OCR
Oxford Cambridge and RSA

GCE

Further Mathematics A

Y544/01: Discrete Mathematics

A Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

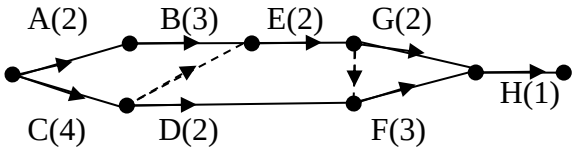
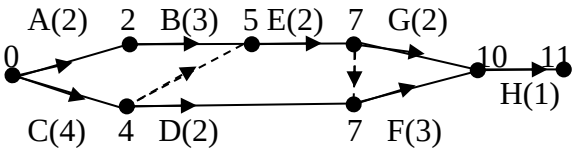
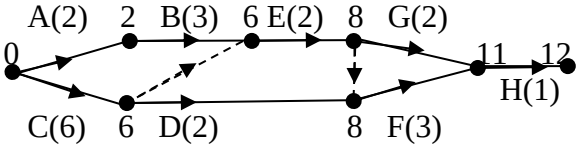
   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

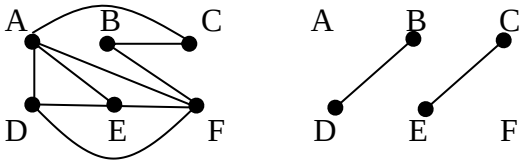
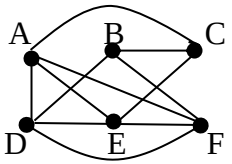
g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)			**M1** **A1** **B1** **[3]**	**3.3** **1.1** **3.3**	Activity on arc Durations not necessary, ignore any working Single start, precedences correct for A, B, C, D Single finish, precedences correct for E, F, G, H Correct use of exactly 2 directed dummy activities
1	(b)		 11 (hours)	**M1** **A1** **[2]**	**3.4** **1.1**	Forward pass attempted for their network Ignore backward pass if shown Or 2+3+2+3+1 seen cao **SC B1** answer 11 without valid method seen
1	(c)		 12 (hours)	**M1** **A1** **[2]**	**3.4** **2.2a**	Appropriate reasoning e.g. C becomes critical delays E by 1 hour o.e. E starts at 6 instead of 5, E finishes at 8, o.e. Or 6 + 2 + 3 + 1 seen cao **SC B1** answer 12 without valid method seen

Question			Answer	Marks	AO	Guidance
2	(a)		e.g. A – C – B – D – E – F – A	B1 [1]	1.1	A valid correct cycle (written)) through all 6 vertices with any starting point (which must also be shown as end point)
2	(b)		e.g. B – C – A – D – E – F – D – B – F – A – E – C	M1 A1 [2]	1.1 1.2	Any route that starts at B and ends at C, including A, D, E, F (or starts at C and ends at B) Any valid correct route (written) that uses every arc exactly once Must have each letter twice
2	(c)		{A, B, E}, {C, D, F} can be used to form $K_{3,3}$ by removing arcs AE and DF $K_{3,3}$ is a subgraph so non-planar	M1 A1	2.4 2.4	Identifying the subsets {A, B, E}, {C, D, F} or the arcs AE, DF $K_{3,3}$ and conclusion
			Alternative method Contraction of BC gives K_5 as a subgraph K_5 is a subgraph so non-planar	 M1 A1		 Contract BC K_5 and conclusion
				[2]		
2	(d)		Non-planar so thickness ≠ 1 Can be drawn using 2 layers e.g.  so thickness ≤ 2 hence thickness = 2	B1 M1 A1 [3]	2.1 2.1 1.1	Thickness must be > 1 (or thickness ≥ 2) from previous result Attempt to show how the graph can be drawn using only 2 layers (11 arcs, allow at most one error or omission) For reference:  A correct labelled diagram or complete description using 2 layers so thickness = 2

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>3</td><td>(a)</td><td></td><td>
<table>
<tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>RHS</td></tr>
<tr><td>1</td><td>0</td><td>2</td><td>-1</td><td>0</td><td>1</td><td>6</td></tr>
<tr><td>0</td><td>0</td><td>-1.5</td><td>1</td><td>1</td><td>-2.5</td><td>5</td></tr>
<tr><td>0</td><td>1</td><td>-0.5</td><td>0</td><td>0</td><td>0.5</td><td>3</td></tr>
</table>
<table>
<tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>RHS</td></tr>
<tr><td>1</td><td>0</td><td>0.5</td><td>0</td><td>1</td><td>-1.5</td><td>11</td></tr>
<tr><td>0</td><td>0</td><td>-1.5</td><td>1</td><td>1</td><td>-2.5</td><td>5</td></tr>
<tr><td>0</td><td>1</td><td>-0.5</td><td>0</td><td>0</td><td>0.5</td><td>3</td></tr>
</table>
</td><td>M1
A1
M1 ft
A1
[4]</td><td>1.1
1.1
1.1
1.1</td><td>Pivot row values correct for first iteration
Constraint row values correct
Second iteration has a valid pivot and pivot row values are correct (from their first iteration, provided not the same pivot as before)
Structure correct – but not matrix from QP (with P = 20)</td></tr>
<tr><td>3</td><td>(b)</td><td></td><td>P = 20, x = 0, y = 0, z = 20, s = 0, t = 6</td><td>B1
B1
[2]</td><td>1.1
1.1</td><td>For any three correct
For all six correct</td></tr>
<tr><td>3</td><td>(c)</td><td></td><td>The next pivot would have to be chosen from the y column
Since -1 is the only negative value in the objective row
But there are no positive entries in the other rows of the y column
So there is no valid pivot choice</td><td>M1
A1
[2]</td><td>2.2a
2.4</td><td>Identifying y
Explaining why there is no valid pivot choice
No other possible pivot col and no valid pivot row</td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>3</td><td>(d)</td><td></td><td>Pivot on k from y column
<table><tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>RHS</td></tr><tr><td>1</td><td></td><td>0</td><td></td><td></td><td>0</td><td>$60/k$</td></tr><tr><td>0</td><td>$5/k$</td><td>1</td><td>$1/k$</td><td>$1/k$</td><td>0</td><td>$20/k$</td></tr><tr><td>0</td><td></td><td>0</td><td></td><td></td><td>1</td><td></td></tr></table></td><td>M1</td><td>3.1.a</td><td>Pivot choice correct e.g. y column or row 2 correct
or implied from written work (e.g. ‘pivot on k’)</td></tr>
<tr><td></td><td></td><td></td><td></td><td>A1</td><td>2.2a</td><td>Pivot row correct, in terms of k</td></tr>
<tr><td></td><td></td><td></td><td></td><td>M1</td><td>3.4</td><td>Value of objective is $60/k$, or implied from $k = 0.12$</td></tr>
<tr><td></td><td></td><td></td><td>$60/k = 500 \Rightarrow k = 60/500 = 3/25 = 0.12$</td><td>A1</td><td>1.1</td><td>$k = 0.12$ (o.e.)</td></tr>
<tr><td></td><td></td><td></td><td>Alternative method</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>Pivot will be chosen from y column, so
after one iteration x and z will still be 0</td><td>M1</td><td></td><td>Pivot from y column so x and z are both 0 (soi)
y becomes basic, x and z are non-basic</td></tr>
<tr><td></td><td></td><td></td><td>$P = -2x + 3y – z$ so $3y = 500 \Rightarrow y = 500/3$</td><td>A1</td><td></td><td>$3y = 500$ (o.e.)</td></tr>
<tr><td></td><td></td><td></td><td>So RHS of pivot row becomes 166.67</td><td>M1</td><td></td><td>Entry in RHS column of pivot row = 500/3 (or 167 or better
or implied from $k = 0.12$</td></tr>
<tr><td></td><td></td><td></td><td>$20/k = 500/3 \Rightarrow k = 60/500 = 3/25 = 0.12$</td><td>A1</td><td></td><td>$k = 0.12$ (o.e.)</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[4]</td><td></td><td></td></tr>
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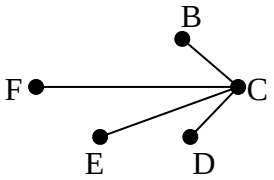
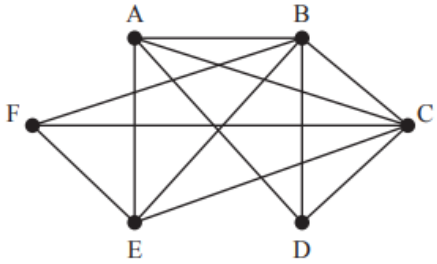
Question			Answer	Marks	AO	Guidance
4	(a)		2 primes from 8 and 3 non-primes from 12 $^8C_2 \times {}^{12}C_3 = 28 \times 220$ $= 6160$	**M1** **A1** **[2]**	**1.1** **1.1**	$^8C_2 \times {}^{12}C_3$ or 28×220 (o.e.) seen (isw) cao
4	(b)		$6160 + (^8C_3 \times {}^{12}C_2) + (^8C_4 \times {}^{12}C_1) + {}^8C_5$ $= 6160 + (56 \times 66) + (70 \times 12) + 56$ $= 6160 + 3696 + 840 + 56$ $= 10752$	**M1*** **M1 ft dep*** **A1 ft**	**3.1a** **1.1** **1.1**	Any of $(^8C_3 \times {}^{12}C_2)$, $(^8C_4 \times {}^{12}C_1)$, $^8C_5$ (o.e.) seen Their attempt at $6160 + 3696 + 840 + 56$ (soi) FT their 6160 10752 or 4592 + (their) 6160 evaluated
			Alternative method $^{20}C_5 - (^8C_1 \times {}^{12}C_4) - {}^{12}C_5$ $= 15504 - (8 \times 495) - 792$ $= 15504 - 3960 - 792$ $= 10752$	 **M1*** **M1 dep *** **A1**	 	 Any of $^{20}C_5$, $(^8C_1 \times {}^{12}C_4)$, $^{12}C_5$ (o.e.) in any form Their attempt at $15504 - 3960 - 792$ (soi) 10752 cao
				[3]		

Question			Answer	Marks	AO	Guidance
4	(c)		{3, 13} and 3 others (but not both 7 and 17) or {7, 17}and 3 others (but not both 3 and 13) or {3, 7, 13, 17}and 1 other $= {}^{18}C_3 + {}^{18}C_3 - {}^{16}C_1$	M1	3.1a	Any of ${}^{18}C_3$, ${}^{16}C_1$, ${}^{17}C_3$, (= 680) or ${}^{16}C_2$ (= 120) o.e. seen Or any of 816, 800, 680, 120 or 16 seen as values to be + or – or implied from final answer
			= 816 + 816 – 16	M1	1.1	Their attempt at 816 + 816 – 16 or 800 + 800 + 16 or 816 + 800 Allow any two of 816, 816, 800, 800 with ± 16 added Or 680 + 680 + 120 + 120 + 16 (o.e.) May be implied from final answer 1600, 1616, 1632, provided first M mark has been awarded
			= 1616	A1 [3]	1.1	cao
4	(d)		Units digit $\in$ {1, 3, 7, 9}	M1	2.5	Identify these four pigeonholes e.g. units digit is not even and is not 5
			5 > 4 so at least two primes with the same units digit, hence result	A1 [2]	2.4	Using pigeonhole principle to draw conclusion

Question			Answer	Marks	AO	Guidance
5	(a)		First pass 1 3 24 8 4 20 30 18	M1	1.1	List starts 1 3 24
				A1	1.1	cao
			Second pass 1 3 8 4 20 18 24 30	M1	1.1	List ends 24 30
				A1	1.1	cao
				[4]		
5	(b)	(i)	7	B1	1.1	cao
				[1]		
5	(b)	(ii)	Sublist lengths roughly half each time	M1	2.1	Allow for 3 or 4 as answer e.g. x x x X x x x x → x X x X x X x x → x X x X x X x X
			3	A1	1.1	cao
				[2]		
5	(c)		0.03×5^2	M1	1.2	A valid calculation seen or implied e.g. $(3\times10^{-6}) \times500^2$, $0.03\times \left(\frac{500}{100}\right)^2$, $\frac{0.03}{100^2} = \frac{t}{500^2}$
			= 0.75 (seconds)	A1	1.1	0.75, $\frac{3}{4}$ or 7.5×10^{-1}
				[2]		

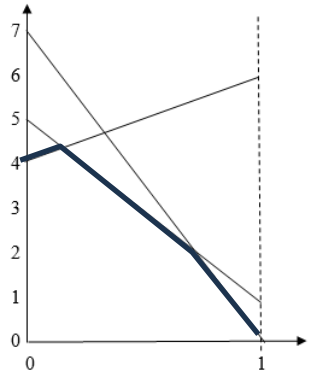
Question			Answer	Marks	AO	Guidance
5	(d)		In the best case no sublist has length 0	B1	2.2a	No sublist of length 0, allow for '2 sublists after first pass'
			After one pass there is 1 fixed value and two sublists with a total of $n - 1$ values to be sorted	M1	2.2a	$n - 1$ values to be sorted after first pass (or 1 value is fixed) May be implied from $n - 1 - 2$ seen (but note that $n - 3$ is given)
			Each of the other values are compared with a pivot, apart from the pivots There are 2 pivots in the second pass Hence $n - 3$	A1	2.2a	2 pivots used in second pass (in best case) Must be explicitly identified as pivots $n - 1 - 2$ is not enough here Reasoning leading to $n - 3$ values to be compared with pivots
						Note: if candidate claims that the best case is when n is odd (or 1 less than a power of 2) and then proceeds with this approach they can get B0, M1, A1 max
			Alternative method 1			Note: n is a given value so it may be odd or even
			After one pass: for odd n, in the best case, the sublists both have length $(n - 1)/2$ Each value in sublists compared with pivot $\Rightarrow$ $2[(n - 1)/2 - 1] = n - 3$ comparisons in second pass	B1		Odd $n \Rightarrow$ two sublists of length $(n - 1)/2$
			After one pass: for even n, in the best case the sublists have length $(n/2)$ and $(n/2) - 1$	M1		Even $n \Rightarrow$ sublist of length $(n/2)$ or $(n/2) - 1$ (either)
			Each value in sublists compared with pivot $\Rightarrow [(n/2) - 1] + [(n/2) - 2] = n - 3$	A1		Both $(n/2)$ and $(n/2) - 1$ Reasoning leading to $n - 3$ values to be compared with pivots
			Alternative method 2			
			Result(s) deduced from specific cases	B1		At least two numerical cases when n is odd, leading to $n - 3$
			[max 2 marks]	SC B1		At least two numerical cases when n is even, leading to $n - 3$
				[3]		

Question			Answer	Marks	AO	Guidance
6	(a)		D – C – F	B1 [1]	1.1	Allow DC, CF but not path reversed
6	(b)		B, C, D and F have odd degree Path starts and ends at A	B1	3.1a	Seen or implied from working
			Pair odd vertices BC = 3 BD = 4 BF = 6 DF = 6 CF = 5 CD = 1 Min total = 7	M1	1.1	Any one pairing seen (e.g. BF, CD) with correct weights or total (e.g. 7)
			56 + 7 Total weight = 63	A1 [3]	1.1	63 from correct use of route inspection algorithm
6	(c)		B has odd degree so end at another odd vertex (C, D or F) CD = 1 CF = 5 DF = 6 Least = 1	M1	2.1	Arcs joining odd degree vertices, excluding B CD = 1 is the minimum or shortest or least
			56 + 1 Total weight = 57	A1	1.1	57 from valid working seen **SCB1** for 57 without enough evidence for M1
			Alternative method From **(b)** starting and finishing at B is 63 Delete an arc from B to one of C, D, F BC = 3, BD = 4, BF = 6 Greatest = 6	M1		Arcs joining B to another odd degree vertex BF = 6 is the maximum or longest or greatest
			63 – 6 = 57 Total weight = 57	A1 ft		57 from valid working seen, or ft (their 63) – 6
				[2]		

Question			Answer	Marks	AO	Guidance
6	(d)	(i)	e.g. Remove vertex A and arcs incident on A DC = 1 CB = 3 CF = 5 CE = 6 (or FE = 6)  	M1	3.1a	Using Prim or Kruskal to construct a tree on 5 or 6 vertices May be implied by any tree through 5 or 6 vertices that does not use any of AF, DE, DF (labelled diagram or arcs written or identified) For reference:  A B C D E F A - 5 3 2 8 - B 5 - 3 4 7 6 C 3 3 - 1 6 5 D 2 4 1 - - - E 8 7 6 - - 6 F - 6 5 - 6 -
			MST with A removed has weight 15	A1	1.1	Finding total weight of MST with a vertex removed (o.e. such as sum seen 1+3+5+6) [B = 14, C = 18, D = 17, E = 11, F = 12]
			AD = 2, AC = 3	M1	3.4	Two least weight arcs from (their) removed vertex May be implied from weights or total weight of these two [B = 7, C = 4, D = 3, E = 12, F = 11]
			2 + 3 + 15 Lower bound = 20	A1 [4]	1.1	A lower bound = 20 (or using their removed vertex) [B = 21, C = 22, D = 20, E = 23, F = 23]
		(ii)	A – D – C – B – F – E – A 2 + 1 + 3 + 6 + 6 + 8	M1	3.4	Nearest neighbour from A written as a cycle or arcs listed in order Allow not closed
			Upper bound = 26	A1 [2]	1.1	Upper bound = 26 **SC B1** final answer 26 without achieving M1

Question			Answer	Marks	AO	Guidance
6	(e)		This is a solution between LB and UB	B1	3.2b	The given route is a solution and could be optimal Allow 'this is upper bound' or '(visits every vertex and) weight is 26' or '(solution with) weight 26'
			There may be a better solution <u>but</u> it is not worth spending time checking every possibility	B1	2.2b	Practical reason why it may be good enough
				[2]		

Question			Answer	Marks	AO	Guidance
7	(a)	(i)	<table><tr><th></th><th>A</th><th>B</th><th>C</th><th>min</th></tr><tr><td>X</td><td>2</td><td>–3</td><td>–4</td><td>–4</td></tr><tr><td>Y</td><td>0</td><td>1</td><td>3</td><td>0</td></tr><tr><td>Z</td><td>–2</td><td>2</td><td>4</td><td>–2</td></tr><tr><td>max</td><td>2</td><td>2</td><td>4</td><td></td></tr></table>	**M1** **M1**	**1.1** **1.1**	–4, 0, –2 as row minima for player 1 2, 2, 4 as column maxima for player 2 (or –2, –2, –4)
			Play-safe strategy for player 1 is Y Play-safe strategy for player 2 is either A or B	**A1** **[3]**	**2.2a**	1 = Y, and 2 = A and/or B
		(ii)	$0 \neq 2$ so not stable	**B1** **[1]**	**2.1**	row maximin ≠ col minimax (o.e.) (0, -2) is not a possible cell in a zero-sum game Or an appropriate description of 'chasing cells'
7	**(b)**		X and Y: 2 > 0 but -3 < 1 (or -4 < 3) X and Z: 2 > -2 but -3 < 2 (or -4 < 4) Y and Z: 0 > -2 but 1 < 2 (or 3 < 4)	**M1** **A1**	**1.1** **2.4**	Showing no dominance between two of the rows (e.g. X and Y) Allow XA > YA, XB < YB etc. 6 appropriate comparisons (or equivalent)
			Alternative method 1 A: X > Y > Z but B,C: Z > Y > X	**M1** **A1**		Either of these, or 'first, second, third' Both together with evidence of reasoning
			Alternative method 2 For player 1 Best in A is X and best in B (or C) is Z Worst in A is Z and worst in B (or C) is X	**M1** **A1**		Use col max to show that none of X, Y, Z is always best (o.e.) Use col min to show that none of X, Y, Z is always worst (o.e.) (or compare Y with both X and Z)
				[2]		

7	(c)		Each entry is increased by 4 to make them all non-negative. Using the augmented values, when player 2 chooses A player 1 expects to win $6x + 4y + 2z$ For each (x, y, z) m is the minimum expected win so $m \leq 6x + 4y + 2z$ The optimum is the maximum value of m over x, y and z	**B1** **B1** **[2]**	**3.3** **3.4**	Add 4 throughout Add 4 to <u>all</u> values Appropriate reference to strategy A or first column
7	(d)		 $4 + 2x = 5 - 4x$ $x = \frac{1}{6}$, $y = \frac{5}{6}$, $z = 0$ $m = 4\frac{1}{3} \Rightarrow M = \frac{1}{3}$ so the value of the game to player 1 is $\frac{1}{3}$	**B1** **M1** **A1** **A1** **[4]**	**3.1a** **3.4** **1.1** **3.2a**	Sketch graph showing $m = 4 + 2x$, $m = 5 - 4x$, $m = 7 - 7x$ for $0 \leq x \leq 1$ Solve as simultaneous equations to find $x \in [0, 1]$ or implied from a correct x value $[x = \frac{1}{6}, x = \frac{1}{3}$ or $x = \frac{2}{3}]$ (o.e. solving for m) cao cao

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