OCR
Oxford Cambridge and RSA

Tuesday 19 October 2021 – Afternoon

A Level Further Mathematics B (MEI)

Y431/01 Mechanics Minor

Time allowed: 1 hour 15 minutes

9 0 5 0 7 1 9 1 8 3

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space, use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2021 [K/508/5595]
DC (ST/FC) 300799/3
OCR is an exempt Charity

stemup.co.uk

Answer **all** the questions.

1 **(a)** State the dimensions of force. **[1]**

The force F required to keep a car moving at constant speed on a circular track is given by the formula

$$F = \frac{mv^2}{r},$$

where

- m is the constant mass of the car,
- v is the speed of the car,
- r is the radius of the circular track.

(b) Verify that the formula is dimensionally consistent. **[2]**

(c) Determine the percentage increase in force required to keep a car moving on a circular track if the speed of the car were to increase by 10% and if the track radius were to decrease by 10%. **[2]**

It is proposed that a new unit of force, the *trackforce* (Tr), should be adopted in motor-racing. 1 Tr is defined as the amount of force required to accelerate a mass of 1 ton at a rate of 1 mile per hour per second.

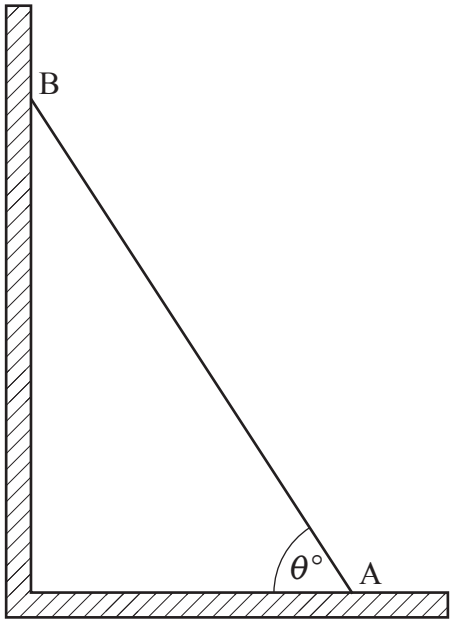
It is given that 1 ton = 1016 kg and 1 mile = 1609 m.

(d) Determine the number of newtons that are equivalent to 1 Tr. **[2]**

© OCR 2021

stemup.co.uk

2 The diagram shows a uniform beam AB that rests with its end A on rough horizontal ground and its end B against a smooth vertical wall. The beam makes an angle of $\theta°$ with the ground.



The weight of the beam is WN.

The beam is in limiting equilibrium and the coefficient of friction between the beam and the ground is μ.

It is given that the magnitude of the contact force at A is 70 N and the magnitude of the contact force at B is 20 N.

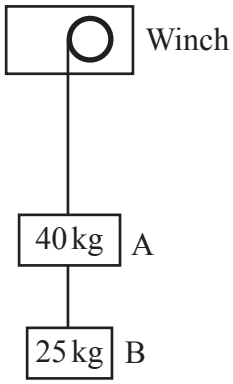
Determine, in any order,

- the value of W,
- the value of μ,
- the value of θ. **[7]**

© OCR 2021

stemup.co.uk

3 The diagram shows an electric winch raising two crates A and B, with masses 40 kg and 25 kg, respectively. The cable connecting the winch to A, and the cable connecting A to B may both be modelled as light and inextensible. Furthermore, it can be assumed that there are no resistances to motion.



Throughout the entire motion, the power P W developed by the winch is constant.

Crates A and B are both being raised at a constant speed $v\,\text{m s}^{-1}$ when the cable connecting A and B breaks. After the cable between A and B breaks, crate A continues to be raised by the winch. Crate A now accelerates until it reaches a new constant speed of $(v+3)\,\text{m s}^{-1}$.

Determine

- the value of v,
- the value of P. **[5]**

© OCR 2021

stemup.co.uk

4 A child throws a ball of mass m kg vertically upwards with a speed of $7.2\,\text{m s}^{-1}$.
The ball leaves the child's hand at a height of 1.6 m above horizontal ground.

(a) Ignoring any possible air resistance, use an energy method to determine the maximum height reached by the ball above the ground. **[2]**

In fact, the ball only reaches a height of 4.1 m above the ground. For the rest of this question you should assume that the air resistance may be modelled as a constant force acting in the opposite direction to the ball's motion.

(b) Show that the ball does $0.568m$ J of work against air resistance per metre travelled. **[2]**

(c) Calculate the speed of the ball just before it hits the ground. **[3]**

The ball bounces off the ground and first comes instantaneously to rest 2.8 m above the ground.

(d) Determine the coefficient of restitution between the ball and the ground. **[3]**

In the first impact between the ball and the ground, the magnitude of the impulse exerted on the ball by the ground is 12 N s.

(e) Determine the value of m. **[2]**

© OCR 2021

stemup.co.uk

5 **Fig. 5.1** shows a solid L-shaped ornament, of uniform density. The ornament is 3 cm thick. The x, y and z axes are shown, along with the dimensions of the ornament. The measurements are in centimetres.

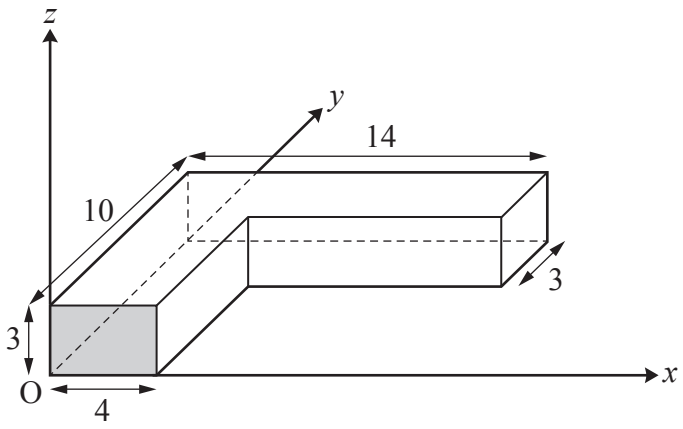


Fig. 5.1

(a) Determine, with reference to the axes shown, the coordinates of the ornament's centre of mass. **[4]**

Fig. 5.2 shows the ornament placed so that the shaded face (indicated in **Fig. 5.1**) is in contact with a plane inclined at $\theta°$ to the horizontal, with the 4 cm edge parallel to a line of greatest slope. The surface of the plane is sufficiently rough so that the ornament will not slip down the plane.

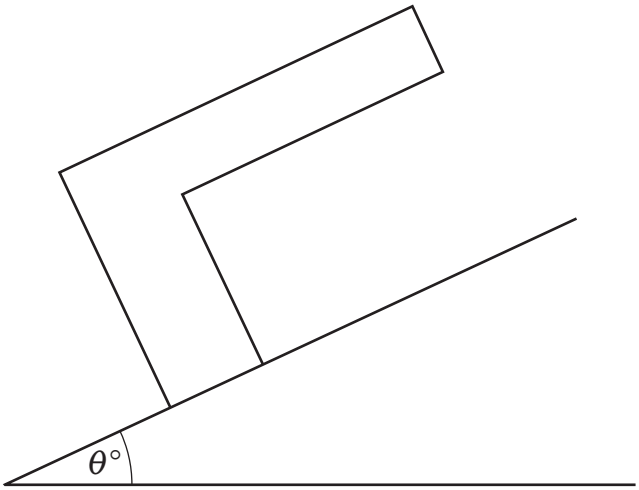


Fig. 5.2

(b) Determine the minimum and maximum possible values of θ for which the ornament does not topple. **[3]**

© OCR 2021

stemup.co.uk

The ornament is now placed with its shaded face in contact with a rough horizontal surface. A force of magnitude PN, acting parallel to the planes of the L-shaped faces, is applied to one of the edges of the ornament, as shown in **Fig. 5.3**. The force is inclined at an angle of 30° to the horizontal. The coefficient of friction between the ornament and the surface is μ.

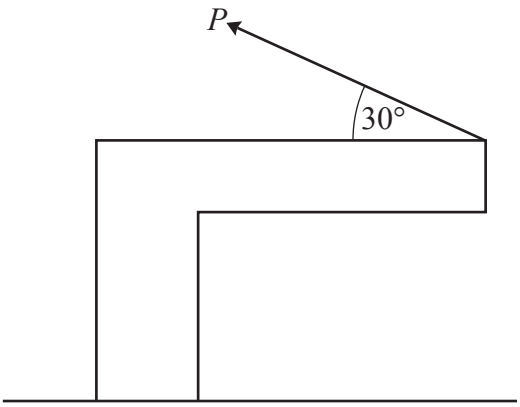


Fig. 5.3

The value of P is gradually increased until the ornament is on the point of toppling but does not slide.

(c) Determine the minimum value of μ. **[7]**

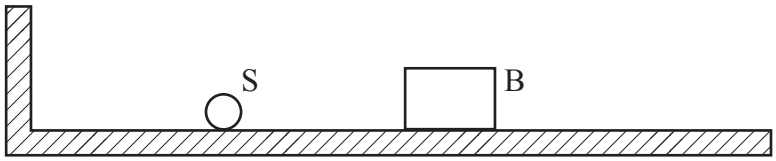
(d) Explain how your answer to part **(c)** would change if the angle between P and the horizontal was less than 30°. **[2]**

stemup.co.uk

6 A block rests on a horizontal surface. The coefficient of friction between the block and the surface is μ.

(a) Show that if the block is given an initial speed of $v\,\text{m}\,\text{s}^{-1}$, it will move a distance of $\frac{v^2}{2\mu g}$ before coming to rest. **[3]**

Block B rests on the same horizontal surface as a sphere S. On the other side of S is a vertical wall, as shown below. The mass of B is 8 times the mass of S.



S is projected directly towards B with speed $u\,\text{m}\,\text{s}^{-1}$ and hits B.
It is given that

- the coefficient of restitution between S and B is 0.8,
- collisions between S and the wall are perfectly elastic,
- the wall is perpendicular to the direction of motion of S and B.

Furthermore, you should model the contact between B and the surface as **rough** and model the contact between S and the surface as **smooth**.

(b) Determine, in terms of u, expressions for

- the speed of S
- the speed of B

immediately after the first collision between S and B. In each case stating the corresponding direction of motion. **[5]**

It is given that B has sufficient time to come to rest before each subsequent collision with S.

Let x_n be the distance B moves after the nth impact between S and B.

(c) Explain why $x_{n+1} = \frac{9}{25}x_n$. **[2]**

(d) Given that $u = 11.2$ and the coefficient of friction between B and the surface is $\frac{1}{7}$, show that B will travel a total distance that cannot exceed 2.8 m. **[3]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© OCR 2021

stemup.co.uk

Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) $[F] = MLT^{-2}$

1 (b) $[m] = M, \quad [v] = LT^{-1}, \quad [r] = L$

$$\left[\frac{mv^2}{r}\right] = \frac{M(LT^{-1})^2}{L} = \frac{ML^2T^{-2}}{L}$$

$$= MLT^{-2} = [F]$$

Both sides have dimensions MLT^{-2}, so the formula is dimensionally consistent.

1 (c) $F \propto \frac{v^2}{r}$: new force $= \frac{(1.1v)^2}{0.9r} \times m = \frac{1.21}{0.9} \times \frac{mv^2}{r}$

$$\frac{1.21}{0.9} = 1.3444\ldots$$

Percentage increase = 34.4%

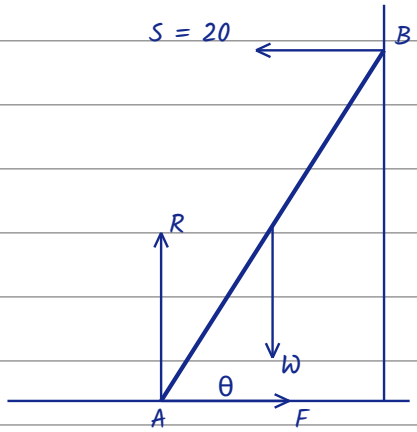
1 (d) 1 mile per hour per second $= \frac{1609}{3600}\ m\ s^{-2}$

$$1\ Tr = 1016 \times \frac{1609}{3600} = 454.09\ldots$$

1 Tr = 454 N (3 s.f.)

Question number

2



Wall smooth, so the force at B is normal: $S = 20$ N horizontally.

At A: friction F (horizontal) and normal reaction R (vertical).

Resolve horizontally: $F = S = 20$

Contact force at A: $F^2 + R^2 = 70^2$

$R = \sqrt{70^2 - 20^2} = \sqrt{4500} = 30\sqrt{5}$

Resolve vertically: $W = R = 30\sqrt{5} = 67.08...$

$W = 67.1$ N (3 s.f.)

Limiting friction: $F = \mu R \Rightarrow \mu = \frac{20}{30\sqrt{5}} = \frac{2\sqrt{5}}{15}$

$\mu = 0.298$ (3 s.f.)

Let AB = 2a. Moments about A:

$W \times a \cos\theta = 20 \times 2a \sin\theta$

$\tan\theta = \frac{W}{40} = \frac{30\sqrt{5}}{40} = \frac{3\sqrt{5}}{4}$

$\theta = 59.2°$ (3 s.f.)

3

Constant speed, no resistance, so the tension in the winch cable equals the weight being raised. $P = Fv$.

Before the cable breaks: $P = 65g \times v = 637v$

Question number

After, at the new constant speed: $P = 40g \times (v + 3) = 392(v + 3)$

P constant: $637v = 392v + 1176$

$245v = 1176$

$v = 4.8$

$P = 637 \times 4.8 = 3057.6$

$P = 3060$ W (3 s.f.)

4 (a) Let h be the height risen above the hand. KE lost = GPE gained:

$\frac{1}{2}m(7.2)^2 = m \times 9.8 \times h$

$h = \frac{25.92}{9.8} = 2.6449...$

Maximum height = 1.6 + 2.6449... = 4.24 m (3 s.f.)

4 (b) Let the work done against air resistance be W joules per metre.

The ball rises 4.1 − 1.6 = 2.5 m. Work-energy principle:

$\frac{1}{2}m(7.2)^2 - 2.5W = m \times 9.8 \times 2.5$

$25.92m - 2.5W = 24.5m$

$2.5W = 1.42m$

$W = 0.568m$ J per metre, as required

4 (c) Let v be the speed just before hitting the ground (falls 4.1 m from rest).

$m \times 9.8 \times 4.1 - 0.568m \times 4.1 = \frac{1}{2}mv^2$

$\frac{1}{2}v^2 = 4.1 \times 9.232 = 37.8512$

$v^2 = 75.7024$

Question number

$v = 8.70 \text{ m s}^{-1}$ (3 s.f.)

4 (d) Let V be the speed just after the bounce (rises 2.8 m to rest).

$\frac{1}{2}mV^2 - 0.568m \times 2.8 = m \times 9.8 \times 2.8$

$\frac{1}{2}V^2 = 2.8 \times 10.368 = 29.0304$

$V^2 = 58.0608 \Rightarrow V = 7.6197...$

$e = \frac{V}{v} = \frac{7.6197...}{8.7007...} = 0.876$ (3 s.f.)

4 (e) Impulse = change in momentum (taking up as positive):

$12 = mV - (-mv) = m(V + v)$

$m = \frac{12}{7.6197... + 8.7007...} = \frac{12}{16.320...}$

$m = 0.735$ (3 s.f.)

5 (a) Uniform thickness 3 cm, so $\bar{z} = 1.5$

Split the L into two cuboids:

Part 1: 4 × 10 × 3, volume 120, centre (2, 5)

Part 2: 10 × 3 × 3, volume 90, centre (9, 8.5)

$210\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 120\begin{pmatrix}2\\5\end{pmatrix} + 90\begin{pmatrix}9\\8.5\end{pmatrix} = \begin{pmatrix}1050\\1365\end{pmatrix}$

$\bar{x} = \frac{1050}{210} = 5, \quad \bar{y} = \frac{1365}{210} = 6.5$

Centre of mass is at (5, 6.5, 1.5)

5 (b) The base is the 4 cm edge along the slope; G is 6.5 cm from the base

Question number

and 5 cm up the slope from the lower edge (1 cm beyond the upper edge).

No toppling when the vertical through G lies over the base:

Toppling up the slope (θ too small): need $\tan\theta \geq \frac{5-4}{6.5}$

$\theta_{min} = \tan^{-1}\frac{1}{6.5} = 8.75°$ (3 s.f.)

Toppling down the slope (θ too big): need $\tan\theta \leq \frac{5}{6.5}$

$\theta_{max} = \tan^{-1}\frac{5}{6.5} = 37.6°$ (3 s.f.)

5 (c) Let O be the left-hand edge of the base, R and F the normal reaction and friction, m the mass. P acts at (14, 10) relative to O.

On the point of toppling about O, so R and F act at O.

Moments about O:

$P\sin 30° \times 14 + P\cos 30° \times 10 = mg \times 5$

$P(7 + 5\sqrt{3}) = 5mg$

$P = \frac{5mg}{7 + 5\sqrt{3}}$

Resolve vertically: $R = mg - P\sin 30°$

Resolve horizontally: $F = P\cos 30°$

Does not slide, so $F \leq \mu R$:

$P\cos 30° \leq \mu(mg - P\sin 30°)$

Substitute for P, multiply by $(7 + 5\sqrt{3})$ and divide by mg:

$5\cos 30° \leq \mu(7 + 5\sqrt{3} - 5\sin 30°)$

$\frac{5\sqrt{3}}{2} \leq \mu(\frac{9}{2} + 5\sqrt{3})$

Question number

$\mu \geqslant \dfrac{5\sqrt{3}}{9 + 10\sqrt{3}} = \dfrac{5}{3\sqrt{3} + 10}$

$\mu \geqslant \dfrac{50 - 15\sqrt{3}}{73}$

Minimum value of $\mu = 0.329$ (3 s.f.)

5 (d) If the angle were smaller, P would have a larger horizontal component (and a smaller vertical component, so R is larger but the turning effect of P about O is smaller, so P at toppling is larger).

With P at angle α: $\mu_{min} = \dfrac{5}{9\tan\alpha + 10}$, which increases as α decreases,

so the minimum value of μ would be larger.

6 (a) Let the mass be m and the distance travelled be x.

Block is sliding, so friction $F = \mu R = \mu mg$.

Work-energy principle: work done by friction = loss of KE

$\mu mg \times x = \frac{1}{2}mv^2 - 0$

$x = \dfrac{v^2}{2\mu g}$, as required

6 (b) Let S have mass m, B mass 8m. Velocities after: v_S, v_B (positive in the direction S was moving).

Conservation of momentum: $mu = mv_S + 8mv_B$

Newton's law of restitution: $v_B - v_S = 0.8u$

$u = v_S + 8(0.8u + v_S) = 9v_S + 6.4u$

$v_S = -0.6u$, $v_B = 0.8u - 0.6u = 0.2u$

Question number

S has speed 0.6u, back towards the wall (reversed direction).

B has speed 0.2u, away from the wall (direction S was moving).

6 (c) S rebounds elastically from the wall, so it returns with the same speed 0.6u and hits B, which is at rest again.

So each impact is the same as the first with the speed of S multiplied by $\frac{3}{5}$, so B leaves each impact with $\frac{3}{5}$ of its previous speed.

By (a), the distance moved is $\frac{v^2}{2\mu g}$, i.e. proportional to v^2,

so $x_{n+1} = (\frac{3}{5})^2 x_n = \frac{9}{25} x_n$

6 (d) After the first impact B has speed $0.2 \times 11.2 = 2.24$ m s^{-1}

$2\mu g = 2 \times \frac{1}{7} \times 9.8 = 2.8$

$x_1 = \frac{2.24^2}{2.8} = \frac{5.0176}{2.8} = 1.792$

Distances form a geometric series with $a = 1.792$, $r = \frac{9}{25}$

Total distance $\leq \sum_{n=1}^{\infty} x_n = \frac{a}{1-r} = \frac{1.792}{0.64} = 2.8$

So B will travel a total distance that cannot exceed 2.8 m, as required