OCR
Oxford Cambridge and RSA

Thursday 7 October 2021 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes

7820681061

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2021 [F/508/5506]
DC (ST) 300066/3
OCR is an exempt Charity

Answer **all** the questions.

1 Two matrices, **A** and **B**, are given by $\mathbf{A} = \begin{pmatrix} 1 & -2 & -1 \\ 2 & -3 & 1 \\ a & 1 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -6 & 3 & -4 \\ -1 & 6 & -4 \\ 8 & -8 & -1 \end{pmatrix}$ where a is a constant.

Find the value of a for which $\mathbf{AB} = \mathbf{BA}$. **[3]**

2 **In this question you must show detailed reasoning.**

The complex numbers z_1 and z_2 are given by $z_1 = 3 - 7\mathrm{i}$ and $z_2 = 2 + 4\mathrm{i}$.

(a) Express each of the following as exact numbers in the form $a + b\mathrm{i}$.

(i) $3z_1 + 4z_2$ **[1]**

(ii) $z_1 z_2$ **[2]**

(iii) $\dfrac{z_1}{z_2}$ **[2]**

(b) Write z_1 in modulus-argument form giving the modulus in exact form and the argument correct to **3** significant figures. **[3]**

© OCR 2021

3 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$.

The plane Π has equation $\mathbf{r}.\begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} = 4$.

(a) Find the position vector of the point of intersection of l_1 and Π. **[3]**

(b) Find the acute angle between l_1 and Π. **[3]**

A is the point on l_1 where $\lambda = 1$.

l_2 is the line with the following properties.

- l_2 passes through A
- l_2 is perpendicular to l_1
- l_2 is parallel to Π

(c) Find, in vector form, the equation of l_2. **[3]**

4 In this question you must show detailed reasoning.

Determine the value of $\sum_{r=1}^{100} (2r+3)^2$. **[3]**

5 In this question you must show detailed reasoning.

(a) Using the definition of $\cosh x$ in terms of exponentials, show that $\cosh 2x \equiv 2\cosh^2 x - 1$. **[2]**

(b) Solve the equation $\cosh 2x = 3\cosh x + 1$, giving all your answers in exact logarithmic form. **[6]**

6 In this question you must show detailed reasoning.

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(a) Define the transformation represented by **A**. **[1]**

(b) Show that the area of any object shape is invariant under the transformation represented by **A**. **[1]**

The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} 7 & 2 \\ 21 & 7 \end{pmatrix}$. You are given that **B** represents the transformation which is the result of applying the following **three** transformations in the given order.

- A shear which leaves the y-axis invariant and which transforms the point (1, 1) to the point (1, 4).
- The transformation represented by **A**.
- A stretch of scale factor p which leaves the x-axis invariant.

(c) Determine the value of p. **[4]**

7 In this question you must show detailed reasoning.

(a) Find the values of A, B and C for which $\dfrac{x^3+x^2+9x-1}{x^3+x^2+4x+4} \equiv A + \dfrac{Bx+C}{x^3+x^2+4x+4}$. **[1]**

(b) Hence express $\dfrac{x^3+x^2+9x-1}{x^3+x^2+4x+4}$ using partial fractions. **[5]**

(c) Using your answer to part **(b)**, determine $\displaystyle\int_0^2 \frac{x^3+x^2+9x-1}{x^3+x^2+4x+4}\,\mathrm{d}x$ expressing your answer in the form $a + \ln b + c\pi$ where a is an integer, and b and c are both rational. **[4]**

© OCR 2021

8 A particle P of mass 2 kg can only move along the straight line segment OA, where OA is on a rough horizontal surface. The particle is initially at rest at O and the distance OA is 0.9 m.

When the time is t seconds the displacement of P from O is x m and the velocity of P is $v\,\text{ms}^{-1}$. P is subject to a force of magnitude 4e^{-2t} N in the direction of A for any $t \geqslant 0$. The resistance to the motion of P is modelled as being proportional to v.

At the instant when $t = \ln 2$, $v = 0.5$ and the resultant force on P is 0 N.

(a) Show that, according to the model, $\frac{\text{d}v}{\text{d}t} + v = 2\text{e}^{-2t}$. **[3]**

(b) Find an expression for v in terms of t for $t \geqslant 0$. **[5]**

(c) By considering the behaviour of v as t becomes large explain why, according to the model, P's speed must reach a maximum value for some $t > 0$. **[2]**

(d) Determine the maximum speed considered in part **(c)**. **[2]**

(e) Determine the greatest value of t for which the model is valid. **[4]**

9 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$.

(a) By considering $\mathbf{A}$, $\mathbf{A}^2$, $\mathbf{A}^3$ and $\mathbf{A}^4$ make a conjecture about the form of the matrix $\mathbf{A}^n$ in terms of n for $n \geqslant 1$. **[2]**

(b) Use induction to prove the conjecture made in part **(a)**. **[4]**

10 In this question you must show detailed reasoning.

(a) By using an appropriate Maclaurin series prove that if $x > 0$ then $\text{e}^x > 1 + x$. **[2]**

(b) Hence, by using a suitable substitution, deduce that $\text{e}^t > \text{e}t$ for $t > 1$. **[1]**

(c) Using the inequality in part **(b)**, and by making a suitable choice for t, determine which is greater, e^π or π^e. **[3]**

END OF QUESTION PAPER

© OCR 2021

BLANK PAGE

BLANK PAGE

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© OCR 2021