OCR
Oxford Cambridge and RSA

Monday 23 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y436/01 Further Pure with Technology

Time allowed: 1 hour 45 minutes

1874740945

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a computer with appropriate software
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2025 [K/508/5600]
DC (PQ/FC) 352593/3
OCR is an exempt Charity

1 A family of curves is given by the cartesian equation

$$\frac{x^2}{a^2}+mxy+\frac{y^2}{b^2}=1$$

where a, b and m are real numbers with a and b non-zero.

(a) In this part of the question $a=2$ and $b=1$.

(i) On the axes in the Printed Answer Booklet, sketch the curve in each of these cases.

- $m=0$
- $m=1$
- $m=2$

[3]

(ii) State **one** feature of the curve for the case $m=0$ that is **not** a feature of the curve in the cases $m=1$ and $m=2$. **[1]**

For the remainder of this question $m=0$.

(b) Verify that the parametric equations of the curve are

$$x(t)=a\cos(t),\ y(t)=b\sin(t),$$

where $0 \leqslant t < 2\pi$ is a parameter. **[1]**

(c) Show that $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{b}{a}\cot(t)$. **[2]**

(d) Show that the equation of the normal to the curve at the point with parameter t is

$$y=\frac{a}{b}\tan(t)x-\left(\frac{a^2-b^2}{b}\right)\sin(t).$$ **[5]**

(e) Show that the parametric equations of the envelope of the normal to the curve are

$$x(t)=\left(\frac{a^2-b^2}{a}\right)\cos^3(t),\ y(t)=\left(\frac{b^2-a^2}{b}\right)\sin^3(t),$$

where $0 \leqslant t < 2\pi$ is a parameter. **[6]**

(f) In this part of the question $a=2$ and $b=1$.

On the axes in the Printed Answer Booklet, sketch the envelope of the normal to the curve. **[1]**

(g) By considering the expression $(ax(t))^{\frac{2}{3}}+(by(t))^{\frac{2}{3}}$ or otherwise, determine a cartesian equation of the envelope of the normal to the curve. **[2]**

© OCR 2025

2 **(a)** **(i)** Write down $7^{10} \pmod{1000}$. **[1]**

Fermat's little theorem states that if p is a prime and x is an integer which is co-prime to p, then $x^{p-1} \equiv 1 \pmod{p}$.

(ii) Explain why Fermat's little theorem implies $2^{12} \equiv 1 \pmod{13}$. **[1]**

(iii) Determine $2^{(12q+1)} \pmod{13}$, where q is a positive integer. **[2]**

(b) In the rest of this question the highest common factor of positive integers m and n is denoted by (m, n).

(i) Write down (354, 27). **[1]**

Euler's totient function $\varphi(n)$, where n is a positive integer, is defined to be the number of integers m with $1 \leqslant m \leqslant n$ such that $(m, n) = 1$.

For example, $\varphi(12) = 4$ since 1, 5, 7 and 11 are all co-prime with 12, but 2, 3, 4, 6, 8, 9, 10, and 12 all share a common factor greater than 1 with 12.

(ii) Create a program which returns the value of $\varphi(n)$ for a given positive integer n. Write out your program in full in the Printed Answer Booklet. **[4]**

(iii) Use your program to find $\varphi(1000)$. **[1]**

Euler's theorem states that if a and n are co-prime positive integers, then $a^{\varphi(n)} \equiv 1 \pmod{n}$.

(iv) Determine $7^{(400r+10)} \pmod{1000}$, where r is a positive integer. **[4]**

(v) Using part **(b)(iv)**, determine the tens digit of 7^{2010}. **[2]**

(c) Suppose that $p \geqslant 3$ is a prime number and that x and y are positive integers.

By considering the equation $p^{2x} + 1 = 2^{2y}$ modulo 4, or otherwise, prove there are **no** integer solutions to the equation $p^{2x} + 1 = 2^{2y}$. **[5]**

3 This question concerns the family of differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x - y + a\mathrm{e}^{x}\sin(y) \qquad (**)$$

where a is a constant.

(a) In this part of the question $a = 0$.

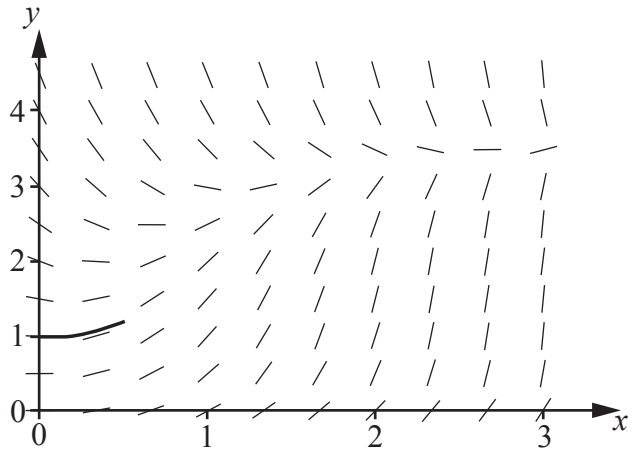
(i) Verify that $y = 2\mathrm{e}^{-x} + x - 1$ is the particular solution of (**) that satisfies $y = 1$ when $x = 0$. **[3]**

The solution $y = 2\mathrm{e}^{-x} + x - 1$ has a minimum value at the point (m, n) where $0 < m < 1$.

(ii) Find the exact value of m. **[2]**

(iii) Sketch the particular solution of (**) given in part **(a)(i)** for $0 \leqslant x \leqslant 3$ on the axes in the Printed Answer Booklet. **[2]**

(b) The figure below shows the tangent field for an unspecified value of a. A sketch of the solution curve $y = \mathrm{g}(x)$ which passes through the point (0, 1) is shown for $0 \leqslant x \leqslant \frac{1}{2}$.



Continue the sketch of the solution curve for $\frac{1}{2} \leqslant x \leqslant 3$ on the axes in the **Printed Answer Booklet**. **[2]**

(c) (i) The standard Runge-Kutta method of order 4 for the solution of the differential equation $\frac{dy}{dx} = f(x, y)$ is as follows.

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Construct a spreadsheet to solve (**) so that the value of a and the value of h can be varied in the case $x_0 = 0$ and $y_0 = 4$. State the formulae you have used in your spreadsheet. **[4]**

(ii) In this part of the question $a = 1$, $x_0 = 0$ and $y_0 = 4$.

Use your spreadsheet with $h = 0.05$ to determine an approximation of the value of y for the solution to (**) when $x = 1$, giving your answer correct to **2** decimal places. **[1]**

(iii) In this part of the question $a = 0.5$, $x_0 = 0$ and $y_0 = 4$.

The solution to (**) has a minimum point (r, s) with $1 < r < 2$.

Use your spreadsheet with suitable values of h to determine the value of r correct to **2** decimal places. **[2]**

(iv) In this part of the question $h = 0.05$, $x_0 = 0$ and $y_0 = 4$.

There is a value of $a > 0$ such that $y(1)$ is 2.47 to 2 decimal places.

By varying the value of a in your spreadsheet, find a suitable value of a correct to **1** decimal place. **[2]**

END OF QUESTION PAPER

© OCR 2025

BLANK PAGE

© OCR 2025

BLANK PAGE

© OCR 2025

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2025

OCR
Oxford Cambridge and RSA

Monday 23 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y436/01 Further Pure with Technology

Printed Answer Booklet

Time allowed: 1 hour 45 minutes

AB

You must have:
- Question Paper Y436/01 (inside this document)
- the Formulae Booklet for Further Mathematics B (MEI)
- a computer with appropriate software
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.

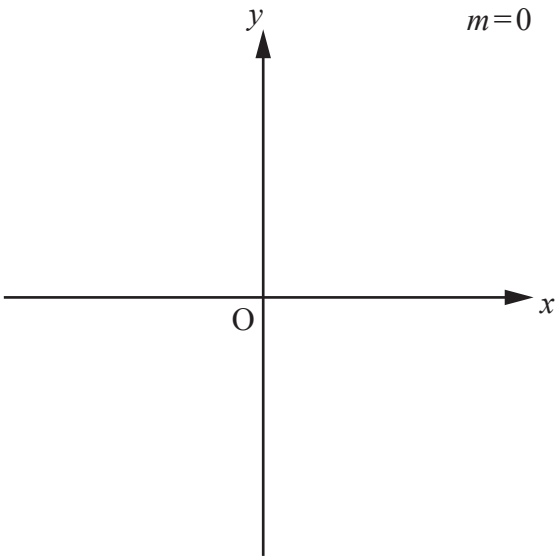
INFORMATION
- This document has **16** pages.

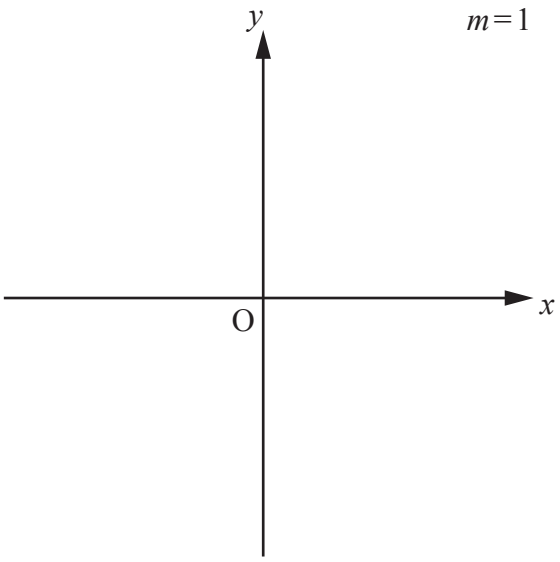
ADVICE
- Read each question carefully before you start your answer.

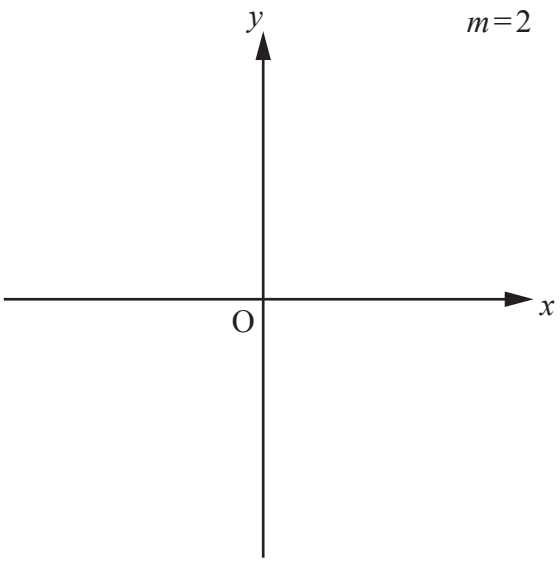
© OCR 2025 [K/508/5600]

OCR is an exempt Charity

1(a)(i)







© OCR 2025

1(a)(ii)	
1(b)	
1(c)	

© OCR 2025

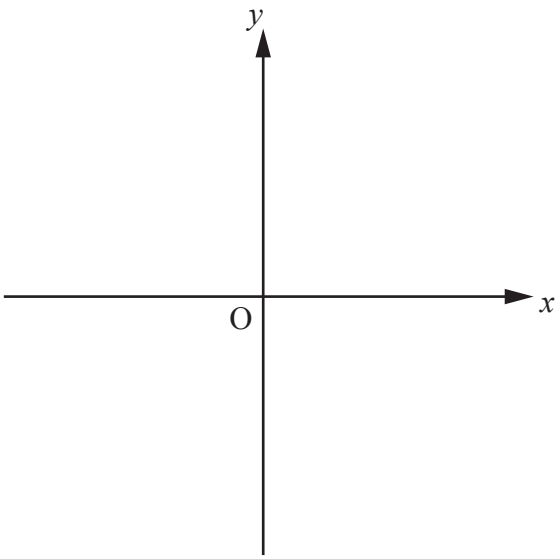
1(d)

© OCR 2025

1(e)

© OCR 2025

1(f)



1(g)

© OCR 2025

2(a)(i)	
2(a)(ii)	
2(a)(iii)	
2(b)(i)	

DO NOT WRITE IN THIS SPACE

© OCR 2025

2(b)(ii)	

2(b)(iv)

2(b)(v)

© OCR 2025

2(c)

© OCR 2025

3(a)(i)

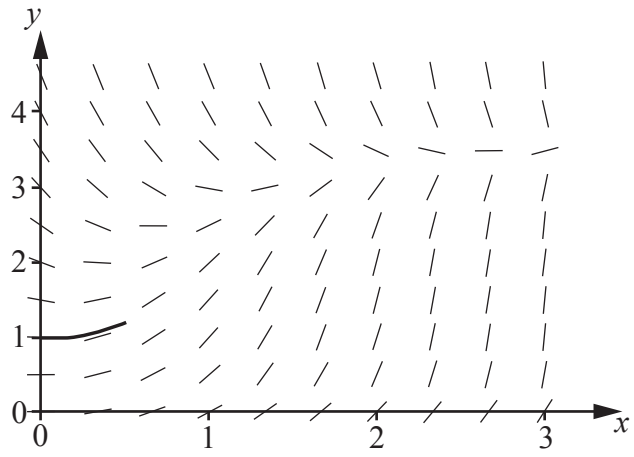
3(a)(ii)

© OCR 2025

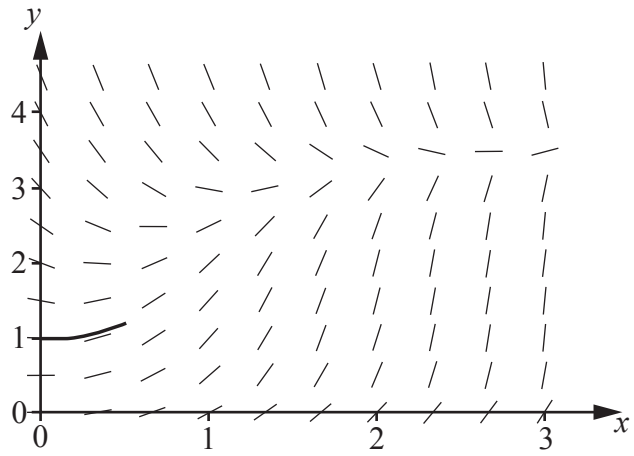
3(a)(iii)



3(b)



A spare copy of the tangent field is provided below.



© OCR 2025

3(c)(i)

3(c)(ii)

© OCR 2025

3(c)(iii)

3(c)(iv)

© OCR 2025

EXTRA ANSWER SPACE

If you need extra space use these lined pages. You must write the question numbers clearly in the margin.

© OCR 2025

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2025