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# Wednesday 8 June 2022 – Afternoon

## A Level Further Mathematics A

**Y541/01** Pure Core 2

**Time allowed: 1 hour 30 minutes**

*9458776900*

**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

**ADVICE**
- Read each question carefully before you start your answer.

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Answer **all** the questions.

**1** **(a)** Find a vector which is perpendicular to both $3\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. **[1]**

The equations of two lines are $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 11\mathbf{j} - 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$.

**(b)** Show that the lines intersect, stating the point of intersection. **[5]**

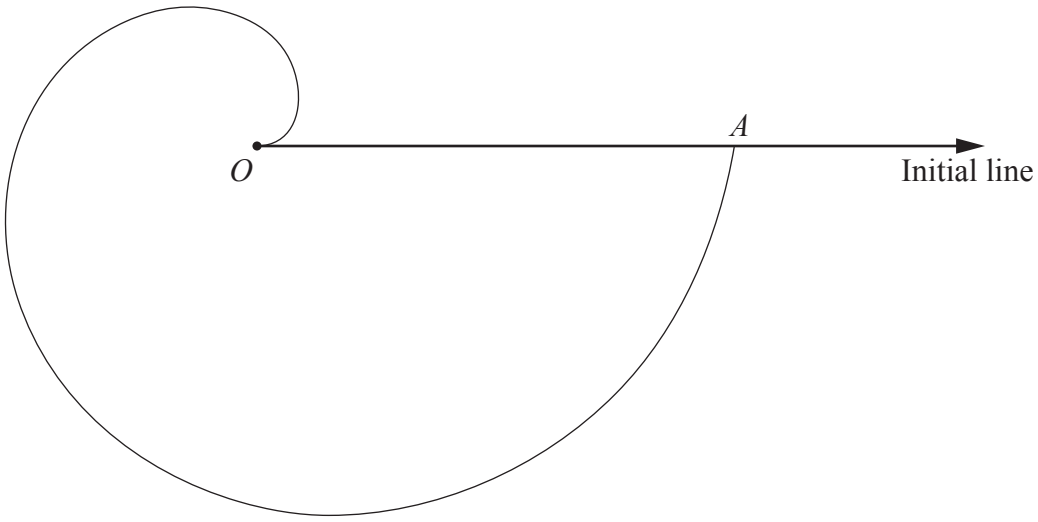
**2** Two polar curves, $C_1$ and $C_2$, are defined by $C_1: r = 2\theta$ and $C_2: r = \theta + 1$ where $0 \leqslant \theta \leqslant 2\pi$.

$C_1$ intersects the initial line at two points, the pole and the point $A$.

**(a)** Write down the polar coordinates of $A$. **[2]**

**(b)** Determine the polar coordinates of the point of intersection of $C_1$ and $C_2$. **[2]**

The diagram below shows a sketch of $C_1$.




**(c)** On the copy of this sketch in the Printed Answer Booklet, sketch $C_2$. **[1]**

**3** **In this question you must show detailed reasoning.**

The roots of the equation $4x^3 + 6x^2 - 3x + 9 = 0$ are $\alpha$, $\beta$ and $\gamma$.

Find a cubic equation with integer coefficients whose roots are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$. **[6]**

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**4** **In this question you must show detailed reasoning.**

Determine the smallest value of $n$ for which $\frac{1^2+2^2+...+n^2}{1+2+...+n} > 341$. **[4]**

**5** **(a)** By using the exponential definitions of $\sinh x$ and $\cosh x$, prove the identity $\cosh 2x \equiv \cosh^2 x + \sinh^2 x$. **[2]**

**(b)** Hence find an expression for $\cosh 2x$ in terms of $\cosh x$. **[1]**

**(c)** Determine the solutions of the equation $5\cosh 2x = 16\cosh x + 21$, giving your answers in exact logarithmic form. **[4]**

**6** A particle, $P$, positioned at the origin, $O$, is projected with a certain velocity along the $x$-axis. $P$ is then acted on by a single force which varies in such a way that $P$ moves backwards and forwards along the $x$-axis.

When the time after projection is $t$ seconds, the displacement of $P$ from the origin is $x$ m and its velocity is $v\,\text{ms}^{-1}$.

The motion of $P$ is modelled using the differential equation $\ddot{x} + \omega^2 x = 0$, where $\omega\,\text{rad s}^{-1}$ is a positive constant.

**(a)** Write down the general solution of this differential equation. **[1]**

$D$ is the point where $x = d$ for some positive constant, $d$. When $P$ reaches $D$ it comes to instantaneous rest.

**(b)** Using the answer to part **(a)**, determine expressions, in terms of $\omega$, $d$ and $t$ only, for the following quantities

- $x$
- $v$

**[3]**

**(c)** Hence show that, according to the model, $v^2 = \omega^2(d^2 - x^2)$. **[1]**

The quantity $z$ is defined by $z = \frac{1}{v}$.

**(d)** Using part **(c)**, determine an expression for $z_m$, the mean value of $z$ **with respect to the displacement**, as $P$ moves directly from $O$ to $D$. **[2]**

One measure of the validity of the model is consideration of the value of $z_m$. If $z_m$ exceeds 8 then the model is considered to be valid.

The value of $d$ is measured as 0.25 to 2 significant figures. The value of $\omega$ is measured as $0.75 \pm 0.02$.

**(e)** Determine what can be inferred about the validity of the model from the given information. **[1]**

**(f)** Find, according to the model, the least possible value of the velocity with which $P$ was initially projected. Give your answer to **2** significant figures. **[2]**

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**7** You are given that $a$ is a parameter which can take only real values.

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 4 & -6 \\ -3 & 10-4a & 9 \\ 7 & 4 & 4 \end{pmatrix}$.

**(a)** Find an expression for the determinant of $\mathbf{A}$ in terms of $a$. **[2]**

You are given the following system of equations in $x$, $y$ and $z$.

$$\begin{aligned} 2x + \quad & 4y - 6z = 6 \\ -3x + \quad & (10-4a)y + 9z = -9 \\ 7x + \quad & 4y + 4z = 11 \end{aligned}$$

The system can be written in the form $\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -9 \\ 11 \end{pmatrix}$.

**(b) (i)** In the case where $\mathbf{A}$ is **not** singular, solve the given system of equations by using $\mathbf{A}^{-1}$. **[5]**

**(ii)** In the case where $\mathbf{A}$ **is** singular describe the configuration of the planes whose equations are the three equations of the system. **[3]**

The transformation represented by $\mathbf{A}$ is denoted by T.
A 3-D object of volume $|5a-20|$ is transformed by T to a 3-D image.

**(c) (i)** Determine the range of values of $a$ for which the orientation of the image is the reverse of the orientation of the object. **[1]**

**(ii)** Determine the range of values of $a$ for which the volume of the image is less than the volume of the object. **[2]**

**8** **In this question you must show detailed reasoning.**

It is given that $\sum_{r=k}^{98} \frac{5r+2}{r(r+1)(r+2)} = \frac{20539}{34650}$ for some $k$.

Determine the value of $k$. **[7]**

**9** **In this question you must show detailed reasoning.**

**(a)** Show that $\mathrm{Re}\left(\mathrm{e}^{4\mathrm{i}\theta}\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)^4\right) = a\cos 4\theta\cos^4\theta$, where $a$ is an integer to be determined. **[3]**

**(b)** Hence show that $\cos\frac{1}{12}\pi = \frac{1}{2}\sqrt[4]{b+c\sqrt{3}}$, where $b$ and $c$ are integers to be determined. **[6]**

**10** The coordinates of the points $A$ and $B$ are $(3, -2, -1)$ and $(13, 10, 9)$ respectively.

- The plane $\Pi_A$ contains $A$ and the plane $\Pi_B$ contains $B$.
- The planes $\Pi_A$ and $\Pi_B$ are parallel.
- The $x$ and $y$ components of any normal to plane $\Pi_A$ are equal.
- The shortest distance between $\Pi_A$ and $\Pi_B$ is 2.

There are **two** possible solution planes for $\Pi_A$ which satisfy the above conditions.

Determine the acute angle between these two possible solution planes. **[8]**

**END OF QUESTION PAPER**

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