OCR
Oxford Cambridge and RSA

Thursday 20 June 2019 – Morning

A Level Further Mathematics A

Y544/01 Discrete Mathematics

Time allowed: 1 hour 30 minutes

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You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

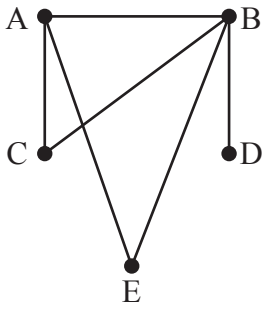
INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

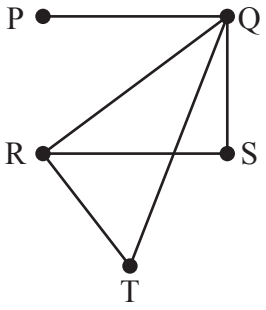
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Answer **all** the questions.

1 Two graphs are shown below.



Graph G1



Graph G2

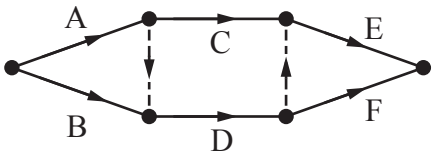
(a) **(i)** Prove that the graphs are isomorphic. **[2]**

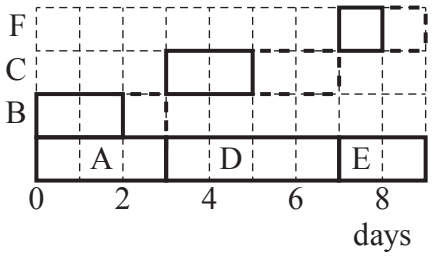
(ii) Verify that Euler's formula holds for graph G1. **[2]**

(b) Describe how it is possible to add 4 arcs to graph G1 to make a non-planar graph with 5 vertices. **[2]**

(c) Describe how it is possible to add a vertex U and 4 arcs to graph G2 to make a connected non-planar graph with 6 vertices. **[2]**

2 A project is represented by the activity network and cascade chart below. The table showing the number of workers needed for each activity is incomplete. Each activity needs at least 1 worker.





Activity	Workers
A	2
B	x
C	
D	
E	
F	

(a) Complete the table in the Printed Answer Booklet to show the immediate predecessors for each activity. **[2]**

(b) Calculate the latest start time for each non-critical activity. **[1]**

The minimum number of workers needed is 5.

(c) What type of problem (existence, construction, enumeration or optimisation) is the allocation of a number of workers to the activities? **[1]**

There are 8 workers available who can do activities A and B.

(d) **(i)** Find the number of ways that the workers for activity A can be chosen. **[1]**

(ii) When the workers have been chosen for activity A, find the total number of ways of choosing the workers for activity B for all the different possible values of x, where $x \geqslant 1$. **[2]**

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3 A problem is represented as the initial simplex tableau below.

P	x	y	z	s	t	RHS
1	−2	0	1	0	0	0
0	1	1	1	1	0	60
0	2	3	4	0	1	60

(a) Write the problem as a linear programming formulation in the standard algebraic form with no slack variables. **[3]**

(b) Carry out one iteration of the simplex algorithm. **[3]**

(c) Show algebraically how each row of the tableau found in part **(b)** is calculated. **[3]**

4 An algorithm must have an input, an output, be deterministic and finite.

(a) Why is a counter sometimes used in an algorithm? **[1]**

A computer takes 0.2 seconds to sort a list of 500 numbers.

(b) How long would you expect the computer to take to sort a list of 5000 numbers? **[1]**

Simon says that he can sort a list of numbers 'just by looking at them'.

(c) Explain to Simon why sorting algorithms are needed. **[2]**

(d) Demonstrate how quick sort works by using it to sort the following list into **increasing** order. You should indicate the pivots used and which values are already known to be in their correct position.

41 17 8 33 29 **[4]**

For an average case the efficiency of quick sort is $O(n \log n)$, where n is the number of items in the list.

(e) Explain why quick sort is typically quicker than bubble sort and shuttle sort. **[1]**

When the number of comparisons made is used as a measure of the efficiency, the worst case for quick sort is no more efficient than the worst case for bubble sort.

An arrangement of the five numbers from part **(d)** makes up a new list that is to be sorted using the bubble sort or the quick sort.

(f) Without writing out all the passes, determine

- the worst case list
- the total number of comparisons for the worst case list

for each of the algorithms in turn. **[3]**

5 A network is represented by the distance matrix below. For this network a direct connection between two vertices is always shorter than an indirect connection.

	A	B	C	D	E	F	G	H
A	-	130	100	-	-	250	-	-
B	130	-	-	50	-	-	170	100
C	100	-	-	-	80	170	-	90
D	-	50	-	-	-	-	120	-
E	-	-	80	-	-	140	-	120
F	250	-	170	-	140	-	-	-
G	-	170	-	120	-	-	-	90
H	-	100	90	-	120	-	90	-

(a) How does the distance matrix show that the arcs are undirected? **[1]**

The shortest distance from A to E is 180.

(b) Write down the shortest route from A to E. **[1]**

(c) Use Dijkstra's algorithm on the distance matrix to find the length of the shortest route **from G** to each of the other vertices. **[4]**

The arcs represent roads and the weights represent distances in metres. The total length of all the roads is 1610 metres.

Emily and Stephen have set up a company selling ice-creams from a van.

(d) Emily wants to deliver leaflets to the houses along **each side** of each road. Find the length of the shortest continuous route that Emily can use. **[1]**

(e) Stephen wants to drive along each road in the ice-cream van.

(i) Determine the length of the shortest route for Stephen if he starts **at B**. **[5]**

(ii) Stephen wants to use the shortest possible route.

- Find the length of the shortest possible route.
- Write down the start and end vertices of this route. **[2]**

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6 The pay-off matrix for a game between two players, Sumi and Vlad, is shown below. If Sumi plays A and Vlad plays X then Sumi gets x points and Vlad gets 1 point.

		Vlad		
		X	Y	Z
Sumi	A	$(x, 1)$	$(4, -2)$	$(2, 0)$
	B	$(3, -1)$	$(6, -4)$	$(-1, 3)$

You are given that cell (A, X) is a Nash Equilibrium solution.

(a) Find the range of possible values of x. **[1]**

(b) Explain what the statement 'cell (A, X) is a Nash Equilibrium solution' means for each player. **[2]**

(c) Find a cell where each player gets their maximin pay-off. **[3]**

Suppose, **instead**, that the game can be converted to a zero-sum game.

(d) Determine the optimal strategy for Sumi for the zero-sum game.
- Record the pay-offs for Sumi when the game is converted to a zero-sum game.
- Describe how Sumi should play using this strategy. **[7]**

7 Sam is making pies.

There is exactly enough pastry to make 7 large pies or 20 medium pies or 36 small pies, or some mixture of large, medium and small pies.

This is represented as a constraint $180x + 63y + 35z \leqslant 1260$.

(a) Write down what x, y and z represent. **[2]**

There is exactly enough filling to make 5 large pies or 12 medium pies or 18 small pies, or some mixture of large, medium and small pies.

(b) Express this as a constraint of the form $ax + by + cz \leqslant d$, where a, b, c and d are integers. **[2]**

The number of small pies must equal the total number of large and medium pies.

(c) Show that making exactly 9 small pies is inconsistent with the constraints. **[2]**

(d) Determine the maximum number of large pies that can be made.
- Your reasoning should be in the form of words, calculations or algebra.
- You must check that your solution is feasible. **[6]**

END OF QUESTION PAPER

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