GCE

Further Mathematics A

Y541/01: Pure Core 2

A Level

Mark Scheme for June 2025

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training: OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are available in RM Assessor

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **required number** of standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone, email or via the RM Assessor messaging system.

5. **Crossed-Out Responses**
   Where a candidate has crossed out a response and provided a clear alternative then the crossed-out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed-out response where legible.

Rubric Error Responses – Optional Questions
Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM Assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

Multiple-Choice Question Responses
When a multiple-choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate).
When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.

Contradictory Responses
When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only one mark per response)
Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

Short Answer Questions (requiring a more developed response, worth two or more marks)
If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space).

Longer Answer Questions (requiring a developed response)
Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there, then add the annotation 'SEEN' to confirm that the work has been seen and mark any responses using the annotations in section 11.

7. There is a NR (**No Response**) option. Award NR (No Response):

   - if there is nothing written at all in the answer space
   - OR if there is a comment which does not in any way relate to the question (e.g., 'can't do', 'don't know')
   - OR if there is a mark (e.g., a dash, a question mark) which is not an attempt at the question.

   Note: Award 0 marks – for an attempt that earns no credit (including copying out the question).

8. The RM Assessor **comments box** is used by your Team Leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**

9. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

10. For answers marked by levels of response: Not applicable in F501
    To determine the level – start at the highest level and work down until you reach the level that matches the answer
    To determine the mark within the level, consider the following

Descriptor	Award mark
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

11. **Annotations**

Annotation	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject Specific Marking Instructions

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.
Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			**DR** $\mathbf{a} \cdot \mathbf{b} = 2 \cdot (-1) + (-4) \cdot 3 + 3 \cdot (-2)$ $= -2 - 12 - 6$ $= -20$	**B1**	1.1	Some evidence of correct calculation must be shown (i.e. any intermediate step before -20) but ignore poor/incorrect notation e.g. $\begin{pmatrix} -2 \\ -12 \\ -6 \end{pmatrix}$ seen in working.	
			$\begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} (-4) \cdot (-2) - 3 \cdot 3 \\ 3 \cdot (-1) - 2 \cdot (-2) \\ 2 \cdot 3 - (-1) \cdot (-4) \end{pmatrix}$ $= \begin{pmatrix} 8 - 9 \\ -3 - (-4) \\ 6 - 4 \end{pmatrix}$	**M1**	1.1	Working must be correct for at least 2 components.	May see components given separately e.g. allow **M1** for any two of: $\begin{vmatrix} -4 & 3 \\ 3 & -2 \end{vmatrix}, \begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix}, \begin{vmatrix} 2 & -1 \\ -4 & 3 \end{vmatrix}$ or for $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 3 \\ -1 & 3 & -2 \end{vmatrix}$
			$= \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$	**A1**	1.1	If **M0** then SCB1 for $\begin{pmatrix} \pm 1 \\ \pm 1 \\ \pm 2 \end{pmatrix}$ with working shown (max 2/3).	If no working for either product (**B0M0**) then **SCB1** for both correct (max 1/3).
				[3]			

Question			Answer	Marks	AO	Guidance	
2	(a)		$-7+5\mathrm{i}$	B1	1.2		
				[1]			
	(b)		$(-7+5\mathrm{i})(-7-5\mathrm{i})=49+25=74$ or $(-7+5\mathrm{i})+(-7+5\mathrm{i})=-14$	M1	1.1	Finding sum or product of known roots (could be embedded)	Or dividing cubic by one of $(x-(-7+5\mathrm{i})),(x-(-7-5\mathrm{i}))$ and the quotient by the other Or by multiplying $\big(x-(-7+5\mathrm{i})\big)\big(x-(-7-5\mathrm{i})\big)=\ldots$
			$\Rightarrow x^2+14x+74$	A1	2.2a		
			$\Rightarrow x-4$ (hence $(x-4)(x^2+14x+74)$)	A1	1.1	No need to reassemble if both factors correct and unambiguous	**SCB1** for correct answer without evidence of use of **(a)** (max 1/3).
			Alternative method: $(-7+5\mathrm{i})(-7-5\mathrm{i})=49+25=74$ $\frac{296}{74}=4$	M1		Finding product of known roots and $\alpha\beta\gamma=-\frac{c}{a}$ to find the real root.	
			$\Rightarrow x-4$	A1			
			$\Rightarrow x^2+14x+74$ (hence $(x-4)(x^2+14x+74)$)	A1		No need to reassemble if both factors correct and unambiguous	
				[3]			

Question			Answer	Marks	AO	Guidance	
3	(a)		$(\mathbf{AB} =) \begin{pmatrix} -6 & -9 \\ -4 & 4 \end{pmatrix}$	**B1**	1.1		
				[1]			
	(b)		$\det(\mathbf{AB}) = -60$ or $\det \mathbf{A} = 20$ or $\det \mathbf{B} = -3$	**M1**	1.1	Correctly finds at least one relevant determinant (including for their '**AB**').	
			$\det(\mathbf{A}) \times \det(\mathbf{B}) = 20 \times (-3)$ $= -60 = \det(\mathbf{AB})$	**A1**	2.1	Complete, connected and convincing argument, including all 3 determinants correct and multiplication giving -60.	Allow $20 \times (-3) = -60$ or $\det(\mathbf{A}) \times \det(\mathbf{B}) = -60$ (provided determinants found first)
				[2]			
	(c)		$\mathbf{BA} = \begin{pmatrix} -15 & -15 \\ 9 & 13 \end{pmatrix}$	**B1***	1.1	Finds **BA**. Or finds one element and explicitly compares to corresponding element in **AB**.	
			$\mathbf{BA} \neq \mathbf{AB}$	**B1 dep***	2.1	Clear concluding statement following correct work.	
				[2]			
	(d)		$\begin{pmatrix} 1 & -3 \\ 4 & 8 \end{pmatrix} \begin{pmatrix} 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 17 \\ -32 \end{pmatrix}$	**M1**	1.1	Attempt to multiply the vector into the matrix, one element correct	
			$\neq \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ (so $(2, -5)$ is not an invariant point)	**A1**	2.4	For clear conclusion from correct working	Values shown must be correct. Could be explained in words.
				[2]			
	(e)		$\mathbf{A}^{-1}$ soi as answer	**M1**	1.2	Recalling the connection between inverse transformation and inverse matrix.	Could be stated in words or implied by an attempt at the inverse e.g. $A^{-1} = \cdots$ or $\frac{1}{n}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ etc.
			$\frac{1}{20}\begin{pmatrix} 8 & 3 \\ -4 & 1 \end{pmatrix}$	**A1FT**	2.2a	FT their det **A** from **(b)** if not recalculated. ISW.	$\begin{pmatrix} \frac{2}{5} & \frac{3}{20} \\ -\frac{1}{5} & \frac{1}{20} \end{pmatrix}$ or $\begin{pmatrix} 0.4 & 0.15 \\ -0.2 & 0.05 \end{pmatrix}$
				[2]			

Question			Answer	Marks	AO	Guidance	
4			**DR** $(\sqrt[3]{512000} =) 80$	**B1**	3.1a	May be seen embedded in a sum	
			$\sum_{r=1}^{80} r^3 = \frac{1}{4} \times 80^2 \times (80+1)^2$	**M1**	1.1	Using the correct formula for sum of cubes with an upper limit (between 26-720 but not 216) substituted	
			$= 10497600$ OR $-\frac{1}{4} \times 5^2 \times 6^2$ OR -225	**A1**	1.1	Any one of these seen correct	May see $1 + 8 + 27 + 64 + 125 = 225$
			Required sum $= 10497375$	**A1**	2.2a		Correct answer with no working is 0/4
			Alternative method: **DR** $(\sqrt[3]{512000} =$ 80 and $80 - 5 =) 75$	**B1**		May be seen embedded in a sum (e.g. 75 as upper limit)	
			Sum is $\left(\sum_{r=6}^{80} r^3\right) = \sum_{R=1}^{75} (R+5)^3 = \sum_{R=1}^{75} R^3 + 15R^2 + 75R + 125$	**M1**		Expressing the required sum as a single sum and multiplying out so that standard formulae can be applied (allow one slip in coefficients). Ignore limits for this mark.	
			$= \frac{1}{4} \times 75^2 \times (75+1)^2$ $+15 \times \frac{75(75+1)(2\times75+1)}{6} + 75 \times \frac{75(75+1)}{2}$ $+125 \times 75$	**A1**		3 out of 4 terms correct (must include 125×75)	
			$= 8122500 + 2151750 + 213750 + 9375$ $= 10497375$	**A1**			Correct answer with no working is 0/4
				[4]			

Question			Answer	Marks	AO	Guidance	
5	(a)		**DR** $(a+b\text{i})^2 = a^2 - b^2 + 2ab\text{i} = -3 + \left(4\sqrt{7}\right)\text{i}$	**M1**	1.1	Setting up the square root, squaring and equating. May be implied by correctly equated real and imaginary parts.	
			$a^2 - b^2 = -3$ **and** $2ab = 4\sqrt{7}$	**M1**	1.1	Equating real **and** imaginary parts	
			$a^2 - \left(\frac{2\sqrt{7}}{a}\right)^2 = -3$ $\therefore (a^2)^2 + 3a^2 - 28 = 0$	**M1**	1.1	Using one equation to eliminate one unknown and rearranging to correct quadratic in the square of the other (condone a^4 or b^4).	$\left(\frac{2\sqrt{7}}{b}\right)^2 - b^2 = -3$ $\therefore (b^2)^2 - 3b^2 - 28 = 0$
			$(a^2+7)(a^2-4) = 0 \Rightarrow a^2 = -7$ or $a^2 = 4$ (a is real so) $a^2 \neq -7$	**A1**	2.3	Solving and rejecting negative root (may be implicit e.g. by asserting that $a^2 > 0$ or not using negative root in subsequent working).	$(b^2-7)(b^2+4) = 0$ $\Rightarrow b^2 = 7$ or $b^2 = -4$ (b is real so) $b^2 \neq -4$
			$a = 2 \Rightarrow b = \sqrt{7}$ and $a = -2 \Rightarrow b = -\sqrt{7}$ (So square roots are $\pm(2+\sqrt{7}\text{i})$)	**A1**	1.1	Root need not be assembled but values must be clearly paired. i.e. $a = \pm 2, b = \pm\sqrt{7}$ is insufficient for **A1**.	$b = \sqrt{7} \Rightarrow a = 2$ and $b = -\sqrt{7} \Rightarrow a = -2$ Correct answers without working is 0/5. Use of polar form 0/5 unless via an equivalent algebraic method.
				[5]			
	(b)		**DR** Their arguments differ by π (radians) oe	**B1**	2.4	Accept 180°. Answer must give a connection that could be used to find one argument from the other. Do not accept e.g.: - 'they each make the same angle with the real axis' B0 - 'one is $-1\times$ the other' B0	Accept e.g.: - 'Arguments have the same tangents' B1 - 'They are π apart' B1 - 'The argument of one is π away from the argument of the other' B1 - 'Rotation by π or 180°' B1
				[1]			

Question			Answer	Marks	AO	Guidance	
6	(a)		$2\theta = \theta^2 \Rightarrow (\theta = 0 \text{ or}) \ \theta = 2$	**M1**	3.1a	Can be implied by correct value in final answer	0 does not have to be explicitly rejected.
			[4,2]	**A1**	2.5	Notation must be correct, accept $(4,2)$ but not $[2,4]$	
				[2]			
	(b)		**DR** Use of $A = \frac{1}{2}\int_0^2 r^2 \mathrm{d}\theta$ at least once	**B1**	3.4	Constant and limits must be correct (BOD for missing $\mathrm{d}\theta$)	
			Required area $= \frac{1}{2}\int_0^2 (2\theta)^2 \mathrm{d}\theta - \frac{1}{2}\int_0^2 (\theta^2)^2 \mathrm{d}\theta$	**M1**	3.1a	Condone integrals switched or missing $\frac{1}{2}$ (ignore limits) Must have $(2\theta)^2$ and $(\theta^2)^2$ and a subtraction for this mark.	
			$= \frac{1}{2}\int_0^2 4\theta^2 - \theta^4 \mathrm{d}\theta = \frac{1}{2}\left[\frac{4}{3}\theta^3 - \frac{1}{5}\theta^5\right]_0^2$	**A1FT**	3.4	Integrating their expression correctly (ignore limits)	FT their integrals.
			$\frac{1}{2}\left(\frac{4}{3}\times 2^3 - \frac{1}{5}\times 2^5 - 0\right) = \frac{32}{15}$ or awrt 2.13	**A1**	1.1	Answer must be positive but condone recovery by removing negative sign at end	
				[4]			

Question			Answer	Marks	AO	Guidance	
7	(a)		**DR** $1^3 - 1^2 + 3 \times 1 - 3 = 0$ so $(x-1)$ is a factor	**M1**	2.2a	Using factor theorem in attempt to factorise numerator. Must come up with $f(a) = 0$ so $(x-a)$ is a factor.	May be implied by correct factorisation seen
			$x^3 - x^2 + 3x - 3 = (x-1)(x^2+3)$	**A1**	1.1	soi by denominators	
			$\dfrac{-4x^2+5x-17}{x^3-x^2+3x-3} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+3}$	**B1**	3.1a	Correct partial fraction form. **B0** if e.g. $+D$ or $(A+Ex)$ unless recovered (i.e. unnecessary constants found to be 0).	
			$-4x^2+5x-17$ $= A(x^2+3) + (Bx+C)(x-1)$	**M1**	1.1	Suitable method for determining constants, including comparing coefficients directly. (Provided two fractions with linear or quadratic denominators and any additional polynomial terms).	Condone minor errors e.g. $Bx + C(x-1)$ or denominator on one side provided intent is clear (can be determined by next step)
			$4A = -16 \Rightarrow A = -4$ $-4 = -4 + B \Rightarrow B = 0$ $-17 = -12 - C \Rightarrow C = 5$ $\left(\text{so } \dfrac{5}{x^2+3} - \dfrac{4}{x-1}\right)$	**A1** **A1**	1.1 1.1	Any constant correct from correct working (allow this mark following B0) All three constants (including $B = 0$) correct from correct working.	Final 3 marks independent of first two marks. ISW once constants found.
				[6]			
	(b)		**DR** $\int \dfrac{'4'}{x-1}\,dx = '4'\ln(x-1)$	**B1FT**	1.1	Soi. May see $\ln\lvert 1-x\rvert$ Ignore "+ c". FT their $\int \frac{a}{bx+c}\,dx = \frac{a}{b}\ln(bx+c)$	
			$\int \dfrac{'5'}{x^2+3}\,dx = k\tan^{-1}\dfrac{x}{\sqrt{3}}$	**M1**	1.1	Recognising the integral as $\tan^{-1}$ with any multiplicative constant. Condone 3 in the denominator. Could be their 'C'.	

			$\therefore \int \frac{5}{x^2+3} - \frac{4}{x-1}\,dx$ $= \frac{5}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 4\ln(x-1)$	**A1**	1.1	All correct. Ignore "+ *c*".	
			$\left[\frac{5}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 4\ln(x-1)\right]_{\sqrt{3}}^{3}$ $= \left(\frac{5}{\sqrt{3}}\tan^{-1}\sqrt{3} - 4\ln(2)\right)$ $\quad - \left(\frac{5}{\sqrt{3}}\tan^{-1}1 - 4\ln\left(\sqrt{3}-1\right)\right)$ $= \frac{5}{\sqrt{3}}\times\frac{\pi}{3} - 4\ln 2 - \frac{5}{\sqrt{3}}\times\frac{\pi}{4} + 4\ln\left(\sqrt{3}-1\right)$ $= \frac{5\pi}{12\sqrt{3}} + 4\ln\left(\frac{\sqrt{3}-1}{2}\right)$	**A1**	1.1	oe e.g. $\frac{5\pi}{6\sqrt{12}} + 2\ln\left(\frac{4-2\sqrt{3}}{4}\right)$ or $\frac{5\sqrt{3}\pi}{36} + \ln\left(\frac{7-4\sqrt{3}}{4}\right)$ or $\frac{5\pi}{\sqrt{432}} - \ln\left(28+16\sqrt{3}\right)$ or $\frac{5\sqrt{3}\pi}{36} - 4\ln\left(1+\sqrt{3}\right)$ but arctans must be evaluated and π terms must be combined into a single fraction and ln terms must be collected (allow $\ln(\ldots)^4$ forms provided there is a single ln term). ISW once a complete, correct, acceptable form seen from correct working.	Correct answers without working is 0/4.
				[4]			

Question			Answer	Marks	AO	Guidance	
8	(a)		Basis case: $\left(\text{LHS} = \frac{\mathrm{d}^0\mathrm{f}}{\mathrm{d}x^0} = \mathrm{f} = x\sinh 2x\right)$ $\text{RHS} = 4^0(x\sinh 2x + 0 \times \cosh 2x)$ $= x\sinh 2x$ $\therefore$ LHS = RHS oe so true for $n = 0$	**B1**	2.1	May see $n = 1$ in addition but this mark is for $n = 0$ only. Must see LHS = RHS $\Rightarrow$ true for $n = 0$ oe (but may appear later e.g. in conclusion)	Throughout this question condone single line slips to sin or cos provided recovered.
			Assume true for $n = k$: $\frac{\mathrm{d}^{2k}\mathrm{f}}{\mathrm{d}x^{2k}} = 4^k(x\sinh 2x + k\cosh 2x)$ Then for $n = k + 1$: $\therefore \frac{\mathrm{d}^{2(k+1)}\mathrm{f}}{\mathrm{d}x^{2(k+1)}} = \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(\frac{\mathrm{d}^{2k}\mathrm{f}}{\mathrm{d}x^{2k}}\right)$ $= \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(4^k(x\sinh 2x + k\cosh 2x)\right)$	**M1**	3.1a	Sets up and uses inductive hypothesis by showing an intent to differentiate twice and using the inductive assumption at least once. Complete statement might not be seen until after first or even second differentiation.	Must have statement in terms of some other variable than *n* in all three places consistently. Condone minor slips in notation e.g. $\frac{\mathrm{d}^2\boldsymbol{f}}{\mathrm{d}x^2}\left(\frac{\mathrm{d}^{2k}\mathrm{f}}{\mathrm{d}x^{2k}}\right)$ for all but the final A mark.
			$= 4^k\frac{\mathrm{d}}{\mathrm{d}x}(\sinh 2x + 2x\cosh 2x + 2k\sinh 2x)$ $\left(= 4^k\frac{\mathrm{d}}{\mathrm{d}x}((2k+1)\sinh 2x + 2x\cosh 2x)\right)$	**M1**	1.1	Differentiates using product rule. Condone sign errors, missing 2s, $\frac{1}{2}$ instead of 2 but not $\frac{1}{2}x^2$ as the derivative of *x*.	May be seen separately. Could be done with exponential form but **M1** awarded on correct return to hyperbolic form (which could be after later differentiation).
			$= 4^k(2(2k+1)\cosh 2x + 2\cosh 2x + 4x\sinh 2x)$	**M1**	1.1	Differentiates again. Same guidance. Or $= 4^k(4k\cosh 2x + 4\cosh 2x + 4x\sinh 2x)$	Allow both differentiations in one step if fully correct.
			$= 4^k(4(k+1)\cosh 2x + 4x\sinh 2x)$ $= (4)^k(4)\left((k+1)\cosh 2x + x\sinh 2x\right)$ $= 4^{k+1}(x\sinh 2x + (k+1)\cosh 2x)$	**A1**	2.1	Rewrites to the correct form.	
			So true for $n = k \Rightarrow$ true for $n = k + 1$ True for $\boldsymbol{n = 0} \Rightarrow$ true for $n \geq 0$	**A1**	2.4	Clear conclusion for induction process with implication stated. Dependent on all previous M and A marks www and fully correct notation throughout.	Allow this mark if the basis case was given as $n = 1$ only but their conclusion must match their basis case (i.e. $n = 1 \Rightarrow n \geq 1$).
				[6]			

	(b)		$n = 4 \Rightarrow \frac{\mathrm{d}^8 \mathrm{f}}{\mathrm{d}x^8}\|_{x=0} = 4^4(0\sinh 0 + 4\cosh 0)$	**M1**	3.1a	$= 1024$ Chooses correct value of *n* and uses formula to find value for appropriate derivative when $x = 0$.	
			Coefficient of x^8 is $\frac{\mathrm{f}^{(8)}(0)}{8!}$	**M1**	2.2a	Using the correct rule for finding the coefficient of x^8. This is independent of the previous mark and $\mathrm{f}^{(8)}(0)$ need not be correct or even evaluated but must be $x = 0$.	
			$= \frac{1024}{40320} = \frac{8}{315}$	**A1**	1.1	Must be simplified. Do not accept eg 0.0254 etc but ISW once 8/315 seen unless it is clear that a different value is intended (eg 8 from multiplying by 315).	Allow with x^8 e.g. $\frac{8}{315}x^8$ Answer only with no working scores 0/3.
				[3]			
	(c)		$\sinh 2x = \frac{e^{2x} - e^{-2x}}{2}$ ...	**B1**	1.1	Uses exponential definition of sinh*x* with 2*x* as argument...	
			... so using general term for given series for e^x, x^7term is $= \frac{1}{2}\left(\frac{(2x)^7}{7!} - \frac{(-2x)^7}{7!}\right)$	**M1**	3.1a	... and given general term (or complete series) for e^x to try to find (coefficient of) x^7term of $\sinh 2x$ (or x^8 term of $x\sinh 2x$).	
			So coefficient of x^8 term in $x\sinh 2x$ is $\frac{1}{2}\left(\frac{2^7}{7!} - \frac{(-2)^7}{7!}\right) = \frac{1}{2}\left(\frac{128+128}{5040}\right) = \frac{128}{5040} = \frac{8}{315}$	**A1**	2.2a	No conclusion necessary. Must be in same form as answer to **(b)** (ie explicit verification) (but allow this mark for a correct answer if no solution in **(b)**).	Allow **SC A1** if (b) and (c) both given as eg 0.0254 Allow with x^8 e.g. $\frac{8}{315}x^8$
				[3]			

Question			Answer	Marks	AO	Guidance	
9	(a)		$y = Ae^{-3t} + Bte^{-3t}$ $\Rightarrow y' = -3Ae^{-3t} + Be^{-3t} - 3Bte^{-3t}$ $\Rightarrow y'' = 9Ae^{-3t} - 3Be^{-3t} - 3Be^{-3t} + 9Bte^{-3t}$	**M1**	3.1a	Differentiating given CF twice using product rule. Allow sign errors in either derivative (but coefficients must be correct).	Condone use of e.g. x instead of t for this mark but not a mixture.
			$\therefore y'' + 6y' + 9y =$ $9Ae^{-3t} - 3Be^{-3t} - 3Be^{-3t} + 9Bte^{-3t}$ $-18Ae^{-3t} + 6Be^{-3t} - 18Bte^{-3t} + 9Ae^{-3t}$ $+9Bte^{-3t} = 0$ (as required for a CF.)	**A1**	2.1	Cancelling or combining terms need not be seen, but all must be present before $= 0$.	Condone arguments leading to e.g. $0 = 0$ provided all terms correct.
				[2]			
	(b)		PI: Try $y = \alpha t^2 e^{-3t}$	**B1**	1.2	Correct form for PI. Allow more general forms e.g. $\alpha t^2 e^{-3t} + \cdots$ if subsequent coefficients β, γ etc. later found to be 0.	Condone use of e.g. x instead of t for this mark and the first 3 M marks (but no further).
			$y' = 2\alpha te^{-3t} - 3\alpha t^2 e^{-3t}$ $y'' = 2\alpha e^{-3t} - 6\alpha te^{-3t} - 6\alpha te^{-3t} + 9\alpha t^2 e^{-3t}$ $(= 2\alpha e^{-3t} - 12\alpha te^{-3t} + 9\alpha t^2 e^{-3t})$	**M1*** **M1 dep***	1.1 1.1	Attempt to differentiate their y twice (finding y' and y''), using the product rule at least once. Correctly differentiating their y to find y' and y''. Condone sign errors.	Allow this mark if their PI is of the form $\alpha p(t)e^{-3t}$ where $p(t)$ is any polynomial in t of degree ≥ 1. Condone missing 3, extra 3 or 1/3 coming down but not t coming down and/or $3t - 1$ as exponent.
			$y'' + 6y' + 9y = 2\alpha e^{-3t} - 12\alpha te^{-3t} + 9\alpha t^2 e^{-3t} + 12\alpha te^{-3t} - 18\alpha t^2 e^{-3t} + 9\alpha t^2 e^{-3t} = 10e^{-3t}$	**M1**	1.1	Substituting their y' and y'' into the DE (with RHS seen or e^{-3t} cancelled from both sides). Allow this mark following M0M0.	This mark may be implied by the next A1 (i.e. a correct GS).
			$\Rightarrow 2\alpha e^{-3t} = 10e^{-3t}$ $\Rightarrow \alpha = 5 \Rightarrow$ GS is $(A + Bt)e^{-3t} + 5t^2 e^{-3t}$	**A1**	3.3	Full form of GS can be implied by later work.	
			$(A + Bt + 5t^2)e^{-3t} = 0$ when $t = 5$ $\Rightarrow A + 5B + 125 = 0$	**M1**	3.4	Substituting $y = 0$ and $t = 5$ into their GS and cancelling any exponential part properly.	May be implied by later working including e^{-3t} if fully correct.

			$A + Bt + 5t^2 \geq 0 \Rightarrow B^2 - 4 \times 5A = 0$	**M1**	3.4	Correctly interpreting and using the 2nd condition which, coupled with the 1st, must mean that the quadratic is a perfect square. May see $3A + 14B = -325$ oe.	Or $B + 10t = 0$ when $t = 5$ (at what must be a minimum point). Or $5\left(t^2 + \frac{Bt}{5}\right) + A = 5\left(t + \frac{B}{10}\right)^2 + A - \frac{B^2}{20} \Rightarrow A - \frac{B^2}{20} = 0$ since $y = 0$.
			$\Rightarrow A = 125$ and $B = -50$ $\therefore$ PS is $y = (5t^2 - 50t + 125)e^{-3t}$	**A1**	3.4	Must be $y =$ or $y(t) =$… May see $5(t^2 - 10t + 25)$ or $5(t-5)^2$	Allow **SCB1** for $A = 125$ and $B = -50$ following M0, provided all 6 of the first marks are gained (7/8).
				[8]			
	(c)		(When $t = 0, y = 125e^{-0} =$) 125	**B1FT**	3.4	FT their PS of the form $p(t)e^{kt}$ where $p(t)$ is any polynomial of non-zero degree.	Must be a value (i.e. not an expression with *A, B,* etc.) and must be ≥ 0.
				[1]			
	(d)		(According to the model) when $t = 1, y = 80e^{-3} = 3.982965$ ...	**M1**	3.4	Finding the value of their y when $t = 1$. FT their PS of the form $p(t)e^{kt}$ where $p(t)$ is any polynomial.	Must be a value (i.e. not an expression with *A, B,* etc.)
			This is < 4 hence the cell can be inferred to be operating properly (but, as the value is very close to 4, there may be a margin of error in the model).	**A1FT**	2.2b	Sensible conclusion in context with correct comparison to 4 for their value (A0 for a value < 0). (i.e. $< 4 \Rightarrow$ operating properly, $> 4 \Rightarrow$ not operating properly). Condone 'it is operating properly'.	Accept e.g.: - 'So it can be inferred that the cell is probably working well although we can't be sure because it is close to 4 and we don't know how well the model works' B1
				[2]			

Question			Answer	Marks	AO	Guidance	
10	(a)		$\left(\frac{\frac{1}{2}e^{-2}}{\sinh 2}\times 100\% =\right)$ awrt 1.87% $\left(\frac{\frac{1}{2}e^{-5}}{\sinh 5}\times 100\% =\right)$ awrt 0.00454%	B1	1.1	For both % symbol not necessary. 1.865736036... % symbol not necessary. $4.540199101...\times10^{-3}$	
				[1]			
	(b)		$\left(\frac{\cosh 2-\sinh 2}{\sinh 2}\times 100\% =\right)$ awrt $\pm$ 3.73% $\left(\frac{\cosh 5-\sinh 5}{\sinh 5}\times 100\% =\right)$ awrt $\pm$ 0.00908%	B1	1.1	For both % symbol not necessary 3.731472073... % symbol not necessary 9.08039820×10^{-3}	
				[1]			
	(c)		**DR** (If n large then) $\sinh(0.01n)\sim 0.5e^{0.01n}$ or $\tanh n\sim 1$ or $\cosh(0.005n)\sim 0.5e^{0.005n}$	B1	3.1a	One correct approximation soi.	
			$0.5e^{0.01n}-2.5e^{0.005n}-9>100$	M1	2.2a	Correct use of approximations to derive an inequality in n.	Accept equations in place of inequalities for all M marks.
			$y=e^{0.005n}\Rightarrow 0.5y^2-2.5y-109>0$ $y^2-5y-218>0$	M1*	3.1a	Rearranging to a 3 term quadratic inequality in $e^{0.005n}$	May see quadratic in $e^{0.005n}$.
			$\therefore e^{0.005n}>17.47$ ...	M1 dep*	2.2a	Correctly solves *their* quadratic equation to find expression for $e^{0.005n}$	17.47497913 ... Or $\left(\frac{5+\sqrt{897}}{2}\right)$ Other root does not need to be explicitly rejected.
			$\therefore n>200\ln 17.47 ... = 572.15 ...$ But n is an integer so $n_{\min}=573$	A1	3.2a		
			f(572) = 99.6 ... < 100 and f(573) = 100.96 ... > 100	A1	1.1	Must see values, not just an assertion.	NB no marks for f(572) = 99.6 ... < 100 and f(573) = 100.96 ... > 100 without justification.

			Alternative method **DR** (If n large then) $\sinh x \sim \cosh x$ or $\tanh n \sim 1$	**B1**	3.1a	One correct approximation soi.	
			$\sinh 0.01n - 5\sinh 0.005n - 9 > 100$	**M1**	2.2a	Correct use of **both** approximations to derive an inequality in sinh or cosh **only**. May see $\cosh 0.01n - 5\cosh 0.005n - 9$ and use of $\cosh x$ throughout.	Accept equations in place of inequalities for all M marks.
			$u = 0.005n \Rightarrow \sinh 2u - 5\sinh u - 9 > 100$ $\Rightarrow 2\sinh u\cosh u - 5\sinh u - 9 > 100$ $\Rightarrow 2\sinh^2 u - 5\sinh u - 9 > 100$ $\Rightarrow 2\sinh^2 u - 5\sinh u - 109 > 0$	**M1***	3.1a	Using an appropriate identity (e.g. $\sinh 2x \equiv 2\sinh x\cosh x$) and using approximation to rearrange to a 3 term quadratic inequality in sinh or cosh only.	May see quadratic in e.g. $y = \sinh u = \sinh 0.005n$ i.e. $2y^2 - 5y - 109 > 0$.
			$\therefore \sinh u > 8.737\ldots$ $[\Rightarrow u > 2.864\ldots]$	**M1 dep***	2.2a	Correctly solves *their* quadratic equation to find expression for $\cosh u$ or $\sinh u$	$y > 8.737489\ldots$ or $\frac{5+\sqrt{897}}{4}$ Other root does not need to be explicitly rejected.
			$\therefore n > 200 \times 2.864\ldots = 572.8057\ldots$ But n is an integer so $n_{\min} = 573$	**A1**	3.2a		
			$f(572) = 99.6\ldots < 100$ and $f(573) = 100.96\ldots > 100$	**A1**	1.1	Must see values, not just an assertion.	NB no marks for $f(572) = 99.6\ldots < 100$ and $f(573) = 100.96\ldots > 100$ without justification.
				[6]			

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