OCR
Oxford Cambridge and RSA

GCE

Further Mathematics A

Y541/01: Pure Core 2

Advanced GCE

Mark Scheme for November 2020

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			**DR** $z = \frac{--20 \pm \sqrt{(-20)^2 - 4\times 4\times 169}}{2\times 4}$	**M1**	1.1	Term by term substituting into formula. If formula quoted, allow one slip … Or correctly completes the square	Condone anything correct of the form $\frac{p \pm \sqrt{q}}{r}$ eg $4\left(\left(z-\frac{5}{2}\right)^2 - \frac{25}{4}\right)+169=0$
			$z = \frac{5 \pm 12\text{i}}{2}$	**A1**	1.1		
			$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{12}{2}\right)^2} = \frac{13}{2}$ oe	**B1ft**	1.1		Ft workings from complex conjugate distinct pair (with real component)
			$\theta = \tan^{-1}\frac{6}{2.5}$ oe	**M1**	1.1	Attempting to find argument using trigonometry	$\theta = \cos^{-1}\frac{2.5}{6.5}$, $\theta = \sin^{-1}\frac{6}{6.5}$
			$\frac{13}{2}(\cos 1.18 + \text{i}\sin 1.18)$	**A1**	2.5	Angle must be in radians.	oe eg $\frac{13}{2}\text{cis}1.18$ or $\left[\frac{13}{2}, 1.18\right]$
			$\frac{13}{2}(\cos(-1.18) + \text{i}\sin(-1.18))$			Argument could be 5.11 but both angles must be the same.	Not 5.10 (rounding error) Not e.g. $\cos(-1.18) + \text{i}\sin(5.11)$
				[5]			

Question			Answer	Marks	AO	Guidance	
2	(a)		**DR** $u = x^2$	**B1**	3.1a	Correct substitution chosen	
			$3\left(\sqrt{u}\right)^3 - 2\left(\sqrt{u}\right)^2 - 5\sqrt{u} - 4\ (=0)$	**M1**	1.1	Oe Attempting to make substitution	or preparation for substitution by removing odd powers. eg $x^2(3x^2 - 5)^2 = (2x^2 + 4)^2$...
			$3u\sqrt{u} - 5\sqrt{u} = 2u + 4 \Rightarrow u(3u-5)^2 = (2u+4)^2$	**M1**	1.1	Rearranging and squaring bs to remove the square root(s)	...and then substituting $u(3u - 5)^2 = (2u + 4)^2$
			$u(9u^2 - 30u + 25) = 4u^2 + 16u + 16 \Rightarrow$ $9u^3 - 34u^2 + 9u - 16 = 0$	**A1**	3.2a	Rearranging to answer	Equation can be in x
			Alternative method **DR**				
			$\alpha^2\beta^2\gamma^2 = (\alpha\beta\gamma)^2 = \left(-\frac{-4}{3}\right)^2 = \frac{16}{9}$	**B1**			Must include one intermediate step
			$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ $= (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$	**M1**		Writing the expression in terms of standard symmetrical forms	NB $\sum\alpha = \frac{2}{3}$, $\sum\alpha\beta = -\frac{5}{3}$, $\alpha\beta\gamma = \frac{4}{3}$
			$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$	**M1**		Writing the expression in terms of standard symmetrical forms	Condone without factorisation of "2"
			$u^3 - \left(\left(\frac{2}{3}\right)^2 - 2\times\frac{-5}{3}\right)u^2 + \left(\left(\frac{-5}{3}\right)^2 - 2\times\frac{4}{3}\times\frac{2}{3}\right)u - \frac{16}{9}$ $= u^3 - \frac{34}{9}u^2 + u - \frac{16}{9} = 0 \Rightarrow 9u^3 - 34u^2 + 9u - 16 = 0$	**A1**		Substituting in and rearranging to answer	NB $\sum\alpha^2 = \frac{34}{9}$, $\sum\alpha^2\beta^2 = 1$
				[4]			
2	(b)		**DR** $\frac{\sum\alpha^2\beta^2}{\alpha\beta\gamma} = \frac{\left(\frac{9}{9}\right)}{\left(\frac{4}{3}\right)}$ or $\frac{1}{\left(\frac{4}{3}\right)}$	**M1**	3.1a	Their $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ from part (a) over $\pm\frac{4}{3}$	Strict ft

			$=\frac{3}{4}$	**A1** **[2]**	1.1		
Question			**Answer**	**Marks**	**AO**	**Guidance**	
3	**(a)**		**DR** $(r+2)(r-1)$	**B1**	1.1	Correct factorisation of denominator soi	
			$\frac{A}{r-1}+\frac{B}{r+2}$	**M1**	1.1	Correct form for partial fractions	
			$A=1, B=-1$	**A1**	1.1		
			$\therefore \sum_{r=5}^{n}\frac{3}{r^2+r-2}=\sum_{r=5}^{n}\frac{1}{r-1}-\sum_{r=5}^{n}\frac{1}{r+2}$ $=\sum_{r=5}^{n}\frac{1}{r-1}-\sum_{r=8}^{n+3}\frac{1}{r-1}$ $=\sum_{r=5}^{7}\frac{1}{r-1}-\sum_{r=n+1}^{n+3}\frac{1}{r-1}$	**M1**	1.1	Using partial fractions, separating into two sums, re-indexing so that the summands have identical form and cancelling central terms.	Might see start and end terms explicitly. eg $\sum_{r=5}^{n}\frac{1}{r-1}-\sum_{r=8}^{n+3}\frac{1}{r-1}$ $=\frac{1}{5-1}+\frac{1}{6-1}+\frac{1}{7-1}+\sum_{r=8}^{n}\frac{1}{r-1}$ $-\left(\sum_{r=8}^{n}\frac{1}{r-1}+\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}\right)$ Might see formal substitution of index. eg Let $R=r+3\Rightarrow r+2=R-1$ $\therefore \sum_{r=5}^{n}\frac{1}{r-1}-\sum_{r=5}^{n}\frac{1}{r+2}$ $=\sum_{r=5}^{n}\frac{1}{r-1}-\sum_{R=8}^{n+3}\frac{1}{R-1}$

			$\therefore \sum_{r=5}^{n} \frac{3}{r^2+r-2} =$ $= \frac{1}{4}+\frac{1}{5}+\frac{1}{6}-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}$ $= \frac{37}{60}-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}$	**A1**	1.1	AG.	

Question			Answer	Marks	AO	Guidance	
			Alternative method for last 2 marks: $=$ $\frac{1}{4} - \frac{1}{7} \quad \ldots \quad - \frac{1}{n-1}$ $\frac{1}{5} - \frac{1}{8} \quad \frac{1}{n-3} - \frac{1}{n}$ $\frac{1}{6} - \frac{1}{9} \quad \frac{1}{n-2} - \frac{1}{n+1}$ $\frac{1}{7} - \ldots \quad \frac{1}{n-1} - \frac{1}{n+2}$	**M1**		At least these terms	M1 can be ft from any A, B having opposite signs For M1, condone omission of $\frac{1}{7}$ or - $\frac{1}{n-1}$
			$= \frac{1}{4}+\frac{1}{5}+\frac{1}{6}-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}$ $= \frac{37}{60}-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}$	**A1**		AG. Correct cancellation to AG.	Requires joined argument Must have either clear diagonal cancellations with one explicit cancellation or less clear cancellation with numerical and algebraic cancellation shown or described
				[5]			
	(b)		$= \frac{37}{60}$ or awrt 0.617	**B1** **[1]**	2.2a		

4	(a)		$\begin{pmatrix}13\\3\\-14\end{pmatrix}.\begin{pmatrix}1\\5\\3\end{pmatrix}=13+15-42=-14$ (so R is on Π)	**B1**	1.1	AG. Intermediate working must be seen	
			eg $7-\mu=13 \Rightarrow \mu=-6 \Rightarrow$ $\mathbf{r}=\begin{pmatrix}7\\9\\-2\end{pmatrix}-6\begin{pmatrix}-1\\1\\2\end{pmatrix}=\begin{pmatrix}13\\3\\-14\end{pmatrix}$ (so R is also on l_2)	**B1**	1.1	AG. Or $9+\mu=3$ or $-2+2\mu=-14$ but must be checked in other two equations.	
			Alternate method $\begin{pmatrix}1\\5\\3\end{pmatrix}.\left(\begin{pmatrix}7\\9\\-2\end{pmatrix}+\mu\begin{pmatrix}-1\\1\\2\end{pmatrix}\right)=46+10\mu=-14\Rightarrow\mu=-6$	**M1**		AG. Substituting in expression of the point into the equation of the plane to find a value for μ	
			$\mu=-6 \Rightarrow$ $\mathbf{r}=\begin{pmatrix}7\\9\\-2\end{pmatrix}-6\begin{pmatrix}-1\\1\\2\end{pmatrix}=\begin{pmatrix}13\\3\\-14\end{pmatrix}$ so R is (13, 3, –14)	**A1**		AG.	Answer in vector form is acceptable.
				[2]			
	(b)		Since lines intersect $\begin{pmatrix}1\\0\\a\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-3\end{pmatrix}=\begin{pmatrix}7\\9\\-2\end{pmatrix}+\mu\begin{pmatrix}-1\\1\\2\end{pmatrix}$ for some λ and μ				
			so $1+2\lambda=7-\mu$ $\lambda=9+\mu$ $(a-3\lambda=-2+2\mu)$	**M1**	3.1a	Equating the lines and deriving 2 useful equations. Ignore attempts at z coefficient equation	
			$\Rightarrow \lambda=5,\ \mu=-4$	**A1**	1.1		Can be BC
			so $a+5\times(-3)=-2+(-4)\times2 \Rightarrow a=5$	**A1ft**	1.1		

			At Q, $\left(\begin{pmatrix}1\\0\\5\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-3\end{pmatrix}\right).\begin{pmatrix}1\\5\\3\end{pmatrix}=-14$ for some λ so		2.1	Substituting general **r** from l_1 into the Π equation and dotting out to form an equation in λ	Accept algebraic expressions in a until final A mark
			$1 + 5\times3 + \lambda(2 + 5 + (-3)\times3) = -14$	***M1**			
			$-2\lambda = -30 \Rightarrow \lambda = 15 \Rightarrow Q(31, 15, -40)$	**A1**	1.1		$\lambda = 7.5+1.5a$ $Q(16+3a,\ 7.5+1.5a,$ $-22.5-3.5a)$
Question			**Answer**	**Marks**	**AO**	**Guidance**	
			$\sqrt{(31-13)^2+(15-3)^2+(-40--14)^2}$	**dep*M1**	2.1	Method fully shown or at least 2 of 3 squared terms correct	Depends on correct method shown to find Q $\sqrt{(3+3a)^2+(4.5+1.5a)^2+(-8.5-3.5a)^2}$
			$=\sqrt{18^2+12^2+26^2}=\sqrt{1144}$	**A1** **[7]**	3.2a	$2\sqrt{286}$	
5	**(a)**		AE: $m^2 - 2m - 15 = 0 \Rightarrow m = 5$ or -3	**M1**	1.1		
			So GS is $Q = Ae^{-3t} + Be^{5t}$	**A1**	1.1		
			Q tends to finite limit as $t \to \infty \Rightarrow B = 0$	**B1**	2.2a	Or dQ/dt tends to zero as $t \to \infty$	www
			$t = 0$, $Q = 100 \Rightarrow A = 100$	**M1**	3.4	Using initial condition to find A (or $A + B$)	
			So $Q = 100e^{-3t}$	**A1**	1.1	soi	
			$t = 0.5 \Rightarrow Q = 100e^{-1.5} = 22.3$	**A1** **[6]**	3.4		
	(b)		(As $t \to \infty$ $e^{-3t} \to 0$ so Q tends to) 0.	**B1** **[1]**	3.4	Only if from $Q = ke^{-at}$, $a > 0$	It must be clear that the limit is 0; "Q is approximately 0" would not be sufficient for B1.

6	(a)		$\ln(1+\sin\theta)=0 \Rightarrow 1+\sin\theta=1 \Rightarrow \sin\theta=0$ so $\alpha=0$ and $\beta=\pi$	**M1** **A1** **[2]**	1.1a 2.2a		
	(b)		$A=\frac{1}{2}\int_0^{\pi}(\ln(1+\sin\theta))^2\,\mathrm{d}\theta$ = 0.4162 (4 sf) cao	**M1** **A1** **[2]**	1.2 1.1	Correct formula for area with r correctly substituted and their limits. Must be unambiguous but can be implied by correct answer/later work BC	Incorrect formula = **M0A0** Condone missing $\mathrm{d}\theta$

Question			Answer	Marks	AO	Guidance	
	(c)		$\theta=\frac{\pi}{2}\Rightarrow r=\ln 2=0.6931$ (4 sf) which would be the diameter, D, of the circle	**M1**	3.1a	or radius R = 0.3466 (4 sf) condone correct R or D without reasoning	It must be clear that the r value would be the diameter of the circle; the calculation alone is insufficient for M1. M1 can be implied by area given as $\pi\left(\frac{\ln 2}{2}\right)^2$
			But A = 0.4162 (4 sf) => D = 0.7280 (4 sf) or R = 0.3640 (4 sf) so the curve is not circular	**A1** **[2]**	3.2a	or R = 0.3466 (4 sf) (or D = 0.6931) => A = 0.3773 (4 sf) which is not 0.4162 (4 sf)	Explanation must include comparison of R's, D's or A's and conclusion . Allow correct working to 3 sf.
7	(a)		det**A** (= 0.6×1.8 – –0.8×2.4) = 3	**B1** **[1]**	1.1		
	(b)		Determinant of rotation = 1 Determinant of rotation × determinant of stretch = 1×sf = 3 => sf = 3	**B1** **B1** **[2]**	1.1 2.2a		

	(c)		Since the second column of **A** contains entries bigger than 1 (in magnitude) the stretch must be parallel to the *y*-axis.	**B1** **[1]**	2.4	Or any correct, complete explanation.	May see $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$ $=\begin{pmatrix}\cos\theta & -3\sin\theta\\ \sin\theta & 3\cos\theta\end{pmatrix}$ or similar
	(d)		$\sin\theta = -0.8$ and $\cos\theta = 0.6$ oe awrt –53° (or –0.93 rads)	**M1** **A1** **[2]**	2.2a 1.1	Condone if only one equation or 53° (0.93 rads) clockwise or 307° (5.36 rads) (anticlockwise).	

Question			Answer	Marks	AO	Guidance	
8	**(a)**		**DR** $r^2 = (-4)^2 + \left(\sqrt{48}\right)^2$ or ($r\cos\theta = -4$ and $r\sin\theta = \sqrt{48}$) or $\tan\theta = -\sqrt{3}$ oe	**M1**	2.1	Correct use of relevant formula(e). Some working must be seen.	Correct answer with no working: M0A0
			$r = 8$ (ie $z = 8e^{i\theta}$) $\theta = 2\pi/3$ (ie $z = re^{i2\pi/3}$)	**A1**	1.1	Not ±8 unless later corrected	or eg $\theta = 8\pi/3$
			$\sqrt[3]{8}$ or 2	**B1ft**	2.1	Modulus of cube root(s) is the cube root of their modulus	
			$\frac{2\pi}{9}$ soi	**B1ft**	2.1	Argument of (principal) cube root is one third of their argument	
			$\frac{2\pi}{3} + 2\pi k$ for $k = 1$ and 2 oe seen	**M1**	2.2a	Considering further arguments at angular distance 2π	
			$2e^{\frac{2}{9}\pi i}$, $2e^{\frac{8}{9}\pi i}$ and $2e^{-\frac{4}{9}\pi i}$	**A1** **[6]**	1.1	or eg $2e^{\frac{2}{9}\pi i}$, $2e^{\frac{8}{9}\pi i}$ and $2e^{\frac{14}{9}\pi i}$	Must be in exponential form, not just r = and θ=. Do not condone any missing i's.
	(b)		**DR** The cube roots form an equilateral triangle which has (3) lines of symmetry, (one) through each vertex	**B1**	2.2a		

			$\theta = \frac{2\pi}{9}$, $\theta = \frac{8\pi}{9}$ and $\theta = -\frac{4\pi}{9}$ soi	**B1** **B1** **[3]**	2.2a 2.2a	for one for all three without extras	ft their angles if $2\pi/3$ apart. If valid alternatives, must come from clear explanation/diagram

Question			Answer	Marks	AO	Guidance	
9	**(a)**		Min value of cosh is 1 (and point on ground is at the minimum) (so $0 = k \times 1 - 1 \Rightarrow$) $k = 1$	**M1** **A1** **[2]**	2.2a 2.2a	Using minimum point of curve and knowledge of cosh graph	Could be derived by differentiation If zero scored then sc1 for k=1 www
	(b)		Passes through (0, 3) $\Rightarrow 3 = \cosh(-b) - 1$ $\Rightarrow b = -\cosh^{-1}(3 + 1)$ $b = (\pm)\ln(4 + \sqrt{(4^2 - 1)})$ $\Rightarrow b = \ln(4 + \sqrt{15})$ Passes through (2, 0) $\Rightarrow 0 = \cosh(2a - b) - 1$ $\Rightarrow b = 2a$ $\Rightarrow a = \frac{1}{2}\ln(4 + \sqrt{15})$	***M1** **dep*M1** **A1** **M1** **A1** **[5]**	3.3 3.1a 1.1 3.3 1.1	Use of (0, 3) to derive an expression for b Correct numerical use of formula Use of (2, 0) to derive $b = 2a$	accept $\cosh(-b) = \frac{4}{k}$ Or rearranges. Could be from **(a)**. Allow ft
	(c)		(By symmetry of both;) (4, 3)	**B1** **[1]**	2.2a		
	(d)		Holly's model; $d_H = 6.75$	**B1**	3.4		Condone 27/4

			Jofra's model: $d_J = \cosh(5a - b) - 1$	**M1**	3.4	Use of $x = 5$ with their values of a and b to predict d. Must have -1	$a = 1.0317\ldots$, $b = 2.0634\ldots$, $d_J = 10.067\ldots$ Condone 10.07 only if clear evidence of production
			AG $d_J - d_H = 10.067\ldots - 6.75 = 3.32$ (3 sf)	**A1** **[3]**	1.1	From correct values	

Question			Answer	Marks	AO	Guidance	
10	(a)	(i)	$f'(x) = \dfrac{1}{\left(1-x^2\right)^{\frac{1}{2}}}$ from the formula book				
			so $f''(x) = -\frac{1}{2}.\dfrac{1}{\left(1-x^2\right)^{\frac{3}{2}}}.(-2x)$	M1	1.1	Formula from the Formula Booklet and attempt differentiation	To within sign error
			$= \dfrac{x}{\left(1-x^2\right)^{\frac{3}{2}}}$	A1	1.1		
				[2]			
	(a)	(ii)	$f(0) = 0,\ f'(0) = 1$ and $f''(0) = 0$	B1	1.1	or $a_0 = 0$, $a_1 = 1$ and $a_2 = 0$	Ignore sign error in $f''(x)$
			$f'''(x) = \dfrac{(1-x^2)^{\frac{3}{2}} - x.\frac{3}{2}(1-x^2)^{\frac{1}{2}}.(-2x)}{(1-x^2)^3}$	M1	3.1a	Differentiate and simplify far enough to be able to justify value 1	Either full derivative or "zero term" denoted as such
			so $f'''(0) = 1$ and $f(x) = x + \frac{1}{6}x^3 + \ldots$	A1	2.1	Condone 3! In place of 6	**Not** BC. If M0 then SC1 for correct expansion
				[3]			
	(a)	(iii)	$\int_0^{\frac{1}{2}} f(x)\,dx \approx \int_0^{\frac{1}{2}} x + \frac{1}{6}x^3\,dx$	M1	1.1	Integral of their 2 term cubic with limits	
			$= 0.127604167\ldots$ $= 0.127604$ to 6 dp	A1	1.1	Could be BC	
				[2]			
	(b)		$\int 1 \times \sin^{-1} x\,dx = x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}\,dx$	M1	3.1a	Attempt integration by parts	ignore limits. Formula for parts must be correct
			$= x\sin^{-1} x + \left(1-x^2\right)^{\frac{1}{2}}$ (+c)	A1	1.1		
			$\int_0^{\frac{1}{2}} f(x) = \dfrac{\pi}{12} + \dfrac{\sqrt{3}}{2} - 1$	A1	1.1		
				[3]			

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