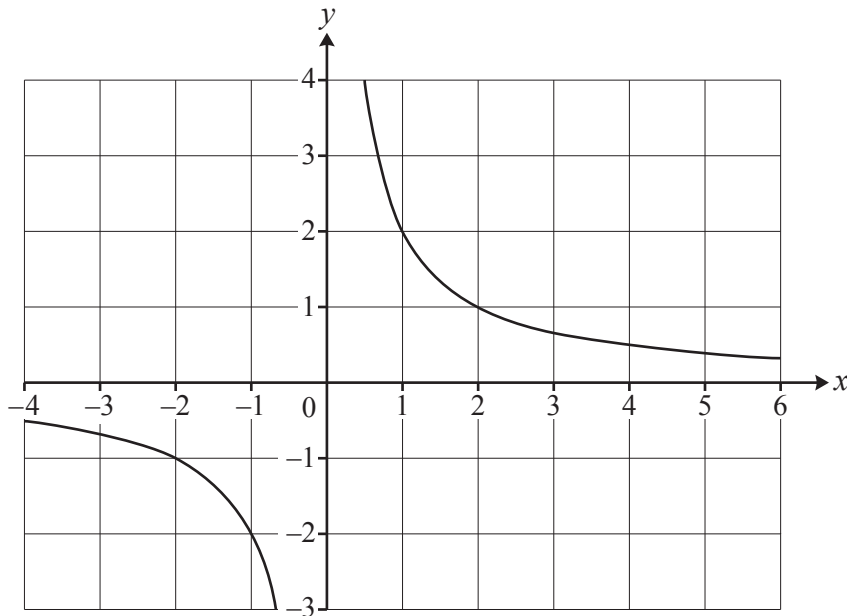
$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

Section A (60 marks)

- 1 A curve for which y is inversely proportional to x is shown below.



Find the equation of the curve.

[2]

- 2 The function $f(x) = \sqrt{x}$ is defined on the domain $x \geq 0$.

The function $g(x) = 25 - x^2$ is defined on the domain $\mathbb{R}$.

(a) Write down an expression for $fg(x)$.

[1]

(b) (i) Find the domain of $fg(x)$.

[3]

(ii) Find the range of $fg(x)$.

[2]

- 3 An infinite sequence $a_1, a_2, a_3, \dots$ is defined by $a_n = \frac{n}{n+1}$, for all positive integers n .

(a) Find the limit of the sequence.

[1]

(b) Prove that this is an increasing sequence.

[3]

4 In this question you must show detailed reasoning.

Determine the exact solutions of the equation $2 \cos^2 x = 3 \sin x$ for $0 \leq x \leq 2\pi$. [5]

5 A curve is defined implicitly by the equation $2x^2 + 3xy + y^2 + 2 = 0$.

(a) Show that $\frac{dy}{dx} = -\frac{4x+3y}{3x+2y}$. [3]

(b) **In this question you must show detailed reasoning.**

Find the coordinates of the stationary points of the curve. [4]

6 A hot drink is cooling. The temperature of the drink at time t minutes is $T^\circ\text{C}$.

The rate of decrease in temperature of the drink is proportional to $(T - 20)$.

(a) Write down a differential equation to describe the temperature of the drink as a function of time. [2]

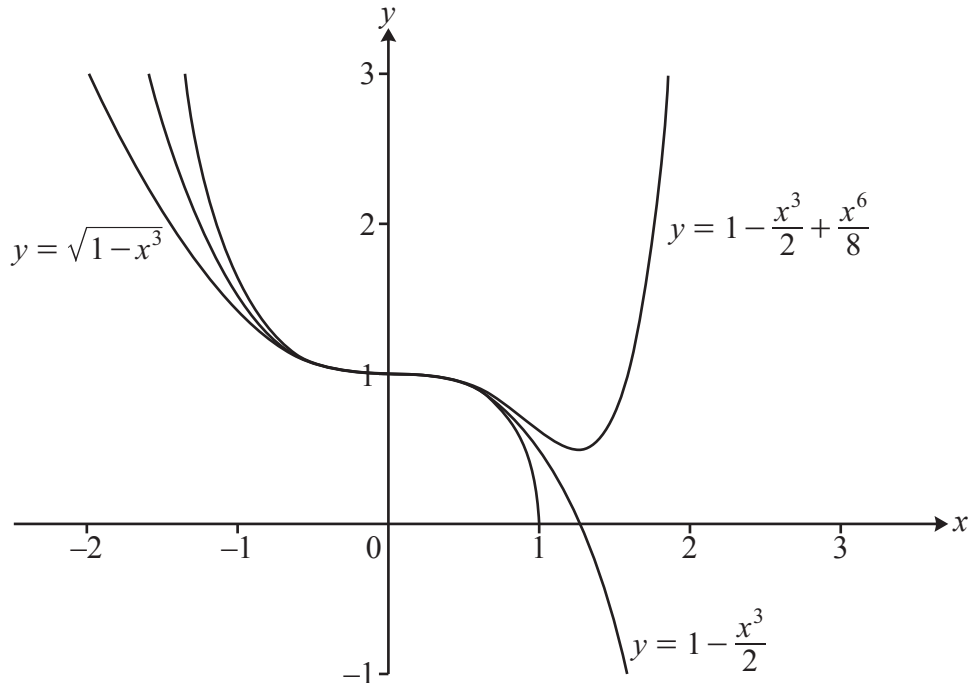
(b) When $t = 0$, the temperature of the drink is 90°C and the temperature is decreasing at a rate of 4.9°C per minute.

Determine how long it takes for the drink to cool from 90°C to 40°C . [6]

7 A student is trying to find the binomial expansion of $\sqrt{1-x^3}$.

She gets the first three terms as $1 - \frac{x^3}{2} + \frac{x^6}{8}$.

She draws the graphs of the curves $y = \sqrt{1-x^3}$, $y = 1 - \frac{x^3}{2}$ and $y = 1 - \frac{x^3}{2} + \frac{x^6}{8}$ using software.



- (a) Explain why $1 - \frac{x^3}{2} + \frac{x^6}{8} \geq 1 - \frac{x^3}{2}$ for all values of x . [1]
- (b) Explain why the graphs suggest that the student has made a mistake in the binomial expansion. [1]
- (c) Find the first **four** terms in the binomial expansion of $\sqrt{1-x^3}$. [3]
- (d) State the set of values of x for which the binomial expansion in part (c) is valid. [1]
- (e) Sketch the curve $y = 2.5\sqrt{1-x^3}$ on the grid in the Printed Answer Booklet. [2]
- (f) **In this question you must show detailed reasoning.**

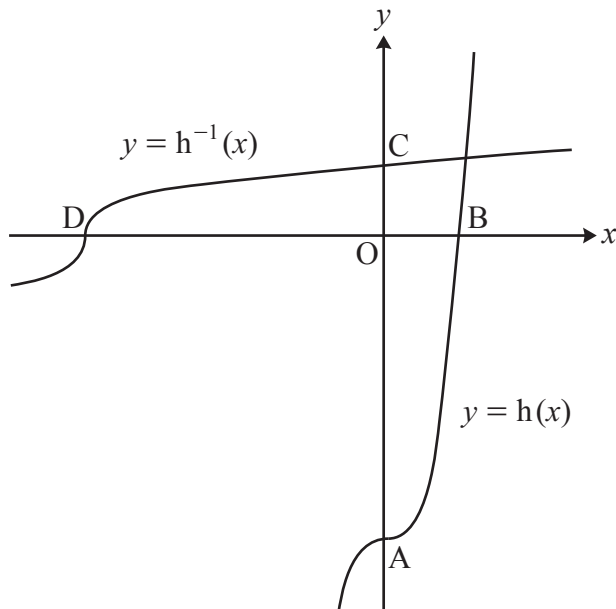
The end of a bus shelter is modelled by the area between the curve $y = 2.5\sqrt{1-x^3}$, the lines $x = -0.75$, $x = 0.75$ and the x -axis. Lengths are in metres.

Calculate, using your answer to part (c), an approximation for the area of the end of the bus shelter as given by this model. [4]

- 8 The curves $y = h(x)$ and $y = h^{-1}(x)$, where $h(x) = x^3 - 8$, are shown below.

The curve $y = h(x)$ crosses the x -axis at B and the y -axis at A.

The curve $y = h^{-1}(x)$ crosses the x -axis at D and the y -axis at C.



- (a) Find an expression for $h^{-1}(x)$. [2]
- (b) Determine the coordinates of A, B, C and D. [5]
- (c) Determine the equation of the perpendicular bisector of AB. Give your answer in the form $y = mx + c$, where m and c are constants to be determined. [4]
- (d) Points A, B, C and D lie on a circle.

Determine the equation of the circle. Give your answer in the form $(x - a)^2 + (y - b)^2 = r^2$, where a , b and r^2 are constants to be determined. [5]

Answer **all** the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

9 Show that $y = x$ has the same gradient as $y = \sin x$ when $x = 0$, as stated in line 5. [2]

10 In this question you must show detailed reasoning.

Fig. C2.2 indicates that the curve $y = \frac{4x(\pi - x)}{\pi^2} - \sin x$ has a stationary point near $x = 3$.

- Verify that the x -coordinate of this stationary point is between 2.6 and 2.7.
- Show that this stationary point is a maximum turning point. [5]

11 Show that, for the angle 45° , the formula $\sin \theta \approx \frac{4\theta(180 - \theta)}{40500 - \theta(180 - \theta)}$ given in line 28 gives the same approximation for the sine of the angle as the formula $\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$ given in line 23. [3]

12 (a) Show that $\cos x = \sin\left(x + \frac{\pi}{2}\right)$. [2]

(b) Hence show that $\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$ gives the approximation $\cos x \approx \frac{\pi^2 - 4x^2}{\pi^2 + x^2}$, as stated in line 31. [3]

END OF QUESTION PAPER

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