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# Thursday 22 May 2025 – Afternoon

## A Level Further Mathematics A

**Y540/01** Pure Core 1

**Time allowed: 1 hour 30 minutes**

*1920517432*

**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

**ADVICE**
- Read each question carefully before you start your answer.

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**1** **(a)** A matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$.

Describe the transformation represented by **M**. **[2]**

**(b)** Write down the $2 \times 2$ matrix that represents a rotation of 90° anticlockwise about the origin. **[1]**

**(c)** Write down the $3 \times 3$ matrix that represents a reflection in the $x$–$z$ plane. **[1]**

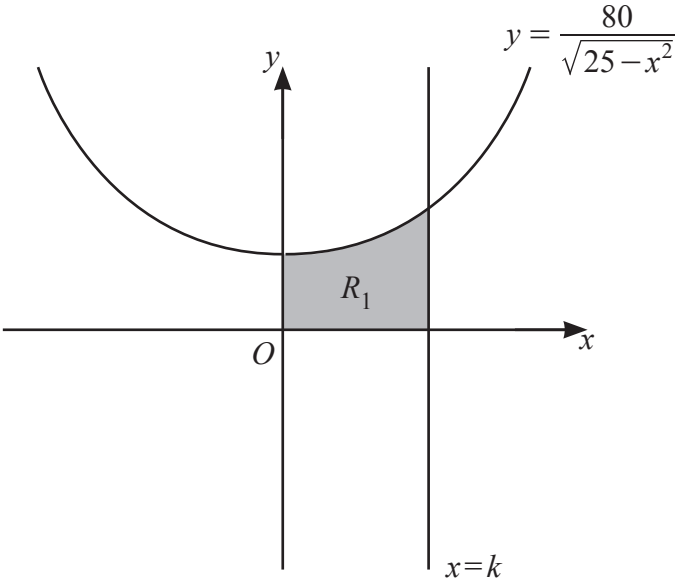
**2** **(a)** Given that $y = \cosh^{-1}\left(\frac{1}{3}x\right)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$. **[1]**

**(b)** Determine an equation of the normal to the curve $y = \cosh^{-1}\left(\frac{1}{3}x\right)$ at the point where $x = 5$.
Give your answer in the form $ax + by = c + \ln d$, where $a$, $b$, $c$ and $d$ are integers. **[4]**

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**3** The region $R_1$ is bounded by the curve $y = \frac{80}{\sqrt{25-x^2}}$, the line $x = k$ and the coordinate axes, as shown in **Fig. 1**.

**Fig. 1**

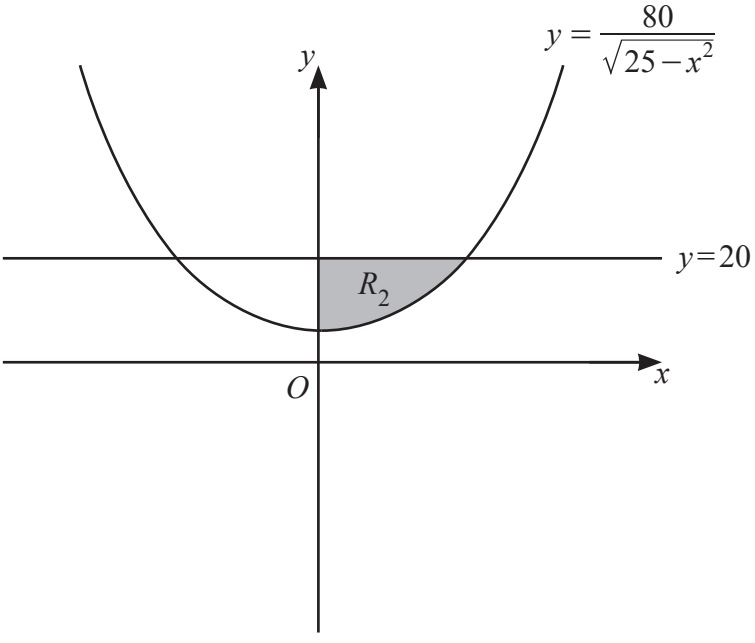



**(a) In this question you must show detailed reasoning.**

Given that the area of $R_1$ is $\frac{40}{3}\pi$, determine the value of $k$. **[3]**

The region $R_2$ is bounded by the curve $y = \frac{80}{\sqrt{25-x^2}}$, the line $y = 20$ and the $y$-axis as shown in **Fig. 2**.

**Fig. 2**




**(b)** Find, in an exact form, the volume of the solid formed when $R_2$ is rotated by $2\pi$ radians about the ***y*-axis**. **[3]**

**4** The equation $5x^3 - 4x^2 + 10 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

**(a)** Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **[2]**

**(b)** By expanding $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$, determine the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. **[3]**

**(c)** By expanding a suitable expression, find the value of $\alpha^2 + \beta^2 + \gamma^2$. **[2]**

**(d)** Hence find a cubic equation with integer coefficients that has roots $\alpha^2$, $\beta^2$, and $\gamma^2$. **[2]**

**5** A vector equation of the plane $\Pi_1$ is $\mathbf{r} = \begin{pmatrix}1\\4\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\-1\end{pmatrix} + \mu\begin{pmatrix}1\\1\\0\end{pmatrix}$.

**(a)** **Verify** that a cartesian equation of $\Pi_1$ is $x - y + 2z = 3$. **[1]**

For some real constant $a$, cartesian equations of planes $\Pi_2$ and $\Pi_3$ are

$\Pi_2$: $\quad x \qquad -3z = 1$
$\Pi_3$: $\quad ax - y - z = 4$

**(b)** By considering a suitable matrix, show that $\Pi_1$, $\Pi_2$ and $\Pi_3$ intersect at a single point for all values of $a$ except $a = 2$. **[3]**

**(c)** Use the matrix from part **(b)** to find the coordinates of the point of intersection of $\Pi_1$, $\Pi_2$ and $\Pi_3$ in the case where $a = 3$. **[2]**

**(d)** In the case where $a = 2$, determine the geometrical arrangement of $\Pi_1$, $\Pi_2$ and $\Pi_3$. **[2]**

**6** **In this question you must show detailed reasoning.**

The series $S_n$ is given by $S_n = \left(\frac{1}{5} \times \frac{1}{15}\right) + \left(\frac{1}{15} \times \frac{1}{25}\right) + \cdots + \left(\frac{1}{10n-5} \times \frac{1}{10n+5}\right)$ for $n \in \mathbb{Z}^+$.

**(a)** Use the method of differences to show that, for all $n \in \mathbb{Z}^+$, $S_n < \frac{1}{50}$. **[5]**

Let $S_\infty = \lim_{n \to \infty} S_n$.

**(b)** Given that, for some value of $k$, $S_\infty = \frac{1}{2450} + S_k$, find the value of $k$. **[3]**

**7** A 3-D coordinate system, whose units are metres, is set up to model a street containing telephone cables $T_1$ and $T_2$.

The cables are modelled as straight lines with vector equations

$$T_1\text{: } \mathbf{r} = \begin{pmatrix}3\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}5\\4\\1\end{pmatrix} \text{ and } T_2\text{: } \mathbf{r} = \begin{pmatrix}8\\2\\4\end{pmatrix} + \mu\begin{pmatrix}2\\-1\\1\end{pmatrix}.$$

**(a)** Show that the cables do not intersect. **[3]**

To access the cables for maintenance, a ladder can be used. The base of the ladder is placed at a fixed point on the ground.

The ladder is modelled as a straight-line segment. The base of the ladder is modelled as being located at the point (4, 5, 0).

**(b)** Determine the minimum length of the ladder required so that it reaches cable $T_1$. Give your answer in centimetres to the nearest centimetre. **[4]**

**(c)** Identify a modelling assumption used in part **(a)** that is unrealistic, **and** which could affect your answer to this part. **[1]**

**8** **In this question you must show detailed reasoning.**

In this question, arguments of complex numbers are in the interval $[0, 2\pi)$.

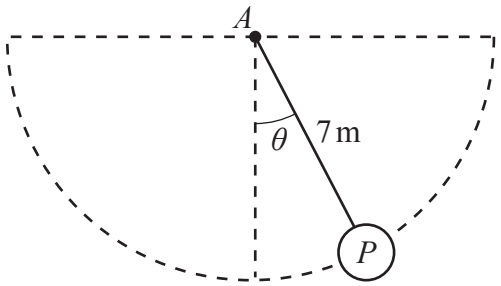
**(a)** Given that $z = -5 + 5\text{i}$, express $z$ in exact exponential form. **[2]**

**(b)** Given that $w = \dfrac{36\cos\left(\frac{1}{7}\pi\right) - 36\text{i}\sin\left(\frac{1}{7}\pi\right)}{27\sin\left(\frac{3}{7}\pi\right) + 27\text{i}\cos\left(\frac{3}{7}\pi\right)}$, express $w$ in exact exponential form. **[5]**

**9** A pendulum comprises an object $P$ of mass $m$ kg and a string of length 7 m. One end of the string is attached to $P$ and the other end is attached to a fixed point $A$.

At time $t$ seconds, $t \geqslant 0$, the string forms an angle of $\theta$ radians, measured anti-clockwise, from the downward vertical through $A$, as shown in the diagram.

When $t = 0$, $\theta = \theta_0 > 0$ and $P$ is released from rest. You may assume that in the subsequent motion $P$ moves along the arc of a circle, centre $A$ and radius 7 m, and that $|\theta| \leqslant \theta_0$ for all $t \geqslant 0$.




The motion of $P$ is modelled by the following differential equation.

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{7}{5}\sin\theta = 0 \ (*)$$

In some situations, it is appropriate to approximate $(*)$ with the following differential equation.

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{7}{5}\theta = 0 \ (**)$$

**(a)** Explain why it would be appropriate to model the motion of $P$ with the differential equation $(**)$ when $\theta_0 = \frac{1}{15}\pi$ but not when $\theta_0 = \frac{1}{3}\pi$. **[1]**

You are now given that $\theta_0 = \frac{1}{15}\pi$.

**(b)** By finding the particular solution to the differential equation $(**)$, determine the total **distance** travelled by $P$ in the first 6 seconds of the motion according to $(**)$. **[6]**

An additional force now acts on $P$. It can be shown that it is now appropriate to model the motion of $P$ with the differential equation $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{k}{m}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{7}{5}\theta = 0$ where $k > 0$.

**(c)** Find the range of values of $m$, in terms of $k$, for which the motion of the pendulum is overdamped. **[2]**

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**10 In this question you must show detailed reasoning.**

**(a)** Use de Moivre's Theorem to show that, if $\cos 5\theta \neq 0$,

$$\tan 5\theta \equiv \frac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{5\tan^4\theta - 10\tan^2\theta + 1}. \quad \textbf{[4]}$$

**(b) (i)** By considering the equation $\tan 5\theta = 1$, use the result in part **(a)** to find the exact roots of the equation

$$t^4 - 4t^3 - 14t^2 - 4t + 1 = 0.$$

Give the roots in the form $t = \tan\phi$ where $0 < \phi < \pi$. **[4]**

**(ii)** By first expressing $t^4 - 4t^3 - 14t^2 - 4t + 1 = 0$ in the form $(t-1)^4 = kt^2$, where $k$ is a constant to be determined, show that

$$\tan\left(\frac{9}{20}\pi\right) = 1 + \sqrt{5} + \sqrt{5 + 2\sqrt{5}}. \quad \textbf{[3]}$$

**END OF QUESTION PAPER**

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**Y540/01** Pure Core 1

Printed Answer Booklet

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AB

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- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

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**INFORMATION**
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**4(d)**

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**5(a)**

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## EXTRA ANSWER SPACE

If you need extra space use these lined pages. You must write the question numbers clearly in the margin.

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