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Friday 17 June 2022 – Afternoon

A Level Further Mathematics A

Y543/01 Mechanics

Time allowed: 1 hour 30 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Answer **all** the questions.

1 A car has mass 1200 kg. The total resistance to the car's motion is constant and equal to 250 N.

(a) The car is driven along a straight horizontal road with its engine working at 10 kW.

Find the acceleration of the car at the instant that its speed is $5\,\text{m}\,\text{s}^{-1}$. **[3]**

The maximum power that the car's engine can generate is 20 kW.

(b) Find the greatest constant speed at which the car can be driven along a straight horizontal road. **[2]**

The car is driven up a straight road which is inclined at an angle θ above the horizontal where $\sin\theta = 0.05$.

(c) Find the greatest constant speed at which the car can be driven up this road. **[2]**

2 The coordinates of two points, A and B, are $(-1, 6)$ and $(5, 12)$ respectively, where the units of the coordinate axes are metres. A particle P moves from A to B under the action of several forces. The force $\mathbf{F} = 7\mathbf{i} - 2\mathbf{j}$ N is one of the forces acting on P.

(a) Calculate the work done by **F** on P as P moves from A to B. **[2]**

At the instant when P reaches B its velocity is $-\mathbf{i} - 5\mathbf{j}\,\text{m}\,\text{s}^{-1}$.

(b) Find the power generated by **F** at the instant that P reaches B. **[2]**

One end of a light elastic string was attached to the origin of the coordinate system and the other to P when P was at A, before it moved to B. The natural length of the string is 8 m and its modulus of elasticity is 24 N.

(c) At the instant that P reaches B, find the following.

- The tension in the string
- The elastic potential energy stored in the string **[3]**

3 A particle P of mass 6 kg moves in a straight line under the action of a single force of magnitude F N which acts in the direction of motion of P.

At time t seconds, where $t \geqslant 0$, F is given by $F = \dfrac{1}{5 - 4\mathrm{e}^{-t^2}}$.

When $t = 0$, the speed of P is $1.9\,\mathrm{m\,s^{-1}}$.

(a) Find the impulse of the force over the period $0 \leqslant t \leqslant 2$. **[2]**

(b) Find the speed of P at the instant when $t = 2$. **[2]**

(c) Find the work done by the force on P over the period $0 \leqslant t \leqslant 2$. **[2]**

4 When two objects are placed a distance apart in outer space each applies a gravitational force to the other. It is suggested that the magnitude of this force depends on the masses of both objects and the distance between them. Assuming that this suggestion is correct, it is further assumed that the magnitude of this force is given by a relationship of the form

$$F = Gm_1^{\alpha} m_2^{\beta} r^{\gamma}$$

where

- F is the magnitude of the force
- m_1 and m_2 are the masses of the two objects
- r is the distance between the two objects
- G is a constant.

(a) Using a dimensional argument based on Newton's third law explain why $\alpha = \beta$. **[1]**

It is given that the magnitude of the gravitational force **is** given by such a relationship and that $G = 6.67 \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$.

(b) Write down the dimensions of G. **[1]**

(c) By using dimensional analysis, determine the values of α, β and γ. **[3]**

You are given that the mass of the Earth is 5.97×10^{24} kg and that the distance of the Moon from the Earth is 3.84×10^{8} m. You may assume that the only force acting on the Moon is the gravitational force due to the Earth.

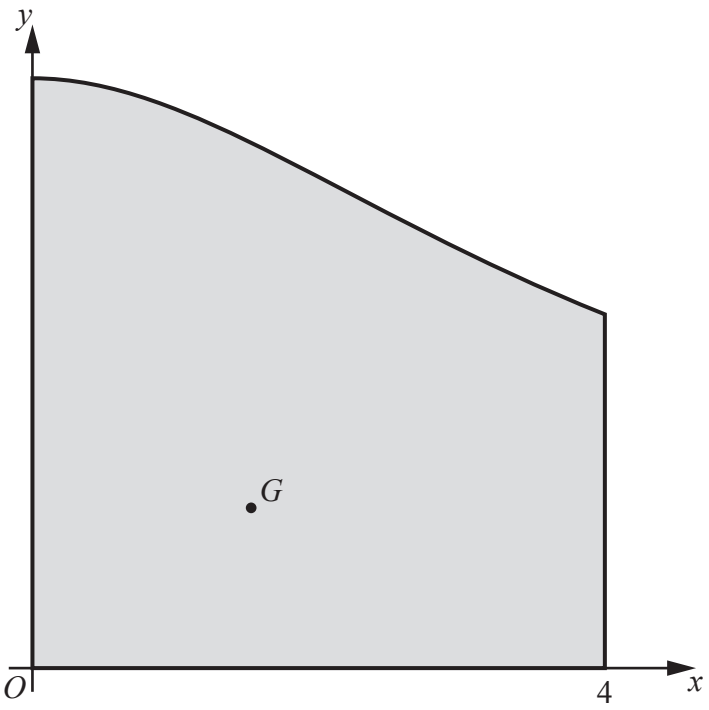
(d) By modelling the Earth as stationary and assuming that the Moon moves in a circular orbit around the Earth, determine the period of the motion of the Moon. Give your answer to the nearest day. **[3]**

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5 In this question you must show detailed reasoning.

The region bounded by the x-axis, the y-axis, the line $x = 4$ and the curve with equation $y = \dfrac{15}{\sqrt{x^2+9}}$ is occupied by a uniform lamina.

The centre of mass of the lamina is at the point $G(\overline{x}, \overline{y})$ (see diagram).



(a) Show that $\overline{x} = \dfrac{2}{\ln 3}$. **[3]**

(b) Determine the value of $\overline{y}$. Give your answer correct to **3** significant figures. **[3]**

P is the point on the curved edge of the lamina where $x = 3$. The lamina is freely suspended from P and hangs in equilibrium in a vertical plane.

(c) Determine the acute angle that the longest straight edge of the lamina makes with the vertical. **[3]**

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6 A particle P of mass 2.5 kg is free to move along the x-axis. When its displacement from the origin is x m its velocity is $v\,\mathrm{m\,s^{-1}}$.

At time $t = 0$ seconds, P is at the point where $x = 1$ and is travelling in the negative x-direction with speed $5\,\mathrm{m\,s^{-1}}$.

At this time an impulse of I Ns is applied to P in the positive x-direction so that P moves in the positive x-direction with speed $18\,\mathrm{m\,s^{-1}}$.

(a) Find the value of I. **[1]**

Subsequently, whenever P is in motion, two forces act on it. The first force acts in the positive x-direction and has magnitude $\frac{5v^2}{x}$ N. The second force acts in the negative x-direction and has magnitude $60v$ N.

(b) Show that the motion of P can be modelled by the differential equation $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{av}{x} + b$ where a and b are constants whose values should be determined. **[2]**

(c) By solving the differential equation derived in part **(b)** find an expression for v in terms of x. **[4]**

You are given that $x = \frac{4}{3\mathrm{e}^{-24t} + 1}$ when $t \geqslant 0$.

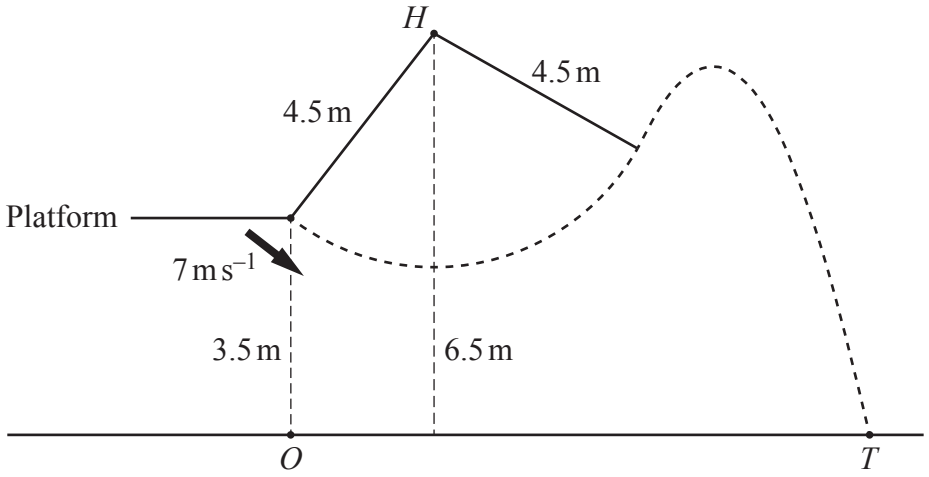
(d) Describe in detail the motion of P when $t \geqslant 0$. **[3]**

7 The training rig for a parachutist comprises a fixed platform and a fixed hook, H. The platform is 3.5 m above horizontal ground level. The hook, which is not directly above the platform, is 6.5 m above the ground.

One end of a light inextensible cord of length 4.5 m is attached to H and the other is attached to a trainee parachutist of mass 90 kg standing on the edge of the platform with the cord straight and taut.

The trainee is then projected off the platform with a velocity of $7\,\text{m}\,\text{s}^{-1}$ perpendicular to the cord in a downward direction. The motion of the trainee all takes place in a single vertical plane and while the cord is attached to H it remains straight and taut.

When the speed of the trainee reaches $5.5\,\text{m}\,\text{s}^{-1}$ the cord is detached from H and the trainee then moves under the influence of gravity alone until landing on the ground (see diagram).



The trainee is modelled as a particle and air resistance is modelled as being negligible.

(a) Show that at the instant before the cord is detached from H, the tension in the cord has a magnitude of 1005.5 N. **[6]**

The point on the ground vertically below the edge of the platform is denoted by O. The point on the ground where the trainee lands is denoted by T.

(b) Determine the distance OT. **[5]**

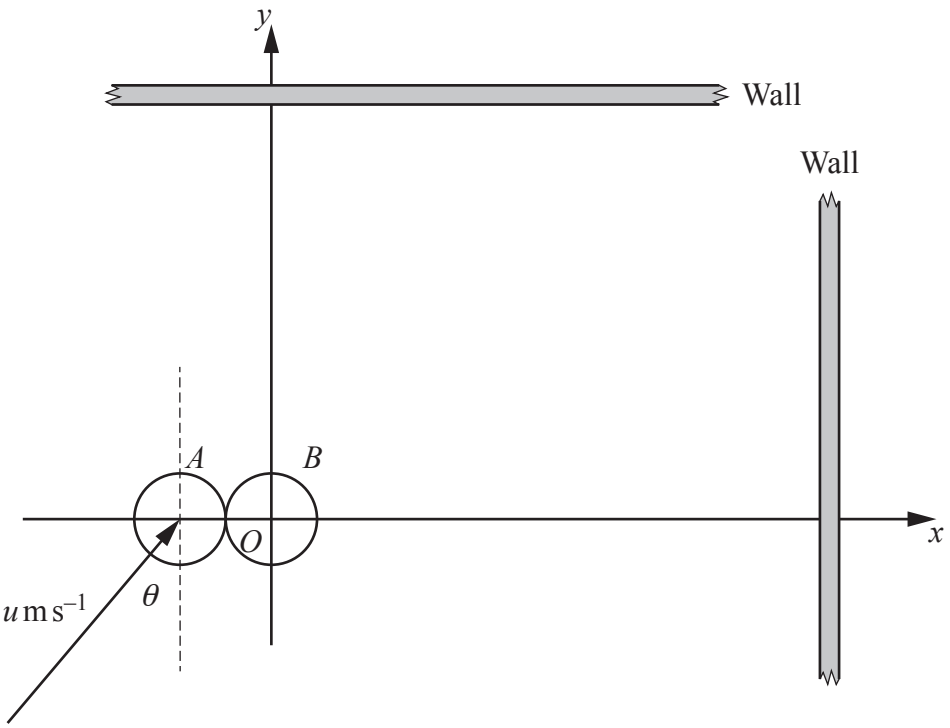
The ground around T is in fact an elastic mat of thickness 0.5 m which is angled so that it is perpendicular to the direction of motion of the trainee on landing. The mat, which is very rough, is modelled as an elastic spring of natural length 0.5 m. It is assumed that the trainee strikes the mat at ground level and is brought to rest once the mat has been compressed by 0.3 m.

(c) Determine the modulus of elasticity of the mat. Give your answer to the nearest integer. **[4]**

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8 Two smooth circular discs, A and B, have equal radii and are free to move on a smooth horizontal plane. The masses of A and B are 1 kg and m kg respectively. B is initially placed at rest with its centre at the origin, O. A is projected towards B with a velocity of $u\,\text{m s}^{-1}$ at an angle of θ to the negative y-axis where $\tan\theta = \frac{5}{2}$. At the instant of collision the line joining their centres lies on the x-axis.

There are two straight vertical walls on the plane. One is perpendicular to the x-axis and the other is perpendicular to the y-axis. The walls are an equal distance from O (see diagram).



After A and B have collided with each other, each of them goes on to collide with a wall. Each then rebounds and they collide again at the same place as their first collision, with disc B again at O.

The coefficient of restitution between A and B is denoted by e. The coefficient of restitution between A and the wall that it collides with is also e while the coefficient of restitution between B and the wall that it collides with is $\frac{5}{9}e$.

It is assumed that any resistance to the motion of A and B may be ignored.

(a) Explain why it must be the case that the collision between A and the wall that it collides with is not inelastic. **[1]**

(b) Show that $e = \frac{1}{m}$. **[4]**

(c) Show that $m = \frac{5}{3}$. **[7]**

(d) State **one** limitation of the model used. **[1]**

END OF QUESTION PAPER

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