OCR
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# Monday 05 October 2020 – Afternoon

## A Level Further Mathematics B (MEI)

### Y420/01 Core Pure

**Time allowed: 2 hours 40 minutes**

*8178476396*

**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **144**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

**ADVICE**
- Read each question carefully before you start your answer.

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**Section A** (36 marks)

Answer **all** the questions.

**1** Using standard summation of series formulae, determine the sum of the first $n$ terms of the series

$(1\times2\times4)+(2\times3\times5)+(3\times4\times6)+\ldots$,

where $n$ is a positive integer. Give your answer in fully factorised form. **[6]**

**2** **(a)** The matrices $\mathbf{M}=\begin{pmatrix}0 & 1 & a\\ 1 & b & 0\end{pmatrix}$ and $\mathbf{N}=\begin{pmatrix}b & -5\\ -1 & c\\ -1 & 1\end{pmatrix}$ are such that $\mathbf{MN}=\mathbf{I}$.

Find $a$, $b$ and $c$. **[5]**

**(b)** State with a reason whether or not **N** is the inverse of **M**. **[1]**

**3** **In this question you must show detailed reasoning.**

Find $\displaystyle\int_0^{\frac{1}{3}}\frac{1}{\sqrt{4-9x^2}}\,\mathrm{d}x$, expressing your answer in terms of $\pi$. **[4]**

**4** The roots of the equation $2x^3-5x+7=0$ are $\alpha$, $\beta$ and $\gamma$.

**(a)** Find $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$. **[4]**

**(b)** Find an equation with integer coefficients whose roots are $2\alpha-1$, $2\beta-1$ and $2\gamma-1$. **[4]**

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**5** Fig. 5 shows the curve with polar equation $r = a(3+2\cos\theta)$ for $-\pi \leqslant \theta \leqslant \pi$, where $a$ is a constant.

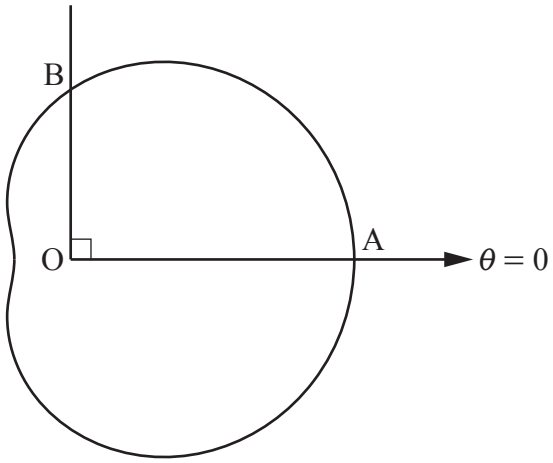



**Fig. 5**

**(a)** Write down the polar coordinates of the points A and B. **[2]**

**(b)** Explain why the curve is symmetrical about the initial line. **[2]**

**(c)** **In this question you must show detailed reasoning.**

Find in terms of $a$ the exact area of the region enclosed by the curve. **[4]**

**6** The complex number $z$ satisfies the equation $z^2 - 4\mathrm{i}z^* + 11 = 0$.

Given that $\mathrm{Re}(z) > 0$, find $z$ in the form $a + b\mathrm{i}$, where $a$ and $b$ are real numbers. **[4]**

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**Section B** (108 marks)

Answer **all** the questions.

**7** Prove by mathematical induction that $\sum_{r=1}^{n}(r \times r!) = (n+1)! - 1$ for all positive integers $n$. **[6]**

**8** **(a)** Given that the lines $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ k \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ meet, determine $k$. **[5]**

**(b)** **In this question you must show detailed reasoning.**

Find the acute angle between the two lines. **[4]**

**9** A linear transformation of the plane is represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & -2 \\ \lambda & 3 \end{pmatrix}$, where $\lambda$ is a constant.

**(a)** Find the set of values of $\lambda$ for which the linear transformation has no invariant lines through the origin. **[5]**

**(b)** Given that the transformation multiplies areas by 5 and reverses orientation, find the invariant lines. **[3]**

**10** **In this question you must show detailed reasoning.**

The region in the first quadrant bounded by curve $y = \cosh \frac{1}{2}x^2$, the $y$-axis, and the line $y = 2$ is rotated through 360° about the $y$-axis.

Find the exact volume of revolution generated, expressing your answer in a form involving a logarithm. **[7]**

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**11 In this question you must show detailed reasoning.**

In Fig. 11, the points A, B, C, D, E and F represent the complex sixth roots of 64 on an Argand diagram. The midpoints of AB, BC, CD, DE, EF and FA are G, H, I, J, K and L respectively.

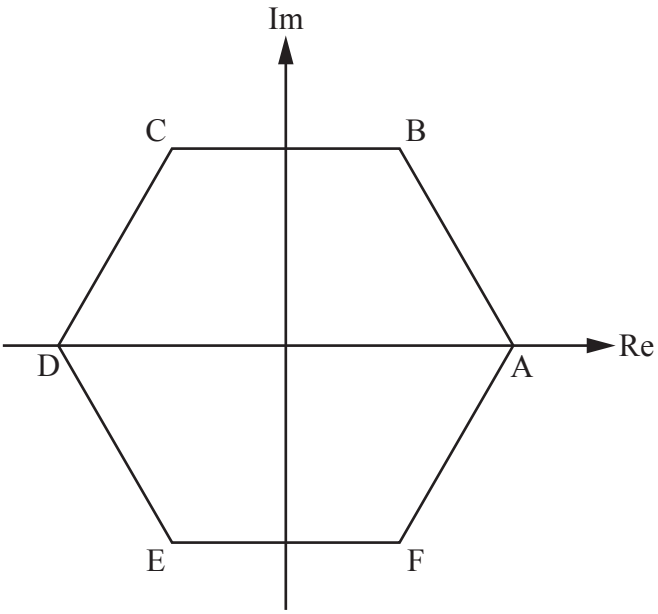



**Fig. 11**

**(a)** Write down, in exponential ($re^{i\theta}$) form, the complex numbers represented by the points A, B, C, D, E and F. **[2]**

**(b)** When these complex numbers are multiplied by the complex number $w$, the resulting complex numbers are represented by the points G, H, I, J, K and L.

Find $w$ in exponential form. **[4]**

**(c)** You are given that G, H, I, J, K and L represent roots of the equation $z^6 = p$.

Find $p$. **[2]**

**12 (a)** Given that $z = \cos\theta + i\sin\theta$, express $z^n + \frac{1}{z^n}$ and $z^n - \frac{1}{z^n}$ in simplified trigonometric form. **[2]**

**(b)** By considering $\left(z + \frac{1}{z}\right)^3\left(z - \frac{1}{z}\right)^3$, find constants $A$ and $B$ such that

$\sin^3\theta\cos^3\theta = A\sin 6\theta + B\sin 2\theta$. **[6]**

**13** **(a)** Using exponentials, prove that $\sinh 2x = 2\cosh x \sinh x$. **[2]**

**(b)** Hence show that if $f(x) = \sinh^2 x$, then $f''(x) = 2\cosh 2x$. **[2]**

**(c)** Explain why the coefficients of odd powers in the Maclaurin series for $\sinh^2 x$ are all zero. **[2]**

**(d)** Find the coefficient of $x^n$ in this series when $n$ is a positive even number. **[3]**

**14** Solve the simultaneous differential equations

$$\frac{dx}{dt} + 2x = 4y, \qquad \frac{dy}{dt} + 3x = 5y,$$

given that when $t = 0$, $x = 0$ and $y = 1$. **[11]**

**15** **(a)** Show that the three planes with equations

$$\begin{aligned} x + \lambda y + 3z &= -12 \\ 2x + y + 5z &= -11 \\ x - 2y + 2z &= -9 \end{aligned}$$

where $\lambda$ is a constant, meet at a unique point except for one value of $\lambda$ which is to be determined. **[3]**

**(b)** In the case $\lambda = -2$, use matrices to find the point of intersection P of the planes, showing your method clearly. **[3]**

The line $l$ has equation $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z+2}{-2}$.

**(c)** Find a vector equation of $l$. **[2]**

**(d)** Find the shortest distance between the point P and $l$. **[4]**

**(e)** **(i)** Show that $l$ is parallel to the plane $x - 2y + 2z = -9$. **[3]**

**(ii)** Find the distance between $l$ and the plane $x - 2y + 2z = -9$. **[2]**

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**16** The population density $P$, in suitable units, of a certain bacterium at time $t$ hours is to be modelled by a differential equation. Initially, the population density is zero, and its long-term value is $A$.

**(a)** One simple model is to assume that the rate of change of population density is directly proportional to $A - P$.

**(i)** Formulate a differential equation for this model. **[1]**

**(ii)** Verify that $P = A(1 - \mathrm{e}^{-kt})$, where $k$ is a positive constant, satisfies

- this differential equation,
- the initial condition,
- the long-term condition. **[3]**

An alternative model uses the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} - \frac{P}{t(1+t^2)} = \mathrm{Q}(t),$$

where $\mathrm{Q}(t)$ is a function of $t$.

**(b)** Find the integrating factor for this differential equation, showing that it can be written in the form $\dfrac{\sqrt{1+t^2}}{t}$. **[8]**

**(c)** Suppose that $\mathrm{Q}(t) = 0$.

**(i)** Show that $P = \dfrac{At}{\sqrt{1+t^2}}$. **[4]**

**(ii)** Find the time predicted by this model for the population density to reach half its long-term value. Give your answer correct to the nearest minute. **[2]**

**(d)** Now suppose that $\mathrm{Q}(t) = \dfrac{t\mathrm{e}^{-t}}{\sqrt{1+t^2}}$.

Show that $P = \dfrac{At - t\mathrm{e}^{-t}}{\sqrt{1+t^2}}$. [You may assume that $\lim\limits_{t\to\infty} t\mathrm{e}^{-t} = 0$.] **[5]**

It is found that the long-term value of $P$ is 10, and $P$ reaches half this value after 37 minutes.

**(e)** Determine which of the models proposed in parts **(c)** and **(d)** is more consistent with these data. **[2]**

**END OF QUESTION PAPER**

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