GCE

Further Mathematics A

Y543/01: Mechanics

A Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader

c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	**(a)**		Initial KE = $\frac{1}{2}m \times 5.3^2$	**B1**	1.1	*14.045m*	*Award if substitution for v seen in energy equation after cancelling the m's*

			Energy at $A = \frac{1}{2}mv^2 + mg\times0.8(1 - \cos(\pi/3))$ $= \frac{1}{2}m\times5.3^2$	**M1**	1.1	**DR** CoE: equating their energy (KE + PE) at the bottom with their energy at *A* (KE + PE). At least one PE must be non-zero. Condone an incorrect height for M1, but do not award A0. Allow sin/ cos interchange	For origin taken as 0 of PE $\frac{1}{2}mv^2 - mg\times0.8\cos(\pi/3))$ $= \frac{1}{2}m\times5.3^2 - 0.8mg$
			$v^2 = 5.3^2 - 2\times0.8\times g(1 - 0.5) = 20.25$ $\Rightarrow v = \sqrt{20.25} = 4.5$ so 4.5 ms^{-1} as required	**A1**	1.1	CoE: equating their energies and attempting to solve for v^2 or v **AG**	Must come from a correct derivation of Δh
				[3]			
	(b)		$a_r = \frac{4.5^2}{0.8}$	**M1**	1.1	Use of $a = \frac{v^2}{r}$	
			awrt 25.3 ms^{-2}...	**A1**	1.1	Allow 405/16	
			... towards *O*	**B1**	1.2	Need to have a clear indication that it is towards the centre, which can rely on a clearly stated angle. Allow "radially inwards" or "centripetally"	Do not allow $\frac{\pi}{6}$above horizontal unless it specifies negative horizontal axis. Do not allow 'inwards' on it's own.
				[3]			
	(c)		$(-)mg\ sin\frac{\pi}{3} = ma_t$	**M1**	1.1	Resolving weight and using *F = ma* in the tangential direction. Condone missing sign.	Allow direct substitution into $g\ sin\frac{\pi}{3} = a_t$
			So magnitude of tangential acceleration is awrt 8.49 ms^{-2}...	**A1**	1.1	$4.9\sqrt{3}$. or $\frac{g\sqrt{3}}{2}$	Do not allow -8.49 for final ***magnitude***
				[2]			

Question	Answer	Marks	AO	Guidance

2	**(a)**		[Strain] = [extension]/[length] = L/L = 1 or L^0 or $M^0L^0T^0$ or dimensionless	**B1**	2.2a	AG. Condone = 0 (or =k) or [L] provided that the ratio is correct and stated as dimensionless	Do not allow use of m (metres). Do not allow L : L
				[1]			
	(b)		[Stress] = $MLT^{-2}\ L^{-2} = ML^{-1}T^{-2}$	**B1**	1.1		
			So because "Strain is dimensionless", $[E]$ = [Stress] = $ML^{-1}T^{-2}$	**B1**	2.2a	oe e.g show divide by 1 when considering dimensions of strain	Allow clear [stress]/[strain] = [stress]
				[2]			
	(c)		$[V] = L^3$ and $[\rho] = ML^{-3}$ and $[c] = LT^{-1}$	**B1**	3.3	All used in the solution	B1 correct dimensions all used in solution
			$c = kE^{\alpha}V^{\beta}\rho^{\gamma}$ (where k is dimensionless) => $LT^{-1} = (ML^{-1}T^{-2})^{\alpha}(L^3)^{\beta}(ML^{-3})^{\gamma}$	**M1***	3.3	Setting up the model with their dimensions (condone missing k here). Must include α,β,γ or equivalent	
			M: $0 = \alpha + \gamma$ L: $1 = -\alpha + 3\beta - 3\gamma$ T: $-1 = -2\alpha$	**M1dep**	1.1	Comparing dimensions on both sides to derive three equations in α, β and γ. Allow one incorrect equation	M2 for correct consideration by division with their dimensions and manipulation of indices leading to their dimensions for c
			$\alpha = ½ \Rightarrow \gamma = -½ \Rightarrow \beta = 0$	**A1**	1.1	Give this if $\beta = 0$ seen in final equation (i.e. no dependence on V)	
			$c = k\sqrt{\frac{E}{\rho}}$	**A1**	1.1	k necessary here. Allow indices also	If correct answer seen from a division method using correct dimensions allow A2. If k not present A1.
				[5]			
	(d)	**(i)**	$500\sqrt{2}\ ms^{-1}$ or awrt $707\ ms^{-1}$	**B1FT**	3.4	Follow through their model from part (c)	**Numerical answers needed**
				[1]			

	(d)	**(ii)**	According to the model there **is no dependency on *V*** so 500 ms^{-1}	**B1**	3.5a	Some explanation involving (or calculation using) the model must be given with reference to V. Allow "volume doesn't matter" so unchanged	Needs to be an explanatory comment, not just "500" No FT in this part
				[1]			
	(e)		There may be other (dimensionless) parameters which affect *c* but have not been, or cannot be, taken into account (eg temperature)	**B1**	3.5b	Allow: can't calculate/find value of dimensionless constant There may be additional terms that are dimensionally consistent	Do not allow "there could be a dimensionless constant" B1 BOD can't identify
				[1]			

Question	Answer	Marks	AO	Guidance

3			$v_{Ay} = u_{Ay} = 4\sin 60°$	**B1**	1.1	Correct perpendicular component of velocity of *A* before collision, which is unchanged by the collision	$= 2\sqrt{3} = 3.46...$ may be seen on the diagram
			$5\times 4\cos 60° + 3\times -6 = 5v_{Ax} + 3v_{Bx}$	**M1**	3.4	Conservation of momentum condone sin/cos need to be mv for all terms	
			$5v_{Ax} + 3v_{Bx} = -8$	**A1**	1.1	Correct rearrangement to a 3 term equation	Check diagram for directions
			$\frac{3}{4} = \frac{v_{Bx} - v_{Ax}}{4\cos 60° - -6}$	**M1**	3.4	Restitution – needs to be consistent	
			$v_{Bx} - v_{Ax} = 6$ oe	**A1**	1.1	Correct rearrangement to a 3 term equation	
			$v_{Ax} = -13/4$ (= –3.25)	**A1**	1.1	$v_{Bx} = 11/4$ (= 2.75)	
			$\alpha = \tan^{-1}\frac{2\sqrt{3}}{3.25} = 46.8°$ (1 dp)	**A1**	2.2a	Could be 133.2° or –46.8° or + 0.817 rad or -0.817 rad or 2.32 rad	
				[7]			

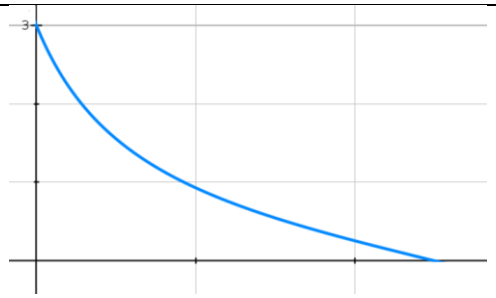
Question			Answer	Marks	AO	Guidance	
4	(a)		(*BD* is a line of) symmetry (of the kite)	B1	2.1		
				[1]			
	(b)		$h_{ABC} = \sqrt{0.37^2 - 0.35^2} = 0.12$ so area = 0.042	B1	3.1b		ALT (considering the CoM of triangle ABD) with *X* as the point where the diagonals meet: $h_{ABX} = 0.12$ so area = 0.021
			$h_{ADC} = \sqrt{0.91^2 - 0.35^2} = 0.84$ so area = 0.294	B1	1.1		$h_{AXC} = 0.84$ so area = 0.147
			Measuring from *B*: $(0.042 + 0.294)\bar{x}$ $= 0.042 \times 0.08 + 0.294 \times (0.12 + 0.28)$	M1	1.1	Attempt to balance moments about any point with at least two non-zero terms each comprising the product of a force or mass with an appropriate distance. May be one error in the distance. Measuring from AC $(0.042 + 0.294)\bar{x}$ $= -0.04 \times 0.042$ $+ 0.28 \times 0.294$ Gives $\bar{x} = \frac{0.08064}{0.336} = 0.24$ Final step: add 0.12 to measure from B	Measuring from *B*: $(0.021 + 0.147)\bar{x}$ $= 0.021 \times \frac{2}{3} \times 0.12$ $+ 0.147$ $\times \left(0.12 + \frac{1}{3} \times 0.84\right)$
			$\bar{x} = \frac{0.12096}{0.336} = 0.36$	A1	1.1	AG. Some intermediate working must be seen.	$\bar{x} = \frac{0.06048}{0.168} = 0.36$ AG
				[4]			
	(b)		**Alternative method**				

			E.g. Taking B as (0,0) $h_{ABC} = \sqrt{0.37^2 - 0.35^2} = 0.12$ so A (0.12, 0.35)	**B1**	1.1	Need to see clear coordinates, may be on diagram. Award if average of trianlge coords method used	
			$h_{ADC} = \sqrt{0.91^2 - 0.35^2} = 0.84$ so D (0.96,0)	**B1**	1.1		
			From B, $\overline{x} = \frac{1}{3}(0 + 0.12 + 0.96)$	**M1**	1.1		
			$= \frac{1.08}{3} = 0.36$ as triangle BAD and triangle BCD are symmetrical	**A1**	1.1	AG. Intermediate working and mention of symmetry along BD must be seen.	
				[4]			
	(c)		$\tan(\angle CAG) = \frac{0.24}{0.35}$	**B1**	1.1	$\angle CAG = 34.4°$ (3 sf)	$\angle AGB = 55.6°$ (3 sf)
			$\theta = \left\|180° - 90° - "34.4°" - \tan^{-1}\frac{0.84}{0.35}\right\|$	**M1**	3.1a	$\angle ACD = tan^{-1}\frac{0.84}{0.35} = tan^{-1}\frac{12}{5}, 67.4°$ (3 sf) Allow a sign error.	May see $\angle BDC = 22.6° = tan^{-1}\frac{0.35}{0.84}$ Allow for method to gain 101.8 degrees with the vertical seen $\left\|180° - "55.6°" - tan^{-1}\frac{0.35}{0.84}\right\|$ Allow for combination of one of ($\angle CAG$ or $\angle AGB$) **and** one of ($\angle ACD$ or $\angle BDC$) to find angle with vertical or horizontal.
			$\theta = 11.8°$ (3 sf)	**A1**	1.1	or –11.8°	
			C is above *D*	**A1**	2.2a	Needs to come from correct working, may have found angle from horizontal or vertical	
				[4]			

Question	Answer	Marks	AO	Guidance

5	**(a)**		$2v\frac{\mathrm{d}v}{\mathrm{d}x} = -\left(v^2+1\right)$	**M1**	3.3	Using *F* = *ma* to derive a differential equation in *v* and x only. Allow a single sign error	Condone m seen in equation (not 2). Do not allow $\frac{dx}{dv}$
			$\int \frac{2v}{v^2+1}\mathrm{d}v = -x + c$	**M1**	1.1	Correctly separating the variables and integrating at least one side	Condone m seen in equation (not 2). Can be given if m substituted later
			$\therefore \ln(v^2+1) = -x + c$	**A1**	1.1	Correct relationship between *v* and *x* (condone missing + c).	Must see brackets for this A mark, and correct numerical m.
			$x = 0,\ v = 3 \Rightarrow c = \ln(u^2+1) = \ln 10$	**M1**	3.4	Using initial condition to find c for an expression of correct format $\ln\big(f(v)\big) = \pm x + c$	May see use of limits $[\ln(v^2+1)]_3^v = [-\mathrm{x}]_0^x$ $\Rightarrow \ln(v^2+1) - \ln(10) = -x$
			$\therefore v^2 + 1 = \mathrm{e}^{-x+c} = 10\,\mathrm{e}^{-x}$ $\therefore v = \sqrt{10\,\mathrm{e}^{-x} - 1}$	**A1**	1.1	The –1 must be clearly under the root sign oe	
				[5]			
	(b)		*v* = 2 => $\ln(2^2 + 1) = -x + \ln 10$	**M1**	3.4	Substituting *v* = 2 into a solution of the correct form, which includes a numerical value for the constant of integration	Or integrating between the correct limits. $[\ln(v^2+1)]_3^2 = [-\mathrm{x}]_0^x$ $\Rightarrow \ln(5) - \ln(10) = -x$
			x = ln10 – ln5 = ln2 so distance is ln2 m or awrt 0.693 m	**A1**	1.1		Award **SC1** if answer seen with no working (need to see v=2)
				[2]			

	(c)		For Q, $F = -1 \Rightarrow a = -0.5 \Rightarrow v = u - 0.5t'$ or $v^2 = u^2 - s$	**M1**	1.1	Calculating the (constant) acceleration and using it in a *suvat* equation	**ALT method 1**: (using F = ma to set up a differential equation): $F = -1 = m\frac{dv}{dt} = 2\frac{dv}{dt}$ $\Rightarrow -t + c = 2v$ At $t = 0, v = 3 \Rightarrow c = 6$ Hence $v = \frac{1}{2}(6 - t)$ **ALT method 2**: $2v\frac{dv}{dx} = -1 \Rightarrow v^2 = -x + c$ At $x = 0, v = 3 \Rightarrow c = 9$ Hence $v^2 = 9 - x$ *oe*
			Q stops when $v_Q = 0$. $v = u + at' \Rightarrow -0.5t' = -3 \Rightarrow t' = 6$ $v^2 = u^2 + 2as \Rightarrow s = 9$	**A1**	2.1	AG Need to see v = 0 used.	**ALT method 1** $v_Q = 0$ when $t' = 6$ $-t' + 6 = 2\frac{dx}{dt'}$ $\Rightarrow -\frac{t'^2}{2} + 6t'(+k) = 2x$ (and $k = 0$) so $x = \frac{1}{2}\left(6t' - \frac{t'^2}{2}\right)$ At $t' = 6$, $x = 9$ **ALT method 2** Q stops when v=0, therefore 9-x = 0, therefore x = 9. *They will also need to use one of the other listed methods to justify the limit on t for this method mark*

			But for P, $F = 1 + v^2 > 1$ (so $\lvert a_P \rvert > 0.5$ for all v) so P will slow down more quickly than Q (and be travelling more slowly at any comparable time) so it must stop before time $t = 6$ and it cannot reach $x = 9$. $\therefore$ $t < 6$ and $x < 9$	**A1**	2.2a	AG Include: • $F = 1 + v^2 > 1$ (explicit comparison) • A reference to P coming to rest more quickly than Q *(comparison). Needs to make the link – a larger resistive force will decelerate P more quickly or a larger resistive force will mean P is travelling more slowly than Q (from starting point of 3)* • Final conclusion: So it comes to rest when $t < 6$ and $x < 9$	Be generous with t and t' confusion
				[3]			
	(d)			**B1**	3.1b	Graph in 1st quadrant only. Strictly decreasing from positive value on vertical (v) axis to the horizontal (t) axis. Negative gradient at vertical axis intercept. Allow concave. Do not allow linear No values required.	***Should come to rest***
				[1]			
	(e)		$v = 0 \Rightarrow \ln 1 = -x + \ln 10$	**M1**	3.4	Substituting $v = 0$ into a solution of the DE needs to be correct form	Or considering $10e^{-x} - 1 \geq 0$
			$x = \ln 10$ so maximum displacement is ln10 m or awrt 2.30 m	**A1**	1.1	Allow -ln(1/10)	Award SC1 if answer seen with no working (need to see v=0)
				[2]			

Question			Answer	Marks	AO	Guidance	
6	(a)	(i)	$\mathbf{v}(0) = (32\sinh 0)\mathbf{i} + (32\cosh 0 - 257)\mathbf{j} = -225\mathbf{j}$	M1	3.1b	**DR required (determine)** Attempt to find **v** at $t = 0$	*Alternative using calculus:* $\boldsymbol{a} = \begin{pmatrix} 64\cosh 2t \\ 64\sinh 2t \end{pmatrix}$ *so* $\boldsymbol{F} = 3\boldsymbol{a} =$ $192\begin{pmatrix} \cosh 2t \\ \sinh 2t \end{pmatrix}$ *seen used in F.v*
			$\mathbf{v}(\ln 2) = (32\sinh(2\ln 2))\mathbf{i} + (32\cosh(2\ln 2) - 257)\mathbf{j}$ $= 60\mathbf{i} - 189\mathbf{j}$	M1	1.1	Attempt to find **v** at $t = \ln 2$	*Uses* $WD = \int F \cdot v\, dt$ $WD = 192\int_{t=0}^{\ln 2} \begin{pmatrix} \cosh 2t \\ \sinh 2t \end{pmatrix} \cdot \begin{pmatrix} 32\sinh 2t \\ 32\cosh 2t - 257 \end{pmatrix} dt$
			$v^2(\ln 2) = 60^2 + (-189)^2 = 39321$	M1	1.1	Attempt to use **v.v** to find v^2 at $t = \ln 2$ ($v = 198.295...$) may see KE = 58981.5J	*Attempts the dot product:* $= 192\int_0^{\ln 2} 32\sinh 4t - 257\sinh 2t\ dt$
			$WD = \frac{1}{2} \times 3\left(39321 - (-225)^2\right)$ J	M1	1.1	Using WD = change in KE $= \Delta\frac{1}{2}m\mathbf{v}.\mathbf{v}$	*Integrates and applies limits:* $= 192\left[8\cosh 4t - \frac{257}{2}\cosh 2t\right]_0^{\ln 2}$
			awrt –17000 J	A1	1.1	-16956 J ISW if replaced with magnitude	
				[5]			
		(ii)	It is negative because over the interval *P* ends up moving slower than when it started	B1	2.4	Condone eg "Because *P* is slowing down", "particle is decelerating", "P is opposing the motion of the particle"	Or "(the overall effect of *F* over this interval is such that) the force *F* is acting in the opposite direction to the motion of *P*" "The particle does work against the force" Allow "F is a resistive force"
				[1]			
	(b)		*P* moving parallel to *x*-axis => $v_y = 0$ => $\cosh 2t = 257/32$	M1	2.2a	Deriving equation for *t* for *P* to be moving parallel to *x*-axis	
			=> $2t = \pm\ln 16$ but $t > 0$ => $t = \ln 4$ (or awrt 1.39)	A1	2.3	May go straight to $t = \ln 4$ (or $2t = \ln 16$). May see $t = 0.5\text{arcosh}(257/32)$	*1.386…*
			$t = \ln 4$ => $\mathbf{v} = (32\sinh(2\ln 4))\mathbf{i} = 255\mathbf{i}$	A1	1.1	or just v_x. If shown, v_y must be 0	

			$\mathbf{F} = m\mathbf{a} = 3\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = (192\cosh 2t)\mathbf{i} + (192\sinh 2t)\mathbf{j}$	**M1***	3.1b	Attempt to differentiate **v** (implied by either (64cosh2*t*)**i** or (64sinh2*t*)**j**) and multiply by *m (may just see the i-component)*	May be seen in part (a), must be used in part (b) to gain this method mark
			t = ln4 => $F_x = 192\cosh(2\ln 4) = 1542$	**M1dep**	1.1	Attempting to find *(i-component)* of **F** at their time. Ignore attempt to find F_y	
			P = **F**.**v** => Power is 1542×255 = awrt 393000 W	**A1**	1.1		3 x 514 x 255=393210
				[6]			

Question			Answer	Marks	AO	Guidance	
7	(a)		For A: $T = 1.25 \times 0.9\omega_A^2$	**M1**	3.3	NII for A using $a = r\omega^2$ or v^2/r or $v\omega$ and correct values for m and r	
			For B: $\updownarrow T\cos\theta = 2g$	**B1**	3.4	Correctly balancing forces in the vertical for B	T = 24.5 = 2.5g $cos\theta = \frac{4}{5}, sin\theta = \frac{3}{5}$
			$r_B = (1.26 - 0.9)\sin\theta$	**B1**	2.2a	Correct expression or value for the radius of B's circle used	$0.36\sin\theta$ or 0.216
			$\leftrightarrow T\sin\theta = 2r_B\omega_B^2$	**M1**	3.4	NII for B using $a = r\omega^2$ or v^2/r or $v\omega$ and correct value for m, $sin\theta$ and their calculated r_B,	T $sin\theta$=1.5g May be seen as one equation for B divided by the other in which case give 1st B1 2nd M1 if correct $tan\theta = \frac{2r_B\omega_B^2}{2g}$
			T = 24.5 => ω_B = 35/6 or awrt 5.83 and ω_A = 14/3 or awrt 4.67	**A1**	1.1		
				[5]			
	(b)		$\therefore$ for strings to realign need $(35/6)t - (14/3)t = k\pi$	**M1FT**	3.1b	Condone 180° or any integer multiple of π rads or 180° . Condone, $T = \frac{2\pi}{\Delta\omega}$, Need to see a calculation of difference of angular speed	Do not award for partially complete common multiple methods. **SC2** for fully correct common multiple or ratio method, finding 4 half turns for one object and 5 half turns for the other leading to $t = 6\pi$ /7 oe. Must come from correct ω.
			$7t/6 = \pi$ => $t = 6\pi/7$ so time is awrt 2.69 s	**A1**	1.1	cao	
				[2]			

Question			Answer	Marks	AO	Guidance	
8			Initial KE = ½×1.75×2.4^2 J	**B1**	3.4	5.04 J	
			Suppose the distance is d m. Initial PE = 1.75×g×$d$sinθ J	**M1**	3.1b	1.05gd or 10.29d. Attempt to use mgh with h different from d $cos\theta = \frac{4}{5}, sin\theta = \frac{3}{5}$	d is the distance moved along the plane. h is the vertical distance between O and the point where P stops.
			EPE/Total energy when P stops = 4.8×(d – 2.1)^2/(2×2.1) J	**M1**	3.4	Correct use of $\frac{\lambda x^2}{2l}$ (but could be in terms of x rather than d).	8(d – 2.1)^2 / 7 or 8x^2 / 7
			C = 1.75$g$cosθ => F_r = 0.732×1.75$g$cosθ	**B1**	1.1	Using law of friction to derive an expression for the frictional force	
			WD against friction = F_r d J	**M1**	3.4	Using WD = Fd to find an expression for the energy lost	1.0248gd or 10.04304d Do not allow sin cos interchange

			$5.04 + 10.29d = 8(d - 2.1)^2/7 + 10.04304d$	**M1**	3.4	Energy budget equation involving initial energy, final energy and energy loss on the correct side, signs correct, in terms of one correct unknown distance. Do not allow sin/cos interchange	Equation could be in terms of extension, x: $5.04 + 10.29(x + 2.1) = 8x^2/7 + 10.04304(x + 2.1)$ **ALT method** Candidates may consider interim energy at 2.1m (giving $v^2 = 6.3527$). Can award B1 B1 for the initial KE and frictional force as per the main scheme. All method marks awarded as per the main scheme for energy consideration involving a correct algebraic distance by considering motion from 2.1m to the point the particle comes to rest. KE (d=2.1) +PE(d=2.1) = EPE stored + WD against Fr $5.558616 + 10.29x = 8x^2/7 + 10.04304x$ *Or* $5.558616 = 8x^2/7 - 0.24696x$
			$d(d - 4.41609) = 0 \Rightarrow (d = 0)$ or d = awrt 4.42 so distance travelled must be awrt 4.42 m	**A1**	1.1	*Could be BC (at least one correct root seen)*	$8x^2 - 1.72872x - 38.910312 = 0$ $(x + 2.1)(8x - 18.52872) = 0$ $x = -2.1$ or awrt 2.32 So: awrt 2.1 + 2.32 = 4.42 m
			Rejection of d = 0 (as start point)	**A1**	3.2a	explicit rejection must be seen, for correct roots only	But $x > 0$ do reject x =-2.1m
				[8]			

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