GCE

Further Mathematics A

Y541/01: Pure Core 2

A Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using

some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
   If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)	(i)	2 by 4 or 2×4	B1	1.2		
				[1]			
		(ii)	$\left(\mathbf{P}^{\mathrm{T}}=\right)\begin{pmatrix}1 & 4\\ 0 & 2\\ -2 & -2\\ 2 & 3\end{pmatrix}$	B1	1.2		condone poor/omitted brackets just here
				[1]			
	(b)		(4 0)	B1	2.5	Do not allow (4, 0)	
				[1]			
	(c)	(i)	5	B1	2.2a		
				[1]			
	(c)	(ii)	6 by 4	B1	2.2a		
				[1]			
	(c)	(iii)	No because the number of columns in **A (** is 5 which) is not equal to the number of rows in matrix **B(**which is 6) (and for the matrices to be conformable these have to be the same.)	B1	2.4	Must include "number of" oe If numbers used, must have a word to imply comparison (eg "while", "but" rather than "and")	Accept $(4\times5)\times(6\times4)$ and "$5\neq6$" numbers given must be correct
				[1]			
	(d)		$\begin{pmatrix}-2 & 3\\ 6 & 10\end{pmatrix}\begin{pmatrix}c & 5\\ 10 & 13\end{pmatrix}=\begin{pmatrix}30-2c & 29\\ 6c+100 & 160\end{pmatrix}$ and $\begin{pmatrix}c & 5\\ 10 & 13\end{pmatrix}\begin{pmatrix}-2 & 3\\ 6 & 10\end{pmatrix}=\begin{pmatrix}30-2c & 3c+50\\ 58 & 160\end{pmatrix}$	M1	1.1	Attempt at multiplication in both directions sufficient to obtain one pair ofequivalent entries in trailing diagonal.	Can be implied by correct linear equation
			$6c + 100 = 58$ or $3c + 50 = 29 \Rightarrow c = -7$	A1	1.1		ignore errors in unused elements
				[2]			

Question			Answer	Marks	AO	Guidance	
2	(a)		**DR** $\lvert -24+7\text{i}\rvert = \sqrt{(-24)^2+7^2}$ or $\arg(-24+7\text{i}) = \tan^{-1}\frac{7}{-24}$ oe	**M1**	1.1	Attempt to find modulus or argument using a correct formula (values must be real)	If 7/24 allow only if supported by explanation, further working or clear diagram. may use alternative trig function
			$\lvert -24+7\text{i}\rvert = 25$ and awrt –0.284 or 2.86	**A1**	1.1	Condone use of degrees for this mark (–16.3° or 163.7°)	accept arctan -7/24
			–24 + 7i = 25(cos2.86 + isin2.86)	**A1**	1.1	Final answer	Accept equivalent notation. E.g cis, (r,θ) or exponential form, not (-C +iS) nor rC + riS.
				[3]			
	(b)		DR $6\text{i}z + 18w = -42\text{i}$ $-6\text{i}z - 5w = 3\text{i} - 13$	***M1**	1.1	Scaling both equations (using $\text{i}^2 = -1$) so that the coefficient of z or w is the same in magnitude.	$-5z + 15\text{i}w = 35$ $-18z + 15\text{i}w = 9 + 39\text{i}$
			$13w = -13 - 39\text{i}$ so $w = -1 - 3\text{i}$	**A1**	1.1		$13z = 26 - 39\text{i}$ so $z = 2 - 3\text{i}$
			$\text{i}z + 3(-1 - 3\text{i}) = -7\text{i}$ or $-6z + 5\text{i}(-1 - 3\text{i}) = 3 + 13\text{i}$ $\text{i}z = 3 + 2\text{i}$ or $-6z = -12 + 18\text{i}$	**dep*M1**	1.1	Substituting back into one equation and attempt to solve by collecting real and imaginary parts.	$\text{i}(2 - 3\text{i}) + 3w = -7\text{i}$ or $-6(2 - 3\text{i}) + 5\text{i}w = 3 + 13\text{i}$ i.e. reaches $kz = a + b$ for real a, b
			$z = 2 - 3\text{i}$	**A1**	1.1		
			Alternative method: $w = \frac{-7\text{i} - \text{i}z}{3}$ or $w = \frac{3 + 13\text{i} + 6z}{5\text{i}}$	**M1**		Using one equation to express one unknown in terms of the other.	$z = \frac{-7\text{i} - 3\text{w}}{\text{i}}$ or $z = \frac{3 + 13\text{i} - 5\text{i}w}{-6}$
			$-6z + 5\text{i}\left(\frac{-7\text{i} - \text{i}z}{3}\right) = 3 + 13\text{i}$ $\therefore -18z + 35 + 5z = 9 + 39\text{i}$	**M1**		Substituting into the other equation and using $\text{i}^2 = -1$ at least once	
			$\therefore (-13z = -26 + 39\text{i}$ so$)\ z = 2 - 3\text{i}$	**A1**			
			$w = \frac{-7\text{i} - \text{i}(2 - 3\text{i})}{3} = -1 - 3\text{i}$	**A1**			

			Alternative method 2: $\mathrm{i}(a+b\mathrm{i})+3(c+d\mathrm{i})=-7\mathrm{i}$ $-6(a+b\mathrm{i})+5\mathrm{i}(c+d\mathrm{i})=3+13\mathrm{i}$	**M1**		replaces z and w with two Cartesian forms in both equations	
			Re:$-b+3c=0, -6a-5d=3$ Im:$a+3d=-7, -6b+5c=13$	**M1**		Takes real and imaginary parts from both complex equations	condone i's left in
			$z=2-3\mathrm{i}$	**A1**		Could be **BC**	$a = 2$, $b = -3$ sufficient
			$w=-1-3\mathrm{i}$	**A1**			$c = -1$, $d = -3$ sufficient
			Alternative method 3: $\begin{pmatrix}\mathrm{i} & 3\\ -6 & 5\mathrm{i}\end{pmatrix}\begin{pmatrix}z\\ w\end{pmatrix}=\begin{pmatrix}-7\mathrm{i}\\ 3+13\mathrm{i}\end{pmatrix}$ $\begin{pmatrix}z\\ w\end{pmatrix}=\begin{pmatrix}\mathrm{i} & 3\\ -6 & 5\mathrm{i}\end{pmatrix}^{-1}\begin{pmatrix}-7\mathrm{i}\\ 3+13\mathrm{i}\end{pmatrix}$	**M1**		Writes in matrix form and derives an equation for $\begin{pmatrix}z\\ w\end{pmatrix}$	Must left-multiply by their inverse matrix
			where $\begin{pmatrix}\mathrm{i} & 3\\ -6 & 5\mathrm{i}\end{pmatrix}^{-1}=\frac{1}{13}\begin{pmatrix}5\mathrm{i} & -3\\ 6 & \mathrm{i}\end{pmatrix}$	**A1**		…with correct inverse matrix	
			$\frac{1}{13}\begin{pmatrix}5\mathrm{i} & -3\\ 6 & \mathrm{i}\end{pmatrix}\begin{pmatrix}-7\mathrm{i}\\ 3+13\mathrm{i}\end{pmatrix}=\frac{1}{13}\begin{pmatrix}35-9-39\mathrm{i}\\ -42\mathrm{i}+3\mathrm{i}-13\end{pmatrix}$	**M1**		Expands…	
			$\Rightarrow z=2-3\mathrm{i}$, $w=-1-3\mathrm{i}$	**A1**		… to correct simplified solution	can be in vector form
				[4]			

Question			Answer	Marks	AO	Guidance	
3	(a)		$y = \sinh^{-1}u \Rightarrow \sinh y = u$ $\Rightarrow \cosh y \frac{dy}{du} = 1 \Rightarrow \frac{dy}{du} = \frac{1}{\cosh y}$ $\therefore \frac{dy}{du} = \frac{1}{\pm\sqrt{1+\sinh^2 y}} = \frac{1}{\pm\sqrt{u^2+1}}$	**M1**	2.1	AG. Taking sinh of both sides, differentiating and using $\cosh^2 y - \sinh^2 y = 1$	Condone missing ± for M1 Accept $\frac{du}{dy}$ at this stage
			But gradient of $y = \sinh^{-1}u$ is never negative so $\frac{dy}{du} = \frac{1}{\sqrt{u^2+1}}$	**A1**	2.4	AG so reason required (accept "always positive")	poor notation can be recovered
			Alternative method: $y = \sinh^{-1}u = \ln\left(u + \sqrt{u^2+1}\right)$ $\therefore \frac{dy}{du} = \frac{1 + \frac{1}{2} \times 2u\left(u^2+1\right)^{-\frac{1}{2}}}{u + \sqrt{u^2+1}}$	**M1**		AG. Using the definition of $\sinh^{-1}$ in logarithmic form and attempting to differentiate using the chain rule on ln function.	
			$= \frac{\left(u^2+1\right)^{\frac{1}{2}} + u}{\left(u^2+1\right)^{\frac{1}{2}}\left(u + \left(u^2+1\right)^{\frac{1}{2}}\right)} = \frac{1}{\left(u^2+1\right)^{\frac{1}{2}}} = \frac{1}{\sqrt{u^2+1}}$	**A1**		AG so some intermediate working must be seen. www	

			Alternative method 2: $I = \int \frac{1}{\sqrt{u^2+1}} \mathrm{d}u$ $u = \sinh v \Rightarrow \mathrm{d}u = \cosh v \, \mathrm{d}v$ $\Rightarrow I = \int \frac{1}{\sqrt{\sinh^2 v + 1}} \cosh v \, \mathrm{dv}$ $= \int \frac{\cosh v}{\sqrt{\cosh^2 v}} \mathrm{d}v = \int 1 \mathrm{d}v = v \ (+c)$ $= \sinh^{-1} u \ (+c)$	**M1**		Correctly integrates RHS	Condone omission of c
			Differentiating both sides wrt u gives $\frac{1}{\sqrt{u^2+1}} = \frac{\mathrm{d}}{\mathrm{d}u}\left(\sinh^{-1} u\right)$	**A1**			
				[2]			

	(b)		$y = \sinh^{-1} 2x$ $\therefore \frac{dy}{dx} = \frac{2}{\sqrt{(2x)^2+1}} = \frac{2}{\sqrt{4x^2+1}}$	**M1**	1.1	Differentiating using chain rule or the formula booklet	Giving $\frac{1}{\sqrt{\frac{1}{4}+x^2}}$
			$x = \sqrt{6} \Rightarrow y = \sinh^{-1} 2\sqrt{6}$ $\left(= \ln\left(5+2\sqrt{6}\right)\right)$	**M1**	1.1	Substituting the *x*-value into the equation to find the *y* coordinate of the point	
			$x = \sqrt{6} \Rightarrow \left.\frac{dy}{dx}\right\|_{x=\sqrt{6}} = \frac{2}{5}$ $\therefore m = -\frac{5}{2}$	**M1**	1.1	Substituting the *x*-value into theirgradient and taking negative reciprocal	
			$y - \ln\left(5+2\sqrt{6}\right) = -\frac{5}{2}\left(x-\sqrt{6}\right)$ $\therefore y = -\frac{5}{2}x + \ln\left(5+2\sqrt{6}\right) + \frac{5\sqrt{6}}{2}$	**A1**	1.1	oe in correct form	
				[4]			

Question			Answer	Marks	AO	Guidance	
4			**DR** $V = \pi\int_{\frac{1}{2}}^{1}\left((3x^2-3x+1)^{-\frac{1}{2}}\right)^2 \mathrm{d}x$	**M1**	3.3	Using $V = \pi\int_a^b y^2\mathrm{d}x$ with limits	Accept squared out expression Condone omission of dx
			$V = \pi\int\frac{1}{3x^2-3x+1}\mathrm{d}x = \frac{1}{3}\pi\int\frac{1}{x^2-x+\frac{1}{3}}\mathrm{d}x$ $= \frac{1}{3}\pi\int\frac{1}{\left(x-\frac{1}{2}\right)^2-\frac{1}{4}+\frac{1}{3}}\mathrm{d}x$ $= \frac{1}{3}\pi\int\frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{1}{12}}\mathrm{d}x$	**M1**	2.2a	Expressing the integral in completed square form	or $\pi\int\frac{1}{3\left(x-\frac{1}{2}\right)^2+\frac{1}{4}}\mathrm{d}x$ or $\pi\int\frac{1}{\left(\sqrt{3}x-\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}}\mathrm{d}x$
			$\int\frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{1}{12}}\mathrm{d}x = \left[\frac{1}{\sqrt{\frac{1}{12}}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\sqrt{\frac{1}{12}}}\right)\right]$	**A1**	1.1	$= \left[\frac{2}{\sqrt{3}}\pi\tan^{-1}\left(\sqrt{3}(2x-1)\right)\right]$ may be equivalent based on their form and their substitution	Could see a substitution eg $u = \sqrt{3}\left(x-\frac{1}{2}\right)$ with $\mathrm{d}u = \sqrt{3}\mathrm{d}x$
			$= \frac{2}{\sqrt{3}}\pi\left[\tan^{-1}\left(\sqrt{3}\right)-\tan^{-1}(0)\right]$ $= \frac{2}{\sqrt{3}}\pi\frac{\pi}{3} = \frac{2\sqrt{3}}{9}\pi^2$ so $\frac{2\sqrt{3}}{9}\pi^2$ cm^3	**A1**	3.4	oe	Do not penalise missing units
				[4]			

Question			Answer	Marks	AO	Guidance	
5	(a)		**DR** $\left(\text{RHS} = 2\sinh x\cosh x =\right) 2\times\frac{e^{x}-e^{-x}}{2}\times\frac{e^{x}+e^{-x}}{2}$	**M1**	2.1	AG. Use of exponential definition of sinh or cosh	Must be **used** on LHS or RHS
			$=\frac{1}{2}\left(e^{2x}+e^{0}-e^{0}-e^{-2x}\right)$ $=\frac{e^{2x}-e^{-2x}}{2}=\sinh 2x=\text{LHS}$	**A1**	2.2a	AG. Soan intermediate step must be shown LHS must be equated to RHS	e.g. $2\left(\frac{e^{2x}-e^{-2x}}{4}\right)$ Accept full reverse argument
				[2]			
	(b)		**DR** $15\sinh x + 16\cosh x - 12\sinh x\cosh x = 20$	**B1**	3.1a	Use of identity in **(a)**.	
			$12sc - 16c - 15s + 20 = 0$ $(3s - 4)(4c - 5) = 0$	**M1**	1.1	Writing as = 0 and factorising	(where $s = \sinh x$ and $c = \cosh x$)
			$x=\sinh^{-1}\left(\frac{4}{3}\right)$ or $x=\cosh^{-1}\left(\frac{5}{4}\right)$	**A1**	1.1	Complete solution in any form (assume that $\cosh^{-1}$ is multi-valued here)	
			$\sinh^{-1}\frac{4}{3}=\ln\left(\frac{4}{3}+\sqrt{\left(\frac{4}{3}\right)^{2}+1}\right)=\ln 3$	**A1**	1.1		
			$\cosh^{-1}\frac{5}{4}=\pm\ln\left(\frac{5}{4}+\sqrt{\left(\frac{5}{4}\right)^{2}-1}\right)=\pm\ln 2$	**A1**	3.2a	Must show ± explicitly (or have both ln2 and ln½)	

			Alternative method: $15\frac{e^x - e^{-x}}{2} + 16\frac{e^x + e^{-x}}{2} - 6\frac{e^{2x} - e^{-2x}}{2} = 20$	**B1**		Use of exponential definitions of sinh$x$, cosh$x$ and sinh2x in equation	Also award if starts with main method before using exponentials
			$\therefore 15e^x - 15e^{-x} + 16e^x + 16e^{-x} - 6e^{2x} + 6e^{-2x} = 40$ $\therefore 15e^{3x} - 15e^x + 16e^{3x} + 16e^x - 6e^{4x} + 6 = 40e^{2x}$ $\therefore 6e^{4x} - 31e^{3x} + 40e^{2x} - e^x - 6 = 0$	***M1**		Multiplying by e^{2x} and collecting like terms to write as quartic equation in e^x	Could see a substitution eg $y = e^x$ leading to $6y^4 - 31y^3 + 40y^2 - y - 6 = 0$ Could use Pythagoras to derive quartic in sinh or cosh
			$y = e^x \Rightarrow 6y^4 - 31y^3 + 40y^2 - y - 6 = 0$ $6\times16 - 31\times8 + 40\times4 - 2 - 6 = 256 - 256 = 0$ $6y^3(y-2) - 19y^2(y-2) + 2y(y-2) + 3(y-2) = 0$ $(y-2)(6y^3 - 19y^2 + 2y + 3) = 0$	***dep*M1**		Using factor theorem to deduce that $e^x = 2$ (or 3 or ½) is a solution and factorising.	or $(6y^2 - y - 1)(y^2 - 5y + 6)$ seen
			$6\times27 - 19\times9 + 2\times3 + 3 = 171 - 171 = 0$ $6y^3 - 19y^2 + 2y + 3 =$ $6y^2(y-3) - y(y-3) - (y-3)$ $= (y-3)(6y^2 - y - 1) = (y-3)(2y-1)(3y+1)$	**dep*M1**		Using factor theorem to find another factor and fully factorising	
			$\therefore y = e^x = 2$, ½, 3 or –⅓. But $e^x > 0$ $\therefore x = \ln2$, $\ln\frac{1}{2}$ (or $-\ln2$) or $\ln3$ only	**A1**		Must reject negative root explicitly for **A1**	**ScB1** for correct solution after B1M1M0M0
				[5]			

Question			Answer	Marks	AO	Guidance	
6	(a)		$\begin{pmatrix}4\\4\\3\end{pmatrix}\times\begin{pmatrix}-2\\3\\1\end{pmatrix}=\begin{pmatrix}-5\\-10\\20\end{pmatrix}=-5\begin{pmatrix}1\\2\\-4\end{pmatrix}$	**B1**	1.1	Finding a normal to Π.	Any valid method; for example using $\begin{pmatrix}1\\a\\b\end{pmatrix}$ and setting the dot product with both vectors in Π to 0.
			$\begin{pmatrix}1\\2\\-4\end{pmatrix}.\begin{pmatrix}2\\0\\3\end{pmatrix}\ (=2-12)$	**M1**	1.1	Any clear attempt to find the angle between the normals (can be implied by dotting the two normals together).	
			$\cos\theta=\dfrac{-10}{\sqrt{1^2+2^2+(-4)^2}\sqrt{2^2+3^2}}$	**M1**	1.1	Using the definition of dot product to find $\cos\theta$ in unsimplified numerical form Could see modulus signs.	This mark can be awarded after **M0**
			$\cos\theta=\dfrac{-10}{\sqrt{273}}$ *so* $\theta=127.2$ so required angle is 52.8° (1 dp)	**A1**	1.1	Or directly to answer. Final answer	awrt 0.921 rads
				[4]			

	(b)		$\begin{pmatrix}-1\\2\\1\end{pmatrix}.\begin{pmatrix}1\\2\\-4\end{pmatrix}=-1+4-4\ \ (=-1)$	**M1**	3.1a	Dotting their normal and a point on Π.	Condone poor notation up to final A mark as long as method clear
			$\left(\begin{pmatrix}9\\-7\\20\end{pmatrix}+\nu\begin{pmatrix}1\\2\\-4\end{pmatrix}\right).\begin{pmatrix}1\\2\\-4\end{pmatrix}=-1$	**M1**	1.1	Forming the equation of the line AF and intersecting with Π to find the value of the parameter for the PoI.	Or **M1** for using formula to find distance AF $\left(=\frac{84}{\sqrt{21}}\right)$, and dividing this by magnitude *their n.*
			$9 - 14 - 80 + (1 + 4 + 16)\nu = -1 \Rightarrow \nu = 4$	**A1**	1.1		
			$\mathbf{r}_F=\begin{pmatrix}9\\-7\\20\end{pmatrix}+4\begin{pmatrix}1\\2\\-4\end{pmatrix}=\begin{pmatrix}13\\1\\4\end{pmatrix}$	**A1**	1.1	Condone presentation as position vector of F.	
			so F is (13, 1, 4)				
			Alternative method				
			$\begin{pmatrix}9\\-7\\20\end{pmatrix}+\nu\begin{pmatrix}1\\2\\-4\end{pmatrix}$ $=\begin{pmatrix}-1\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}4\\4\\3\end{pmatrix}+\mu\begin{pmatrix}-2\\3\\1\end{pmatrix}$	**M1**		Forms equations	
			so $4\lambda - 2\mu - \nu = 10$ $4\lambda + 3\mu - 2\nu = -9$ $3\lambda + \mu + 4\nu = 19$ $(\lambda = 2\ \mu = -3)\nu = 4$	**M1A1**		**BC**	If not **BC**, then M1 for two equations in two unknowns
			F is (13, 1, 4)	**A1**			

			2nd **Alternative method** F is a point on Π so $\overrightarrow{AF} = \overrightarrow{AO} + \overrightarrow{OF} = \begin{pmatrix} -9 \\ 7 \\ -20 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} -10 \\ 9 \\ -19 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ $\overrightarrow{AF}.\begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} = 0 \Rightarrow -61 + 41\lambda + 7\mu = 0$ $(41\lambda + 7\mu = 61)$ $\overrightarrow{AF}.\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} = 0 \Rightarrow 28 + 7\lambda + 14\mu = 0$ $(\lambda + 2\mu = -4)$ $\Rightarrow \lambda = 2,\ \mu = -3$ $\overrightarrow{OF} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} - 3\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \\ 4 \end{pmatrix}$ So F is (13, 1, 4)	**M1** **M1** **A1** **A1**		Using F, a general point on Π, equates $\overrightarrow{AF}$.**b** or $\overrightarrow{AF}$.**c** to 0 2 equations in λ and μ Solves (**BC)**	

			3rd Alternative method D, perp dist from A to Π is given by $D = \dfrac{\left\lvert \begin{pmatrix} 9 \\ -7 \\ 20 \end{pmatrix} . \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} - (-1) \right\rvert}{\sqrt{1^2 + 2^2 + (-4)^2}} = \dfrac{\lvert 9 - 14 - 80 + 1 \rvert}{\sqrt{1+4+16}} = \dfrac{\lvert -84 \rvert}{\sqrt{21}} = \dfrac{84}{21}\sqrt{21} = 4\sqrt{21}$ $\therefore \overrightarrow{AF} = 4\sqrt{21}\,\hat{\mathbf{n}} = 4\sqrt{21} \times \dfrac{1}{\sqrt{21}} \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ -16 \end{pmatrix}$ $\therefore \overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{AF} = \begin{pmatrix} 9 \\ -7 \\ 20 \end{pmatrix} + \begin{pmatrix} 4 \\ 8 \\ -16 \end{pmatrix} = \begin{pmatrix} 13 \\ 1 \\ 4 \end{pmatrix}$ So co-ords of F are (13, 1, 4) Or could see $\overrightarrow{AF} = \lambda \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ $\therefore \sqrt{\lambda^2 + (2\lambda)^2 + (-4\lambda)^2} = \sqrt{21}\lambda = 4\sqrt{21} \Rightarrow \lambda = 4$	**M1** **A1** **M1** **A1**		Finds perpendicular distance Finds normal vector from A to Π Uses their normal vector to find F	
				[4]			

Question			Answer	Marks	AO	Guidance	
7	(a)		**DR** $\frac{5r+6}{r^3+r^2}=\frac{A}{r}+\frac{B}{r^2}+\frac{C}{r+1}$	***M1**	3.1a	Correct PF expansion used in solution. Allow extraneous terms only if stated/evaluated as 0	Accept $\frac{Ar+B}{r^2}+\frac{C}{r+1}$ but not with additional $\frac{D}{r}$ unless recovered later
			$=\frac{Ar(r+1)+B(r+1)+Cr^2}{r^2(r+1)}$ Consider $r=-1$	***M1**	1.1	Recombining and use valid method to find coefficients (eg appropriate choice of r or comparing coefficients in r^2, r or r^0).	Indep of 1st M1 if from PF terms involving 3^+ unknowns and their factors seen in a denominator
			$C = 1$	**A1**	1.1	Any one correct non-zero coefficient	
			$r = 0 \Rightarrow B = 6$ and $A + C = 0 \Rightarrow A = -1$	**A1**	1.1	Other two coefficients by valid method	
			$\therefore \sum_{r=1}^{n}\frac{5r+6}{r^3+r^2}=\sum_{r=1}^{n}\left(\frac{-1}{r}+\frac{6}{r^2}+\frac{1}{r+1}\right)$ $=\sum_{r=1}^{n}\frac{6}{r^2}+\sum_{r=1}^{n}\left(\frac{1}{r+1}-\frac{1}{r}\right)$ $=\sum_{r=1}^{n}\frac{6}{r^2}+\frac{1}{2}-\frac{1}{1}+\frac{1}{3}-\frac{1}{2}+\frac{1}{4}-\frac{1}{3}+\ldots$ $\ldots+\frac{1}{n}-\frac{1}{n-1}+\frac{1}{n+1}-\frac{1}{n}$	**dep*M1**	1.1	Separating and expressing sum in form in which cancellation pattern is clear	$r' = r+1$ $\therefore \sum_{r=1}^{n}\frac{1}{r+1}-\sum_{r=1}^{n}\frac{1}{r}=\sum_{r'=2}^{n+1}\frac{1}{r'}-\sum_{1}^{n}\frac{1}{r}$ $=\frac{1}{n+1}+\sum_{r=2}^{n}\left(\frac{1}{r}-\frac{1}{r}\right)-\frac{1}{1}=\frac{1}{n+1}-1$
			$=\frac{1}{n+1}-1+6\sum_{r=1}^{n}\frac{1}{r^2}$ so $a = 1$, $b = -1$, $c = 6$	**A1**	2.2a	Complete argument with all detail. Allow embedded answers	Minimum of first and last cancellation terms shown (If algebraic approach, must see full argument)
				[6]			
	(b)		$\lim_{n\to\infty}\frac{1}{n+1}=0$	**M1**	3.1a	AG. Considering the limit as n tends to infinity of $\frac{1}{n+1}$ term	not $\frac{1}{\text{infinity}}$

			$\sum_{r=1}^{\infty}\frac{5r+6}{r^3+r^2}=\lim_{n\to\infty}\left\{\sum_{r=1}^{n}\frac{5r+6}{r^3+r^2}\right\}$ $=\lim_{n\to\infty}\left\{\frac{1}{n+1}-1+6\sum_{r=1}^{n}\frac{1}{r^2}\right\}=\pi^2-1$ $=(\pi-1)(\pi+1).$	**A1**	2.2a	AG. Joined up argument Some intermediate working must be shown	Condone poor, but clear limit notation,
				[2]			

Question			Answer	Marks	AO	Guidance	
8	(a)		$\left(2t-t^2\right)\dfrac{\mathrm{d}I}{\mathrm{d}t}=\left(2t-t^2\right)^{\frac{3}{2}}-2(t-1)I$ $\therefore \dfrac{\mathrm{d}I}{\mathrm{d}t}=\left(2t-t^2\right)^{\frac{1}{2}}-\dfrac{2(t-1)}{2t-t^2}I$ $\therefore \dfrac{\mathrm{d}I}{\mathrm{d}t}+\dfrac{2(t-1)}{2t-t^2}I=\left(2t-t^2\right)^{\frac{1}{2}}$	***M1**	3.3	Rearranging to the form $\dfrac{\mathrm{d}I}{\mathrm{d}t}+P(t)I=Q(t)$	
			$\text{IF}=\mathrm{e}^{\int\frac{2(t-1)}{2t-t^2}\mathrm{d}t}=\mathrm{e}^{-\int\frac{2-2t}{2t-t^2}\mathrm{d}t}=\mathrm{e}^{-\ln(2t-t^2)}=\dfrac{1}{2t-t^2}$	**A1**	2.2a	Finding correct integrating factoras an expression not involving exponentials and logs.	Ignore unnecessary constant multiplier here
			$\therefore\left(2t-t^2\right)^{-1}\dfrac{\mathrm{d}I}{\mathrm{d}t}+2(t-1)\left(2t-t^2\right)^{-2}I=\left(2t-t^2\right)^{-\frac{1}{2}}$ $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\left(2t-t^2\right)^{-1}I\right)=\left(2t-t^2\right)^{-\frac{1}{2}}$	***dep*M1**	1.1	Multiplying both sides by IF and recognising LHS as exact derivative of $\left(2t-t^2\right)^{-1}I$	Can be implied by next **M1** Can be awarded from slip in initial rearrangement if their IF works for their rearrangement
			$\left(2t-t^2\right)^{-1}I=\int\left(2t-t^2\right)^{-\frac{1}{2}}\mathrm{d}t=\int\dfrac{1}{\sqrt{1-(t-1)^2}}\mathrm{d}t$	**dep*M1**	1.1	Taking integral of both sides and attempt to complete square in order to express RHS in a standard form (or using the substitution $u = t - 1$ including $\mathrm{d}u = \mathrm{d}t$).	Condone $\pm1\pm(t-1)^2$ for **M1** $=\int\dfrac{1}{\sqrt{1-u^2}}\mathrm{d}u$
			$=\sin^{-1}(t-1)+c$	***A1**	1.1	For correctly integrating RHS to a function of t. "+ c" not necessary here.	
			$t=1,\ I=5\Rightarrow(2-1)^{-1}5=\sin^{-1}(1-1)+c$ $\therefore c=5$ $\therefore\left(2t-t^2\right)^{-1}I=\sin^{-1}(t-1)+5$ $\therefore I=(2t-t^2)(\sin^{-1}(t-1)+5)$	**dep*A1**	3.3	AG so use of relevant condition must be explicit. Verification of AG by substitution is not sufficient; value of c must be derived.	Ignore workings using other condition ($t = 0$, $I = 0$)
				[6]			

	(b)		$I = 0 \Rightarrow (2t - t^2)(\sin^{-1}(t-1) + 5) = 0$ $\therefore t(2-t)(\sin^{-1}(t-1) + 5) = 0$ $\therefore t = 2$ So length of surge is 2 – 0 = 2 (seconds)	**B1**	3.1a	If no other later comment or statement to the contrary then accept just $t = 2$	
			or $\sin^{-1}(t-1) + 5 = 0$ $\therefore \sin^{-1}(t-1) = -5$ which is not possible (since $-\frac{1}{2}\pi \leq \sin^{-1}(t-1) \leq \frac{1}{2}\pi$.)	**B1**	2.4	Do not accept incorrect explanation eg $-1 \leq \sin^{-1}(t-1) \leq 1$	If **B0B0** then **Sc1** if length of surge determined to be awrt 1.96 s from sin(–5rads) + 1 after $t = 2$ found
				[2]			
	(c)		$\left(I = (2t - t^2)(\sin^{-1}(t-1) + 5)\right) \pm 1 \pm (t-1)^2$ & $\left(2t - t^2\right)\frac{dI}{dt} = \left(2t - t^2\right)^{\frac{3}{2}} - 2(t-1)I$ $\therefore \frac{dI}{dt} = \frac{\left(2t - t^2\right)^{\frac{3}{2}} - 2(t-1)(2t - t^2)(\sin^{-1}(t-1) + 5)}{2t - t^2}$	***M1**	3.4	Finding an expression for $\frac{dI}{dt}$ in terms of t by either eliminating given I from given DE or differentiating given $I(t)$ using product rule.	$\left(I = (2t - t^2)(\sin^{-1}(t-1) + 5) \Rightarrow\right) \frac{dI}{dt} =$ $2(1-t)(\sin^{-1}(t-1) + 5) + \frac{(2t - t^2)}{\sqrt{1-(t-1)^2}}$
			$\therefore \frac{dI}{dt} = \left(2t - t^2\right)^{\frac{1}{2}} - 2(t-1)(\sin^{-1}(t-1) + 5)$	**M1 dep ***	2.2a	Simplifying their $\frac{dI}{dt}$ so that there is no denominator which is zero at $t = 0$	$\therefore \frac{dI}{dt} =$ $2(1-t)(\sin^{-1}(t-1) + 5) + \frac{(2t - t^2)}{\sqrt{2t - t^2}}$ $= (2 - 2t)(\sin^{-1}(t-1) + 5) + (2t - t^2)^{\frac{1}{2}}$
			$\therefore t = 0$ $\Rightarrow \frac{dI}{dt} = 0 - 2(-1)(\sin^{-1}(-1) + 5)$ $= 2\left(5 - \frac{1}{2}\pi\right) = 10 - \pi$ (units/s)	**A1**	3.4	Must be in a simplified non-trigonometric form. Ignore units.	If M1M0 or M0M0, then **Sc B1** for correct answer
				[3]			

Question			Answer	Marks	AO	Guidance	
9	(a)		$\frac{dy}{dt} = \frac{a}{1+at} = a(1+at)^{-1}$	M1	1.1	Differentiating correctly first derivative	or $\frac{dy}{dt} = \frac{a}{1+at} = \left(a^{-1}+t\right)^{-1}$ etc
			$\frac{d^2y}{dt^2} = -a^2(1+at)^{-2}$ and $\frac{d^3y}{dt^3} = 2a^3(1+at)^{-3}$ and $\frac{d^4y}{dt^4} = 2(-3)a^4(1+at)^{-4}$ oe	A1	1.1	For all; constants need not be evaluated and may be unsimplified. eg Could see $(-1)^2$ or $(-1)^3$	
			$\frac{d^ny}{dt^n} = (-1)^{n-1}a^n(n-1)!(1+at)^{-n}$	A1	2.2b	Could be $(-1)^{n+1}$oe.	$(-1)^{n-1}(n-1)!\left(a^{-1}+t\right)^{-n}$ Omission of factorial term can score a possible B1M1 in part **(b)**
				[3]			

	(b)		Basis case: $n = 1$ $\frac{d^1y}{dt^1} = (-1)^{1-1}a^1(1-1)!(1+at)^{-1}$ $= (-1)^0 a \times 0! \times (1+at)^{-1} = a(1+at)^{-1}$ But $\frac{d^1y}{dt^1} = \frac{dy}{dt} = a(1+at)^{-1}$ So true for $n = 1$	***B1ft**	2.1	Convincingly showing that their conjecture works for $n = 1$ for their 1st derivative in part **(a)**	accept omission of working shown in 1st line here
			Assume true for $n = k$ ie $\frac{d^k y}{dt^k} = (-1)^{k-1}a^k(k-1)!(1+at)^{-k}$ $\therefore \frac{d^{k+1}y}{dt^{k+1}} = \frac{d}{dt}\left(\frac{d^k y}{dt^k}\right) = \frac{d}{dt}\left((-1)^{k-1}a^k(k-1)!(1+at)^{-k}\right)$	**M1**	3.1a	Forming the inductive hypothesis and making it clear that the $(k+1)^{th}$ derivative is the derivative of the k^{th} derivative	
			$= (-1)^{k-1}a^k(k-1)! \times (-k)a(1+at)^{-k-1}$ $= (-1)^{k-1}(-1) \times a \times a^k \times k(k-1)! \times (1+at)^{-(k+1)}$ $= (-1)^k a^{k+1} k!(1+at)^{-(k+1)}$ $\left(= (-1)^{(k+1)-1}a^{k+1}((k+1)-1)!(1+at)^{-(k+1)}\right)$ (which is the formula with $n = k + 1$)	***A1**	2.2a	Differentiating and rewriting into correct form. Some intermediate working and/or justification must be seen	Could see substitution $u = 1 + at$etc
			So true for $n = k$ implies true for $n = k + 1$. But true for $n = 1$. Therefore true for all integers $n \geq 1$	**dep*A1**	2.4	full correct argument	
				[4]			
	(c)		$\frac{d}{dt}\left(\frac{d^6y}{dt^6}\right) = \frac{d^7y}{dt^7} = (-1)^6 a^7(7-1)!(1+at)^{-7}$	**M1**	3.1a	Considering the seventh derivative.	
			$a = 2$, $t = \frac{3}{2}$ $\Rightarrow \frac{d^7y}{dt^7} = 2^7 \times 720 \times (1+3)^{-7} = \frac{720}{128} = \frac{45}{8}$	**A1**	1.1	5.625. Ignore attempt at units.	
				[2]			

Question			Answer	Marks	AO	Guidance	
10			**DR** C_1 and C_2 intersect when $5 = 3\cosh\theta$ $\therefore \theta = \cosh^{-1}\frac{5}{3}$ so intersection at $(\pm)\cosh^{-1}\frac{5}{3}$ or $(\pm)\ln 3$ or awrt $(\pm)1.10$	**B1**	3.1a	Correct condition for intersection of curves leading to a correct expression or value for an angle at a PoI	
			$A_{\text{sector}} = \frac{1}{2}5^2(\theta_2 - \theta_1) = 25\ln 3$ or $25\cosh^{-1}\frac{5}{3}$ or awrt 27.5	**B1ft**	2.2a	For finding (±) the correct area of the sector (or half of it) either using $\frac{1}{2}r^2\theta$ or $\frac{1}{2}\int_{\theta_1}^{\theta_2}(5)^2\,\mathrm{d}\theta$. Accept lower limit of 0. May be embedded in a calculation for total area.	Here $\theta_1 = -\cosh^{-1}\frac{5}{3}$, $\theta_2 = \cosh^{-1}\frac{5}{3}$ or $(\theta_1 = -\ln 3), (\theta_2 = \ln 3)$ i.e. may later evaluate integral… for some k
			$\frac{1}{2}\int_{\theta_1}^{\theta_2}(3\cosh\theta)^2\,\mathrm{d}\theta$ or $\frac{1}{2}\int_{0}^{\theta_2}(3\cosh\theta)^2\,\mathrm{d}\theta$	**M1**	3.4	Correct use of area formula with ±their value for limits. Condone lower limit of 0	Here $\theta_1 = -\cosh^{-1}\frac{5}{3}$, $\theta_2 = \cosh^{-1}\frac{5}{3}$ Limits can be seen later. Later doubling may be seen

			$\int \cosh^2\theta \, d\theta = \int \left(\frac{e^{\theta} + e^{-\theta}}{2} \right)^2 d\theta =$ $= \frac{1}{4}\int e^{2\theta} + e^{-2\theta} + 2 \, d\theta$	***M1**	3.1a	Correct conversion of $\cosh^2\theta$ into a form which can be integrated.	or $\int \cosh^2\theta \, d\theta = \frac{1}{2}\int 1 + \cosh 2\theta \, d\theta$
			$\int_{-\ln 3}^{\ln 3} e^{2\theta} + e^{-2\theta} + 2 \, d\theta$ $= \left[\frac{1}{2}e^{2\theta} - \frac{1}{2}e^{-2\theta} + 2\theta \right]_{-\ln 3}^{\ln 3}$ $= \frac{1}{2} \times 9 - \frac{1}{2} \times \frac{1}{9} + 2\ln 3 - \left(\frac{1}{2} \times \frac{1}{9} - \frac{1}{2} \times 9 - 2\ln 3 \right)$ $= \frac{80}{9} + 4\ln 3$ or awrt 13.3	**dep*M1**	1.1	Correctly integrating $\cosh^2\theta$ and substituting their limits. Could be embedded. NB $A_{C_2} = \frac{9}{2} \times \frac{1}{4}\left(\frac{80}{9} + 4\ln 3 \right) = 10 + \frac{9}{2}\ln 3$	or $\int_{-\ln 3}^{\ln 3} \cosh^2\theta \, d\theta = \frac{1}{2}\int_{-\ln 3}^{\ln 3} 1 + \cosh 2\theta \, d\theta$ $= \frac{1}{2}\left[\theta + \frac{1}{2}\sinh 2\theta \right]_{-\ln 3}^{\ln 3} = \frac{20}{9} + \ln 3$
			$\therefore$ trapped area $= 25\ln 3 - \left(10 + \frac{9}{2}\ln 3 \right) = \frac{41}{2}\ln 3 - 10$ or awrt 12.5 (12.521...)	**A1**	1.1	Accept unsimplified form	If B1B1M1M0M0 can score **ScB1** both here and in the next line for correct answers
			12.52 / 0.5 = 25.04 so 26 tins of paint are needed	**A1**	3.2a		
				[7]			

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