OCR
Oxford Cambridge and RSA

# GCE

# Further Mathematics A

## Y545/01: Additional Pure Mathematics

A Level

# Mark Scheme for June 2023

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

© OCR 2023

## Text Instructions

### 1. Annotations and abbreviations

| Annotation in RM assessor | Meaning |
|---|---|
| ✓and ✗ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| BP | Blank Page |
| Seen | |
| Highlighting | |
| | |
| **Other abbreviations in mark scheme** | **Meaning** |
| dep* | Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| AG | Answer given |
| awrt | Anything which rounds to |
| BC | By Calculator |
| DR | This question included the instruction: In this question you must show detailed reasoning. |

## 2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

**M**
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

**A**
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question | | Answer | Marks | AO | Guidance |
|---|---|---|---|---|---|
| 1 | (a) | A straight line with positive gradient | M1 | 2.2a | i.e. with correct axes (labelled or otherwise shown or implied by an equation in *y*-*z*) |
| | | $z = 2y + 1$ clearly shown in the *y*-*z* plane | A1 | 1.1 | By intercepts $(-\frac{1}{2}, 0)$ and $(0, 1)$ or any other means |
| | | | [2] | | |
| | (b) | A ∪-shaped parabola | M1 | 2.2a | i.e. with correct axes (labelled or otherwise shown or implied by an equation in *x*-*z*) |
| | | $z = x^2 + 2x$ clearly shown in the *x*-*z* plane | A1 | 1.1 | By intercepts (-2,0) and (0,0) or any other means |
| | | | [2] | | |
| 2 | (a) | $1+\left(\frac{dy}{dx}\right)^2 = 1+\left(\frac{1}{2}(1+x^2)^{-\frac{1}{2}}.2x\right)^2$ | M1 | 1.1 | Including attempt at $\frac{dy}{dx}$ using the Chain Rule |
| | | $= 1+\frac{x^2}{1+x^2}$ or $\frac{1+2x^2}{1+x^2}$ | A1 | 1.1 | |
| | | $A = 2\pi\int_0^1 \left(\sqrt{1+x^2}\times\sqrt{\frac{1+2x^2}{1+x^2}}\right)dx$ | M1 | 1.1 | Use of SA formula with their appropriate terms |
| | | $= 2\pi\int_0^1 \left(\sqrt{1+2x^2}\right)dx$ | A1 | 1.1 | i.e. $k = 2$ |
| | | | [4] | | |
| | (b) | Finding a numerical answer for *A* (BC) **and** two of {7.92, awrt 7.95, awrt 8.03, 8.08} | M1 | 3.1b | **Note:** $A = 7.987\,649\ldots$<br>$\{8\cdot 0.99, A\cdot 0.995, A\cdot 1.005, 8\cdot 1.01\}$ |
| | | Comparison of correct numbers and correct conclusion (*yes*) | A1 | 1.1 | CAO from fully correct working |
| | | | [2] | | |

| Question | | | Answer | Marks | AO | Guidance |
|---|---|---|---|---|---|---|
| 3 | (a) | | $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 1 \\ p \\ q \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2p-3q \\ 2q-2 \\ 3-2p \end{pmatrix}$ | **B1** | **3.1a** | At least 2 entries correct |
| | | | Solving for $p$, $q$ in $\begin{pmatrix} 2p-3q \\ 2q-2 \\ 3-2p \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -11 \end{pmatrix}$ | **M1** | **1.1** | Equating their $\mathbf{a} \times \mathbf{b}$ to the RH vector and solving attempt |
| | | | $p = 7$, $q = 4$ | **A1** | **1.1** | No need to check for consistency of the unused component |
| | | | | **[3]** | | |
| | (b) | i | Use of $\frac{1}{6}(\mathbf{a} \times \mathbf{b}) \bullet \mathbf{c} = (\pm)7$ oe | **M1** | **1.2** | Tetrahedron formula used; allow = +7 or –7 only |
| | | | $\begin{pmatrix} 2 \\ 6 \\ -11 \end{pmatrix} \bullet \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \pm 42 \Rightarrow 2d + 6e - 11f = \pm 42$ | **A1** | **1.1** | **CAO** (up to non-zero multiples)<br>Allow $\lvert 2d + 6e - 11f \rvert = 42$. |
| | | | | **[2]** | | |
| | (b) | ii | (One/Two) plane(s) … | **M1** | **2.2a** | Must be clearly the plane(s) $C$ belongs to (not OAB) |
| | | | … parallel to the plane of $OAB$<br>(at the correct suitable distance) | **A1** | **2.5** | Distance not required |
| | | | | **[2]** | | |

| Question | | Answer | Marks | AO | Guidance |
|---|---|---|---|---|---|
| 4 | | $I_{n+2} = \int cos^{n+1} x \ . \ cos\, x \ \mathrm{d}x = [cos^{n+1} x \quad sin\, x] \ \ldots$ | M1* | 3.1a | Correct use of integration by parts. Allow sign errors in der/int of $\cos x$.<br>Or starting from $I_n$ |
| | | $-\int (n+1)\, cos^{n} x\, (-sin\, x) \,.\, sin\, x \ \mathrm{d}x$ | A1 | 1.1 | Fully correct first stage |
| | | $= (0 +)\ (n+1)(I_n - I_{n+2})$ | M1dep | 2.1 | Use of $s^2 = 1 - c^2$ to express the integral(s) correctly in terms of $I_k$ and limits substituted. |
| | | $\Rightarrow\ (n+2)\, I_n = (n+1)\, I_{n+2}$ | A1 | 1.1 | Correct reduction formula |
| | | Then $A_n = \dfrac{I_{n+2}}{I_n} = \dfrac{n+1}{n+2}$ or $1 - \dfrac{1}{n+2}$<br>$\to 1$ as $n \to \infty$ i.e. $A = 1$ | B1 | 2.2a | **SC1** for arriving at correct $A_n$ with correct evidence for at least three consecutive terms e.g. $A_0 = \frac{1}{2}, A_1 = \frac{2}{3}, A_2 = \frac{3}{4}$.<br>From consideration of the relevant ratio of terms |
| | | Since $A_{n+1} = 1 - \dfrac{1}{n+3} > 1 - \dfrac{1}{n+2} = A_n$ (for all $n$) | M1 | 2.1 | Or considering $A_{n+1} - A_n = \dfrac{1}{(n+2)(n+3)} > 0$<br>Or considering $\frac{A_{n+1}}{A_n} > 1$ oe<br>Condone $\geq$ |
| | | it follows that $\{A_n\}$ is monotonic increasing | A1 | 3.2a | Correct answer from appropriate reasoning |
| | | | [7] | | |

| Question | | | Answer | Marks | AO | Guidance |
|---|---|---|---|---|---|---|
| 5 | (a) | i | $\times_{32}$ \| 1 \| 9 \| 17 \| 25<br>1 \| 1 \| 9 \| 17 \| 25<br>9 \| 9 \| **17** \| **25** \| **1**<br>17 \| 17 \| **25** \| **1** \| **9**<br>25 \| 25 \| **1** \| **9** \| **17** | **B1**<br><br>**B1**<br>**[2]** | **1.1**<br><br>**1.1** | Any two bold entries (in shaded squares) correct<br><br>All bold entries (in shaded squares) correct |
| | | ii | $G \cong C_4$<br>Since **either** $G$ has a generator (9 or 25)<br>**or** from the main diagonal, only 2 of $G$'s elements<br>are self-inverse, so $G \ncong K_4$ | **B1**<br><br>**B1**<br><br>**[2]** | **2.2a**<br><br>**2.4** | Allow statement that $G$ is cyclic<br><br>Either generator noted<br>Or statement that only 1 non-identity element is self-inverse |
| | **(b)** | **i** | 1, 9, 25, 17 | **B1**<br>**B1**<br>**[2]** | **1.2**<br>**1.1** | Any one noted (other than 1)<br>All four noted (and no extras) |

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td></td><td></td><td>ii</td><td>$n^6 + 3n^4 + 7n^2 = (n^2)^3 + 3(n^2)^2 + 7n^2$<br>or<br>$n^6 + 3n^4 + 7n^2 - 11 = (n^2)^3 + 3(n^2)^2 + 7n^2 - 11$</td><td>M1</td><td>3.1a</td><td>Substitution of $n^2$ with a quadratic residue other than 1 (one of 1, 9, 17, 25) or in equivalent factorised forms or substituting 2k+1 for n.<br>NB (mod 32)<table><tr><td>$n^2$</td><td>1</td><td>9</td><td>17</td><td>25</td></tr><tr><td>$n^4$</td><td>1</td><td>17</td><td>1</td><td>17</td></tr><tr><td>$n^6$</td><td>1</td><td>25</td><td>17</td><td>9</td></tr></table></td></tr>
<tr><td></td><td></td><td></td><td></td><td>M1</td><td>1.1</td><td>Substitutions of all four residues or the powers of 2k+1 expanded. Allow numerical errors.</td></tr>
<tr><td></td><td></td><td></td><td>Then $n^6 + 3n^4 + 7n^2 \equiv 11, 139, 139, 235$<br>Or $n^6 + 3n^4 + 7n^2 \equiv 11, 1035, 5899, 17675$<br>Or $n^6 + 3n^4 + 7n^2 =$<br>$64k^6 + 192k^5 + 288k^4 + 256k^3 + 160k^2 + 64k + 11$</td><td>A1</td><td>2.2a</td><td>Evaluating the expression for all cases (other correct values are possible) or simplifying expression in terms of k<br>www</td></tr>
<tr><td></td><td></td><td></td><td>All options correctly shown to be<br>$n^6 + 3n^4 + 7n^2 \equiv 11$ (mod 32)</td><td>A1<br><br>[4]</td><td>2.4</td><td>All working correct</td></tr>
</table>

| Question | | Answer | Marks | AO | Guidance |
|---|---|---|---|---|---|
| 6 | (a) | $\frac{\partial z}{\partial x} = \sin y - \frac{y}{x^2}$ $\quad \frac{\partial z}{\partial y} = x\cos y + \frac{1}{x}$ | **B1**<br>**B1** | **1.1**<br>**1.1** | |
| | | $\frac{\partial^2 z}{\partial x^2} = \frac{2y}{x^3}$ and $\frac{\partial^2 z}{\partial y^2} = -x\sin y$ | **B1** | **1.1** | Both correct |
| | | $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = \cos y - \frac{1}{x^2}$ | **B1** | **1.1** | Only one needs to be seen |
| | | $\lvert \mathbf{H} \rvert = \begin{vmatrix} \frac{2y}{x^3} & \cos y - \frac{1}{x^2} \\ \cos y - \frac{1}{x^2} & -x\sin y \end{vmatrix} =$ | **M1** | **1.1** | **H** attempted with their correct partial derivatives (as functions) in the appropriate places |
| | | $-\frac{2y\sin y}{x^2} - \left(\cos y - \frac{1}{x^2}\right)^2$ | **A1**<br>**[6]** | **1.1** | Any correct (unsimplified) form www |
| | **(b)** | *P* is a saddle-point | **B1** | **2.2a** | |
| | | Since \|H\|<0 (for x, y in the given domain).<br>Both *y* and sin *y* are positive (in (0, π)) and the square is non-negative | **B1**<br>**[2]** | **2.4** | Convincing argument that \|H\| is *always* negative. |
| | **(c)** | $\frac{\partial z}{\partial x} = \sin y - \frac{y}{x^2} = 0$ and $\frac{\partial z}{\partial y} = x\cos y + \frac{1}{x} = 0$ | **M1*** | **1.1** | Setting both their 1st partial derivatives to zero |
| | | $\Rightarrow \frac{1}{x^2} = \frac{\sin y}{y} = -\cos y$ | **M1dep** | **1.1** | Valid method for eliminating the *x*'s and use of $y = \beta$ at some stage |
| | | $\Rightarrow \beta + \tan\beta = 0$ | **A1**<br>**[3]** | **1.1** | **AG** www |

| Question | | | Answer | Marks | AO | Guidance |
|---|---|---|---|---|---|---|
| 7 | (a) | | $\alpha + \beta = 1$ and $\alpha\beta = -1$ | B1<br>[1] | 1.1 | |
| | (b) | i | $S_2 = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 3$<br>$S_3 = \alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2+\beta^2) - \alpha\beta(\alpha+\beta)$<br>$= 4$ | B1<br>M1<br>A1<br>[3] | 1.1<br>3.1a<br>1.1 | Note that $S_1 = 1$ from **(a) (i)**<br>**NB** This is $S_3 = S_1S_2 + S_1$ since $\alpha\beta = -1$<br>Using the exact values of $\alpha, \beta$ to find $S_i$ is accepted. |
| | | ii | $S_{n+2} = \alpha^{n+2} + \beta^{n+2}$<br>$= (\alpha+\beta)(\alpha^{n+1} + \beta^{n+1}) - \alpha^{n+1}\beta - \alpha\beta^{n+1}$<br>$= (\alpha+\beta)(\alpha^{n+1} + \beta^{n+1}) - \alpha\beta(\alpha^n + \beta^n)$<br>$=(1)S_n - (-1)S_{n-1} = S_n + S_{n-1}$ | M1<br>A1<br>[2] | 3.1a<br>2.2a | Or using $\alpha^2 - \alpha - 1 = 0,\ \beta^2 - \beta - 1 = 0$<br>**AG** |
| | | iii | Since $S_1$ and $S_2$ are integers, the Rec. Reln. of **(a) (iii)** $\Rightarrow S_n$ an integer for all positive integers $n$ (by induction) | B1<br>[1] | 2.4 | Explanation of the inductive nature of this scenario |
| | (c) | i | It fails to give an integer | B1<br>[1] | 3.5b | |
| | | ii | $S_n = \begin{cases} \text{INT}(\alpha^n + 1) & n \text{ even} \\ \text{INT}(\alpha^n) & n \text{ odd} \end{cases}$<br>or as a carefully-defined "nearest integer" function | M1<br>A1<br>[2] | 3.5c<br>3.5c | Use of the INT function<br>Both correct. Note: INT($x$ + 1) = INT($x$) + 1 |

| Question | | | Answer | Marks | AO | Guidance |
|---|---|---|---|---|---|---|
| 8 | (a) | i | $2121_n = 2n^3 + n^2 + 2n + 1$<br>$= (2n^3 + 2n) + (n^2 + 1) = (2n + 1)(n^2 + 1)$ | M1<br>A1<br>[2] | 1.2<br>1.1 | Seen, or implied by correct later algebra<br>Correct factorisation. Noting that a(n), $b(n) > 1$ is not required here. |
| | | ii | Since f($n$) is the product of two integers **greater than 1** (**Allow "not equal to 1"**), it is composite | B1<br>[1] | 2.4 | Properly justified non-primality of f($n$) |
| | (b) | i | $h \mid \{n(2n + 1) – 2(n^2 + 1)\}$ | M1 | 3.1a | Attempt at any linear combination of their a($n$) and b($n$) |
| | | | i.e. $(h \mid) \pm(n – 2)$ | A1 | 2.2a | |
| | | | $h \mid \{(2n + 1) – 2(n – 2)\}$ | M1 | 2.1 | Second step of this procedure attempted |
| | | | i.e. $h \mid 5$ and $h = 1$ or 5 | A1 | 2.4 | **AG** obtained from fully correct working |
| | | | **ALT.** $h \mid \{4(n^2 + 1) – (2n – 1)(2n + 1)\}$ | M1 | | Attempt at any linear combination of their a($n$) and b($n$) |
| | | | i.e. $h \mid 4n^2 + 4 – (4n^2 – 1)$ | A1 | | |
| | | | i.e. $h \mid 5$ | M1 | | |
| | | | $h = 1$ or 5 | A1 | | |
| | | | | [4] | | |
| | | ii | From **(b) (i)**, we could look for $(n – 2)$ a multiple of 5 | M1 | 3.1a | Or solving $2n + 1 \equiv 0 \pmod 5 \Rightarrow n \equiv 2 \pmod 5$ and/or $n^2 + 1 \equiv 0 \pmod 5 \Rightarrow n \equiv 2 \pmod 5$ |
| | | | $\Rightarrow n = 7$ | A1 | 1.1 | **NB** $n = 12$ gives $25 \times 145$, etc. |
| | | | | [2] | | |

| Question | | Answer | Marks | AO | Guidance |
|---|---|---|---|---|---|
| 9 | (a) | Require $b$ s.t. $a \oplus b = a$ i.e. $\dfrac{a+b}{ab+1} = a$ | M1 | 1.1 | Including attempt to solve (e.g. by multiplying across), possibly implied by correct answer |
| | | $\Leftrightarrow\ b = 0$ | A1<br>[2] | 2.2a | ("since $a^2 \neq 1$" need not be mentioned)<br>**SC1** verifying that 0 is the identity element |
| | (b) | Require $b$ s.t. $a \oplus b = 0$ | M1 | 2.1 | (Or for $x$) |
| | | $\Rightarrow\ b = -a$ i.e. $x^{-1} = -x$ | A1<br>[2] | 1.1 | (**NB** 0 is 'self-inverse') |
| | (c) | $(a \oplus b) \oplus c = \dfrac{a+b}{ab+1} \oplus c$ attempted | M1 | 1.1 | |
| | | $a \oplus (b \oplus c) = a \oplus \dfrac{b+c}{bc+1}$ attempted | M1 | 1.1 | **OR** noting symmetry in ($a$, $b$, $c$) of answer |
| | | Both shown equal to $\dfrac{a+b+c+abc}{ab+ac+bc+1}$ | A1<br>[3] | 1.1 | Or any equivalently correct equal forms |
| | (d) | The set $C$ is not closed under $\oplus$ since $a \oplus b$ undefined for any $(a, b) = \left(x, -\dfrac{1}{x}\right)$, $x \neq 0, \pm 1$ | B1<br>[1] | 2.3 | Closure shown not to apply (i.e. *with* example, general or specific) |
| | (e) | "Subgroup" of order 3 must be of the form $\{0, a, -a\}$ | B1 | 2.1 | Must be made clear from the working; may state $\{0, a, a \oplus a\}$ initially, instead |
| | | Require $a \oplus a = -a$ or $a \oplus a \oplus a = 0$ | M1 | 3.1a | oe |
| | | $a = \pm i\sqrt{3}$ from solving cubic equation ($a \neq 0$) | A1<br>[3] | 3.2a | |

**Need to get in touch?**

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

**Call us on**

**01223 553998**

**Alternatively, you can email us on**

**support@ocr.org.uk**

**For more information visit**

**ocr.org.uk/qualifications/resource-finder**

**ocr.org.uk**

**Twitter/ocrexams**

**/ocrexams**

**/company/ocr**

**/ocrexams**

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT

OCR is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored. © OCR 2023 Oxford Cambridge and RSA Examinations is a Company Limited by Guarantee. Registered in England. Registered office The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA.

Registered company number 3484466. OCR is an exempt charity.

OCR operates academic and vocational qualifications regulated by Ofqual, Qualifications Wales and CCEA as listed in their qualifications registers including A Levels, GCSEs, Cambridge Technicals and Cambridge Nationals.

OCR provides resources to help you deliver our qualifications. These resources do not represent any particular teaching method we expect you to use. We update our resources regularly and aim to make sure content is accurate but please check the OCR website so that you have the most up-to-date version. OCR cannot be held responsible for any errors or omissions in these resources.

Though we make every effort to check our resources, there may be contradictions between published support and the specification, so it is important that you always use information in the latest specification. We indicate any specification changes within the document itself, change the version number and provide a summary of the changes. If you do notice a discrepancy between the specification and a resource, please contact us.

Whether you already offer OCR qualifications, are new to OCR or are thinking about switching, you can request more information using our Expression of Interest form.

Please get in touch if you want to discuss the accessibility of resources we offer to support you in delivering our qualifications.