OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics
Question Paper

Wednesday 6 June 2018 – Morning
Time allowed: 2 hours

7020513453

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

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Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h\to 0}\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

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Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \,|\, A) = \mathrm{P}(B)\mathrm{P}(A \,|\, B)$ **or** $\mathrm{P}(A \,|\, B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

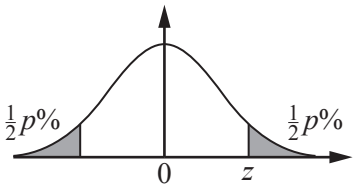
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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Answer **all** the questions

Section A (23 marks)

1 Show that $(x-2)$ is a factor of $3x^3 - 8x^2 + 3x + 2$. **[3]**

2 By considering a change of sign, show that the equation $e^x - 5x^3 = 0$ has a root between 0 and 1. **[2]**

3 **In this question you must show detailed reasoning.**

Solve the equation $\sec^2\theta + 2\tan\theta = 4$ for $0° \leqslant \theta < 360°$. **[4]**

4 Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to $1.2\,\text{m}\,\text{s}^{-1}$ as it travels 1.8 m.

(i) Calculate the acceleration of the box. **[2]**

(ii) Find the magnitude of the force that Rory applies. **[2]**

5 The position vector **r** metres of a particle at time t seconds is given by

$$\mathbf{r} = (1 + 12t - 2t^2)\mathbf{i} + (t^2 - 6t)\mathbf{j}.$$

(i) Find an expression for the velocity of the particle at time t. **[2]**

(ii) Determine whether the particle is ever stationary. **[2]**

6 Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

(i) Calculate how much she saves in two years. **[2]**

Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

(ii) Explain why the amounts Baraka saves each month form a geometric sequence. **[1]**

(iii) Determine whether Baraka saves more in two years than Aleela. **[3]**

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Answer **all** the questions

Section B (77 marks)

7 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force F N applied x m below the top of the rod as shown in Fig. 7.

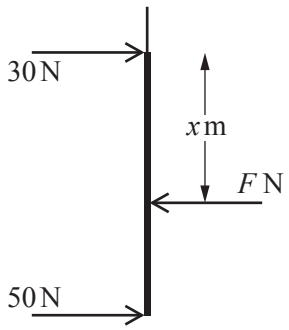


Fig. 7

(i) Find the value of F. **[1]**

(ii) Find the value of x. **[2]**

8 **(i)** Show that $8\sin^2 x\cos^2 x$ can be written as $1-\cos 4x$. **[3]**

(ii) Hence find $\int \sin^2 x\cos^2 x\,\mathrm{d}x$. **[3]**

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9 A pebble is thrown horizontally at $14\,\text{m s}^{-1}$ from a window which is 5 m above horizontal ground. The pebble goes over a fence 2 m high d m away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the x-axis horizontal in the direction in which the pebble is thrown and the y-axis vertically upwards.

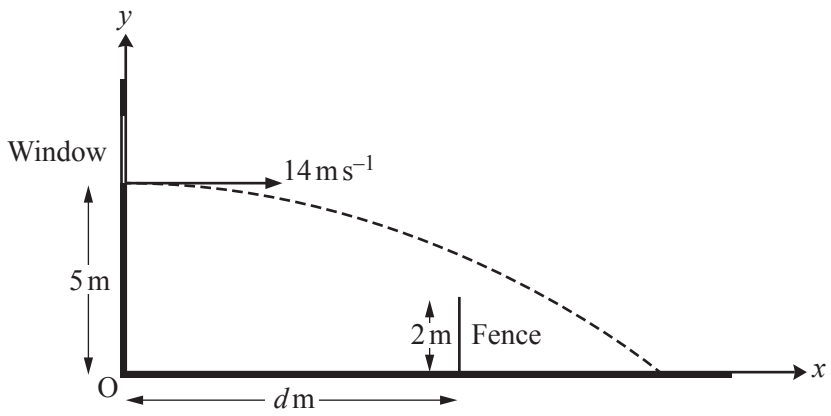


Fig. 9

(i) Find the time the pebble takes to reach the ground. **[3]**

(ii) Find the cartesian equation of the trajectory of the pebble. **[4]**

(iii) Find the range of possible values for d. **[3]**

10 Fig. 10 shows the graph of $y = (k-x)\ln x$ where k is a constant ($k > 1$).

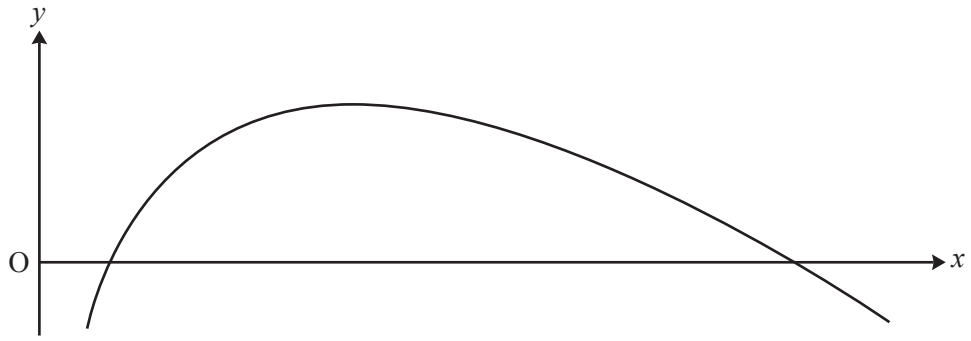


Fig. 10

Find, in terms of k, the area of the finite region between the curve and the x-axis. **[8]**

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11 Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

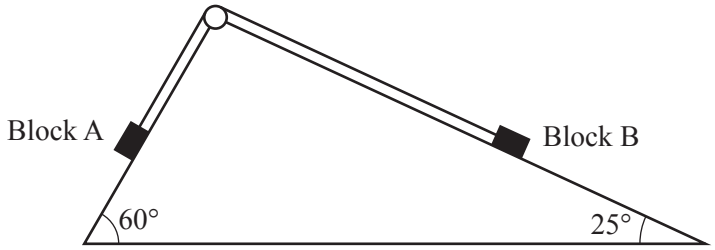


Fig. 11

(i) Show that the tension in the string is 39.9 N correct to 3 significant figures. **[2]**

(ii) Find the coefficient of friction between the rough plane and Block B. **[5]**

12 Fig. 12 shows the circle $(x-1)^2+(y+1)^2=25$, the line $4y=3x-32$ and the tangent to the circle at the point A (5, 2). D is the point of intersection of the line $4y=3x-32$ and the tangent at A.

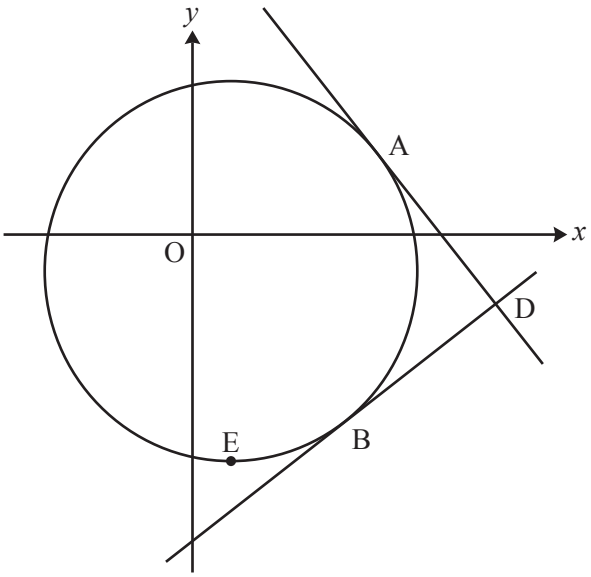


Fig. 12

(i) Write down the coordinates of C, the centre of the circle. **[1]**

(ii) (*A*) Show that the line $4y=3x-32$ is a tangent to the circle. **[4]**

(*B*) Find the coordinates of B, the point where the line $4y=3x-32$ touches the circle. **[1]**

(iii) Prove that ADBC is a square. **[3]**

(iv) The point E is the lowest point on the circle. Find the area of the sector ECB. **[5]**

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13 The function $f(x)$ is defined by $f(x) = \sqrt[3]{27-8x^3}$. Jenny uses her scientific calculator to create a table of values for $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(i) Use calculus to find an expression for $f'(x)$ and hence explain why the calculator gives an error for $f'(1.5)$. **[3]**

(ii) Find the first three terms of the binomial expansion of $f(x)$. **[3]**

(iii) Jenny integrates the first three terms of the binomial expansion of $f(x)$ to estimate the value of $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) **[2]**

(iv) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. **[3]**

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. The graph of $y = f(x)$ is shown in Fig. 13.

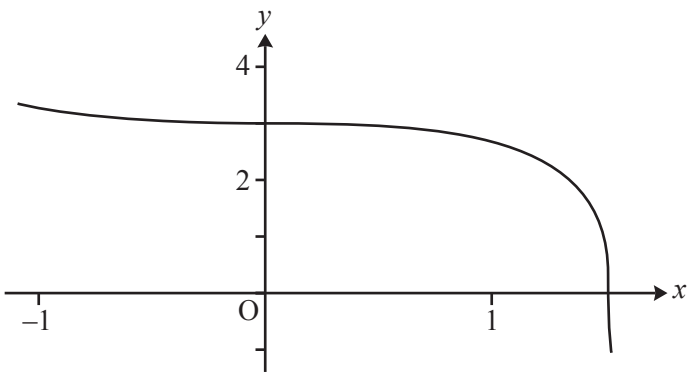


Fig. 13

(v) Explain why the trapezium rule gives an underestimate. **[1]**

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14 The velocity of a car, $v\,\text{m}\,\text{s}^{-1}$ at time t seconds, is being modelled. Initially the car has velocity $5\,\text{m}\,\text{s}^{-1}$ and it accelerates to $11.4\,\text{m}\,\text{s}^{-1}$ in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(i) Find an expression for the velocity of the car at time t using this model. **[3]**

(ii) Explain why this model is not appropriate in the long term. **[1]**

Model A is refined so that the velocity remains constant once the car reaches $17.8\,\text{m}\,\text{s}^{-1}$.

(iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. **[3]**

(iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. **[3]**

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

(v) Show that this model gives an appropriate value for v when $t = 4$. **[1]**

(vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. **[3]**

(vii) Show that model B gives the same value as model A for the displacement at time 20 s. **[3]**

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 $f(x) = 3x^3 - 8x^2 + 3x + 2$

$f(2) = 3 \times 2^3 - 8 \times 2^2 + 3 \times 2 + 2$

$= 24 - 32 + 6 + 2 = 0$

$f(2) = 0$, so by the factor theorem $(x - 2)$ is a factor of $3x^3 - 8x^2 + 3x + 2$.

2 Let $g(x) = e^x - 5x^3$

$g(0) = e^0 - 5 \times 0^3 = 1 > 0$

$g(1) = e^1 - 5 \times 1^3 = e - 5 = -2.28... < 0$

g is continuous and there is a change of sign between 0 and 1, so $e^x - 5x^3 = 0$ has a root between 0 and 1.

3 $\sec^2\theta = 1 + \tan^2\theta$

$1 + \tan^2\theta + 2\tan\theta = 4$

$\tan^2\theta + 2\tan\theta - 3 = 0$

$(\tan\theta - 1)(\tan\theta + 3) = 0$

$\tan\theta = 1 \Rightarrow \theta = 45° \text{ or } \theta = 180° + 45° = 225°$

$\tan\theta = -3 \Rightarrow \tan^{-1}(-3) = -71.565...°$

$\theta = 180° - 71.57° = 108.4° \text{ or } \theta = 360° - 71.57° = 288.4°$

$\theta = 45°, 108.4°, 225°, 288.4°$

Question number	
4 (i)	$u = 0,\ v = 1.2,\ s = 1.8$
	$v^2 = u^2 + 2as$
	$1.2^2 = 0 + 2 \times a \times 1.8$
	$1.44 = 3.6a$
	$a = 0.4\ m\ s^{-2}$
4 (ii)	Newton's second law horizontally: $F - 19 = 2.8 \times 0.4$
	$F = 19 + 1.12 = 20.12$
	Rory's force = 20.1 N (3 s.f.)
5 (i)	$v = dr/dt = (12 - 4t)\ i + (2t - 6)\ j$
5 (ii)	i component zero: $12 - 4t = 0 \Rightarrow t = 3$
	j component zero: $2t - 6 = 0 \Rightarrow t = 3$
	Both components of velocity are zero at the same time, $t = 3$,
	so $v = 0\ i + 0\ j$: yes, the particle is stationary when $t = 3$ s.
6 (i)	Arithmetic sequence: $a = 50,\ d = 20,\ n = 24$
	$S_{24} = \frac{1}{2} \times 24 \times (2 \times 50 + 23 \times 20)$
	$= 12 \times 560 = £6720$
6 (ii)	Each month he saves 12% more than the month before, so each
	amount = previous amount × 1.12, a constant ratio $r = 1.12$.
	So the amounts are geometric with $a = 50$, $r = 1.12$.
6 (iii)	$S_{24} = 50(1.12^{24} - 1) / (1.12 - 1)$
	$= 50 \times 14.17862... / 0.12 = 5907.762...$
	Baraka saves £5907.76 in two years.
	£5907.76 < £6720, so no: Baraka does not save more than Aleela.

Question number	
7 (i)	Horizontal equilibrium: $F = 30 + 50$
	$F = 80$
7 (ii)	Moments about the top of the rod:
	$F \times x = 50 \times 2$
	$80x = 100$
	$x = 1.25$
8 (i)	$8 \sin^2 x \cos^2 x = 2(2 \sin x \cos x)^2$
	$= 2 \sin^2 2x$ using $\sin 2A = 2 \sin A \cos A$
	$= 2 \times \frac{1}{2}(1 - \cos 4x)$ using $\cos 2A = 1 - 2 \sin^2 A$, $A = 2x$
	$= 1 - \cos 4x$ as required
8 (ii)	$\int \sin^2 x \cos^2 x \, dx = \frac{1}{8} \int (1 - \cos 4x) \, dx$
	$= \frac{1}{8}(x - \frac{1}{4} \sin 4x) + c$
	$= \frac{1}{8} x - (1/32) \sin 4x + c$
9 (i)	Vertically (taking up as positive): $u = 0$, $a = -9.8$, $s = -5$
	$s = ut + \frac{1}{2}at^2$
	$-5 = 0 - 4.9t^2$
	$t^2 = 5 / 4.9 = 1.0204...$
	$t = 1.0101... = 1.01$ s
9 (ii)	Horizontally: $x = 14t$
	Vertically: $y = 5 - 4.9t^2$
	$t = x / 14$
	$y = 5 - 4.9(x / 14)^2$
	$y = 5 - x^2/40$

Question number	
9 (iii)	The pebble must be higher than 2 m when it passes the fence.
	When $y = 2$: $2 = 5 - x^2/40$
	$x^2/40 = 3 \Rightarrow x^2 = 120$
	$x = \sqrt{120} = 10.954...$
	$y > 2$ for $0 < x < 10.954...$, so
	$0 < d < 10.95$ m $(d < 11.0$ m$)$
10	Curve meets the x-axis when $(k - x) \ln x = 0$
	$k - x = 0$ or $\ln x = 0 \Rightarrow x = k$ or $x = 1$
	Area $= \int_1^k (k - x) \ln x \, dx$
	By parts: $u = \ln x$, $dv/dx = k - x$
	$du/dx = 1/x$, $v = kx - \frac{1}{2}x^2$
	Area $= [(kx - \frac{1}{2}x^2) \ln x]_1^k - \int_1^k (1/x)(kx - \frac{1}{2}x^2) \, dx$
	$= [(kx - \frac{1}{2}x^2) \ln x]_1^k - \int_1^k (k - \frac{1}{2}x) \, dx$
	$= [(kx - \frac{1}{2}x^2) \ln x - (kx - \frac{1}{4}x^2)]_1^k$
	$= (k^2 - \frac{1}{2}k^2) \ln k - (k^2 - \frac{1}{4}k^2) - [(k - \frac{1}{2}) \ln 1 - (k - \frac{1}{4})]$
	$= \frac{1}{2}k^2 \ln k - \frac{3}{4}k^2 - (-k + \frac{1}{4})$
	Area $= \frac{1}{2}k^2 \ln k - \frac{3}{4}k^2 + k - \frac{1}{4}$
11 (i)	Block A is in equilibrium on a smooth plane.
	Component of weight of A down the plane $= 4.7g \sin 60°$
	Resolving parallel to the plane for A: $T = 4.7g \sin 60°$
	$T = 4.7 \times 9.8 \times \sin 60° = 39.889...$
	So the tension is 39.9 N correct to 3 s.f.
11 (ii)	Block B, resolving perpendicular to the plane:
	$R = 4g \cos 25° = 4 \times 9.8 \times \cos 25° = 35.527...$
	B is on the point of sliding up the plane, so friction F acts
	down the plane. Resolving up the plane for B:

Question number	
	$T - F - 4g \sin 25° = 0$
	$F = 39.889... - 16.566... = 23.322...$
	On the point of sliding (limiting friction): $F = \mu R$
	$\mu = (4.7g \sin 60° - 4g \sin 25°) / (4g \cos 25°)$
	$= 23.322... / 35.527... = 0.6564...$
	$\mu = 0.656$ (3 s.f.)
12 (i)	C is $(1, -1)$
12 (ii)(A)	$4y = 3x - 32 \Rightarrow y = \frac{3}{4}x - 8$
	Substitute into $(x - 1)^2 + (y + 1)^2 = 25$:
	$(x - 1)^2 + (\frac{3}{4}x - 8 + 1)^2 = 25$
	$x^2 - 2x + 1 + (9/16)x^2 - (21/2)x + 49 = 25$
	$(25/16)x^2 - (25/2)x + 25 = 0$
	$\times 16/25$: $x^2 - 8x + 16 = 0$
	$(x - 4)^2 = 0$ (discriminant $= (-8)^2 - 4 \times 1 \times 16 = 0$)
	Repeated root, so the line meets the circle at exactly one point:
	the line $4y = 3x - 32$ is a tangent to the circle.
12 (ii)(B)	$x = 4$, $4y = 3 \times 4 - 32 = -20 \Rightarrow y = -5$
	B is $(4, -5)$
12 (iii)	$\angle CAD = \angle CBD = 90°$ (radius is perpendicular to the tangent)
	Gradient of AC $= (2 - (-1)) / (5 - 1) = \frac{3}{4}$
	Gradient of BC $= (-1 - (-5)) / (1 - 4) = -4/3$
	$\frac{3}{4} \times (-4/3) = -1$, so AC is perpendicular to BC: $\angle ACB = 90°$
	So ADBC has three right angles, so it is a rectangle.
	AC = BC = 5 (both radii), so adjacent sides are equal,
	therefore ADBC is a square.

Question number	
12 (iv)	E is the lowest point: $E = (1, -1 - 5) = (1, -6)$
	CE is vertically down from C; B is 3 across and 4 down from C.
	So $\angle ECB = \theta$ where $\tan\theta = 3/4$
	$\theta = \arctan(3/4) = 0.6435...$ rad
	Area of sector ECB $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times 0.6435...$
	$= 8.0437... = 8.04$ (3 s.f.)
13 (i)	$f(x) = (27 - 8x^3)^{1/3}$
	$f'(x) = \frac{1}{3}(27 - 8x^3)^{-2/3} \times (-24x^2)$ (chain rule)
	$= -8x^2 / (27 - 8x^3)^{2/3}$
	At $x = 1.5$: $27 - 8 \times 1.5^3 = 27 - 27 = 0$
	$f'(1.5) = -8 \times 1.5^2 / 0 = -18 / 0$, and you cannot divide by zero,
	so the calculator gives an error.
13 (ii)	$(27 - 8x^3)^{1/3} = 27^{1/3}(1 - (8/27)x^3)^{1/3} = 3(1 - (8/27)x^3)^{1/3}$
	$= 3[1 + (\frac{1}{3})(-8x^3/27) + ((\frac{1}{3})(-\frac{2}{3})/2!)(-8x^3/27)^2 + ...]$
	$= 3[1 - 8x^3/81 - 64x^6/6561 + ...]$
	$f(x) \approx 3 - (8/27)x^3 - (64/2187)x^6$
13 (iii)	The expansion is valid for $\lvert -8x^3/27 \rvert < 1$
	$\Rightarrow \lvert x^3 \rvert < 27/8 \Rightarrow \lvert x \rvert < 1.5$
	The limits 0 and 1 (and all x between) lie completely inside
	$-1.5 < x < 1.5$, so the expansion is valid for the whole integral.
13 (iv)	$h = 0.25$, ordinates at $x = 0, 0.25, 0.5, 0.75, 1$
	$\int_0^1 f(x)\,dx \approx (0.25/2)[3 + 2.6684 + 2(2.9954 + 2.9625 + 2.8694)]$
	$= 0.125 \times 23.3230$
	$= 2.9154$ (2.92 to 3 s.f.)

Question number

13 (v) The curve is concave (curving downwards) for $0 \leq x \leq 1$, so the tops of the trapezia lie below the curve: the area between the curve and the trapezia is missed out, so the estimate is too small.

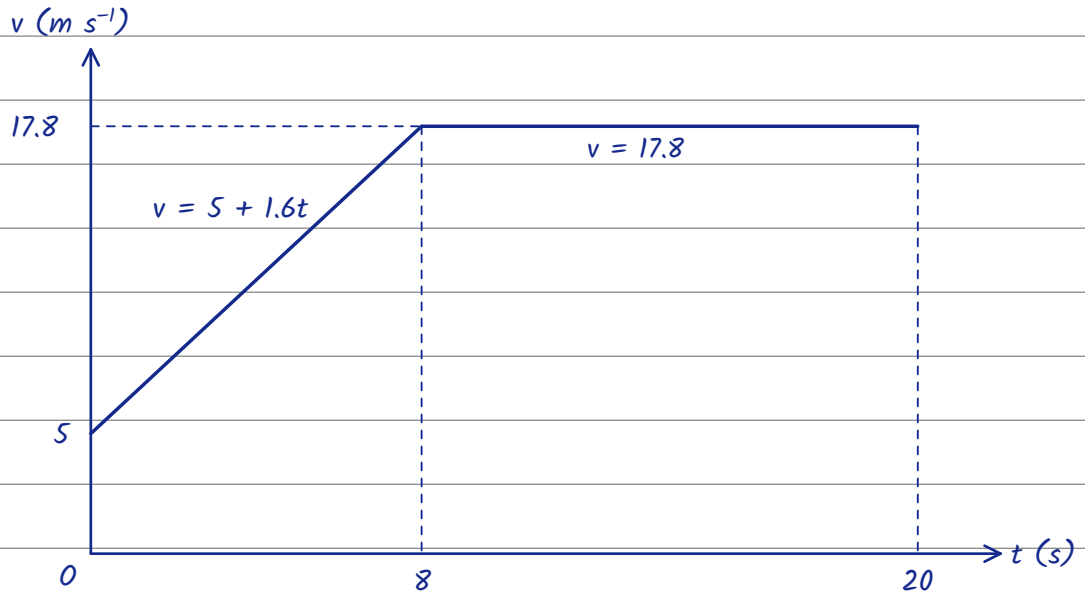
14 (i) $u = 5$, $v = 11.4$, $t = 4$

$a = (v - u)/t = (11.4 - 5)/4 = 1.6$

$v = u + at \quad \Rightarrow \quad v = 5 + 1.6t$

14 (ii) Under model A the velocity keeps increasing without limit; the car cannot accelerate for ever, its velocity would become too large.

14 (iii) When $v = 17.8$: $17.8 = 5 + 1.6t \quad \Rightarrow \quad t = 12.8/1.6 = 8$ s



14 (iv) $0 \leq t \leq 8$: area of trapezium $= \frac{1}{2}(5 + 17.8) \times 8 = 91.2$

$8 \leq t \leq 20$: area of rectangle $= 12 \times 17.8 = 213.6$

Displacement $= 91.2 + 213.6 = 304.8$ m

Question number

14 (v)

When $t = 4$: $v = 5 + 0.6 \times 4^2 - 0.05 \times 4^3$

$= 5 + 9.6 - 3.2 = 11.4 \text{ m s}^{-1}$

This matches the given velocity of 11.4 m s^{-1} at $t = 4$, so it is appropriate.

14 (vi)

$a = dv/dt = 1.2t - 0.15t^2$ for $0 \leqslant t \leqslant 8$

When $t = 8$: $a = 1.2 \times 8 - 0.15 \times 8^2 = 9.6 - 9.6 = 0$

In model B the acceleration decreases smoothly to zero at $t = 8$, exactly when the velocity becomes constant, so there is no sudden change.

In model A the acceleration jumps suddenly from 1.6 m s^{-2} to 0 at $t = 8$, which is unrealistic, so model B is likely to be better.

14 (vii)

$s = \int_0^8 (5 + 0.6t^2 - 0.05t^3)\, dt$

$= [5t + 0.2t^3 - 0.0125t^4]_0^8$

$= 5 \times 8 + 0.2 \times 8^3 - 0.0125 \times 8^4 - 0$

$= 40 + 102.4 - 51.2 = 91.2 \text{ m}$ (same as model A for the first 8 s)

$8 < t \leqslant 20$ at constant 17.8 m s^{-1}: $12 \times 17.8 = 213.6 \text{ m}$

Total displacement $= 91.2 + 213.6 = 304.8 \text{ m}$, the same as model A.