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Monday 23 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y436/01 Further Pure with Technology

Time allowed: 1 hour 45 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a computer with appropriate software
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 A family of curves is given by the cartesian equation

$$\frac{x^2}{a^2}+mxy+\frac{y^2}{b^2}=1$$

where a, b and m are real numbers with a and b non-zero.

(a) In this part of the question $a=2$ and $b=1$.

(i) On the axes in the Printed Answer Booklet, sketch the curve in each of these cases.

- $m=0$
- $m=1$
- $m=2$

[3]

(ii) State **one** feature of the curve for the case $m=0$ that is **not** a feature of the curve in the cases $m=1$ and $m=2$. **[1]**

For the remainder of this question $m=0$.

(b) Verify that the parametric equations of the curve are

$$x(t)=a\cos(t),\ y(t)=b\sin(t),$$

where $0\leqslant t<2\pi$ is a parameter. **[1]**

(c) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{b}{a}\cot(t)$. **[2]**

(d) Show that the equation of the normal to the curve at the point with parameter t is

$$y=\frac{a}{b}\tan(t)x-\left(\frac{a^2-b^2}{b}\right)\sin(t).$$ **[5]**

(e) Show that the parametric equations of the envelope of the normal to the curve are

$$x(t)=\left(\frac{a^2-b^2}{a}\right)\cos^3(t),\ y(t)=\left(\frac{b^2-a^2}{b}\right)\sin^3(t),$$

where $0\leqslant t<2\pi$ is a parameter. **[6]**

(f) In this part of the question $a=2$ and $b=1$.

On the axes in the Printed Answer Booklet, sketch the envelope of the normal to the curve. **[1]**

(g) By considering the expression $(ax(t))^{\frac{2}{3}}+(by(t))^{\frac{2}{3}}$ or otherwise, determine a cartesian equation of the envelope of the normal to the curve. **[2]**

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2 **(a)** **(i)** Write down $7^{10} \pmod{1000}$. **[1]**

Fermat's little theorem states that if p is a prime and x is an integer which is co-prime to p, then $x^{p-1} \equiv 1 \pmod{p}$.

(ii) Explain why Fermat's little theorem implies $2^{12} \equiv 1 \pmod{13}$. **[1]**

(iii) Determine $2^{(12q+1)} \pmod{13}$, where q is a positive integer. **[2]**

(b) In the rest of this question the highest common factor of positive integers m and n is denoted by (m, n).

(i) Write down (354, 27). **[1]**

Euler's totient function $\varphi(n)$, where n is a positive integer, is defined to be the number of integers m with $1 \leqslant m \leqslant n$ such that $(m, n) = 1$.

For example, $\varphi(12) = 4$ since 1, 5, 7 and 11 are all co-prime with 12, but 2, 3, 4, 6, 8, 9, 10, and 12 all share a common factor greater than 1 with 12.

(ii) Create a program which returns the value of $\varphi(n)$ for a given positive integer n. Write out your program in full in the Printed Answer Booklet. **[4]**

(iii) Use your program to find $\varphi(1000)$. **[1]**

Euler's theorem states that if a and n are co-prime positive integers, then $a^{\varphi(n)} \equiv 1 \pmod{n}$.

(iv) Determine $7^{(400r+10)} \pmod{1000}$, where r is a positive integer. **[4]**

(v) Using part **(b)(iv)**, determine the tens digit of 7^{2010}. **[2]**

(c) Suppose that $p \geqslant 3$ is a prime number and that x and y are positive integers.

By considering the equation $p^{2x} + 1 = 2^{2y}$ modulo 4, or otherwise, prove there are **no** integer solutions to the equation $p^{2x} + 1 = 2^{2y}$. **[5]**

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3 This question concerns the family of differential equations

$$\frac{dy}{dx} = x - y + ae^{x}\sin(y) \qquad (**)$$

where a is a constant.

(a) In this part of the question $a = 0$.

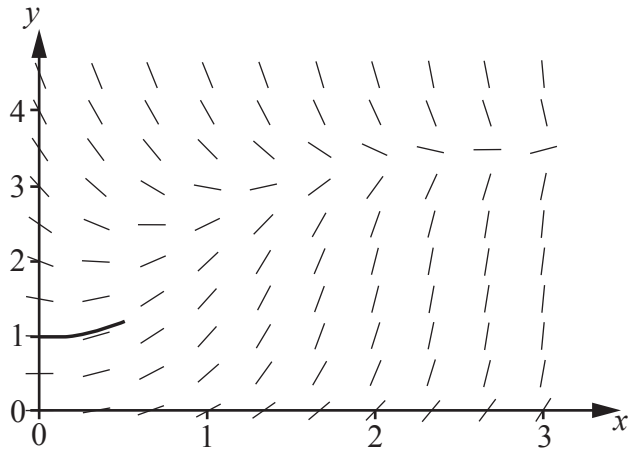
(i) Verify that $y = 2e^{-x} + x - 1$ is the particular solution of (**) that satisfies $y = 1$ when $x = 0$. **[3]**

The solution $y = 2e^{-x} + x - 1$ has a minimum value at the point (m, n) where $0 < m < 1$.

(ii) Find the exact value of m. **[2]**

(iii) Sketch the particular solution of (**) given in part **(a)(i)** for $0 \leqslant x \leqslant 3$ on the axes in the Printed Answer Booklet. **[2]**

(b) The figure below shows the tangent field for an unspecified value of a. A sketch of the solution curve $y = g(x)$ which passes through the point (0, 1) is shown for $0 \leqslant x \leqslant \frac{1}{2}$.



Continue the sketch of the solution curve for $\frac{1}{2} \leqslant x \leqslant 3$ on the axes in the **Printed Answer Booklet**. **[2]**

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(c) (i) The standard Runge-Kutta method of order 4 for the solution of the differential equation $\frac{dy}{dx} = f(x, y)$ is as follows.

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Construct a spreadsheet to solve (**) so that the value of a and the value of h can be varied in the case $x_0 = 0$ and $y_0 = 4$. State the formulae you have used in your spreadsheet. **[4]**

(ii) In this part of the question $a = 1$, $x_0 = 0$ and $y_0 = 4$.

Use your spreadsheet with $h = 0.05$ to determine an approximation of the value of y for the solution to (**) when $x = 1$, giving your answer correct to **2** decimal places. **[1]**

(iii) In this part of the question $a = 0.5$, $x_0 = 0$ and $y_0 = 4$.

The solution to (**) has a minimum point (r, s) with $1 < r < 2$.

Use your spreadsheet with suitable values of h to determine the value of r correct to **2** decimal places. **[2]**

(iv) In this part of the question $h = 0.05$, $x_0 = 0$ and $y_0 = 4$.

There is a value of $a > 0$ such that $y(1)$ is 2.47 to 2 decimal places.

By varying the value of a in your spreadsheet, find a suitable value of a correct to **1** decimal place. **[2]**

END OF QUESTION PAPER

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Centre number | | | | | | Candidate number | | | | |

First name(s) StemUp Model Solution

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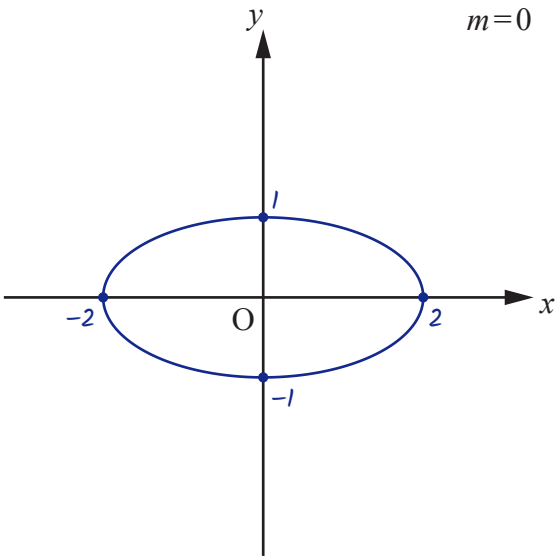
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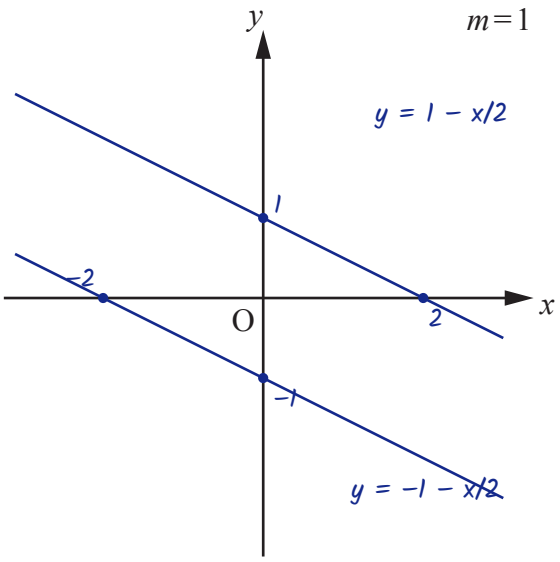
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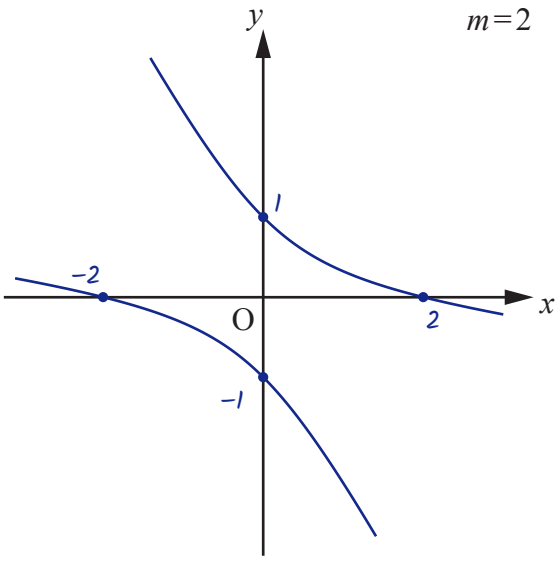
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1(a)(i)







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1(a)(ii)

When $m = 0$ the curve is bounded (it is a closed curve, an ellipse).

For $m = 1$ and $m = 2$ the curve is unbounded (it goes off to infinity).

1(b)

With $m = 0$ the curve is $x^2/a^2 + y^2/b^2 = 1$.

Substitute $x = a\cos(t)$, $y = b\sin(t)$:

$x^2/a^2 + y^2/b^2 = (a\cos t)^2/a^2 + (b\sin t)^2/b^2$

$= a^2\cos^2 t \,/\, a^2 + b^2\sin^2 t \,/\, b^2$

$= \cos^2 t + \sin^2 t$

$= 1$ for all t, so these are parametric equations of the curve.

1(c)

$x = a\cos t \Rightarrow dx/dt = -a\sin t$

$y = b\sin t \Rightarrow dy/dt = b\cos t$

$dy/dx = (dy/dt) \div (dx/dt)$

$= (b\cos t) \,/\, (-a\sin t)$

$= -(b/a) \times (\cos t \,/\, \sin t)$

$= -(b/a)\cot(t)$ as required

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1(d)	Gradient of tangent at t: $dy/dx = -(b/a) \cot t$
	Gradient of normal $= -1 \div (dy/dx) = a / (b \cot t)$
	$= (a/b) \tan t$
	Normal passes through the point $(a \cos t, b \sin t)$:
	$y - b \sin t = (a/b) \tan t (x - a \cos t)$
	$y = (a/b) \tan t\, x - (a^2/b) \tan t \cos t + b \sin t$
	Since $\tan t \cos t = \sin t$:
	$y = (a/b) \tan t\, x - (a^2/b) \sin t + (b^2/b) \sin t$
	$y = (a/b) \tan(t)\, x - ((a^2 - b^2)/b) \sin(t)$ as required

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1(e)

Write the family of normals as $f(x, y, t) = 0$, where

$f(x, y, t) = y - (a/b)\tan(t)\,x + ((a^2 - b^2)/b)\sin(t)$

The envelope satisfies $f = 0$ and $\partial f/\partial t = 0$.

$\partial f/\partial t = -(a/b)\sec^2(t)\,x + ((a^2 - b^2)/b)\cos(t)$

$\partial f/\partial t = 0 \Rightarrow (a/b)\sec^2(t)\,x = ((a^2 - b^2)/b)\cos(t)$

$x = (a^2 - b^2)\cos(t) / (a\sec^2(t))$

$x(t) = ((a^2 - b^2)/a)\cos^3(t)$ as required

Substitute into $f = 0$:

$y = (a/b)\tan(t) \times ((a^2 - b^2)/a)\cos^3(t) - ((a^2 - b^2)/b)\sin(t)$

$= ((a^2 - b^2)/b)\sin(t)\cos^2(t) - ((a^2 - b^2)/b)\sin(t)$

$= ((a^2 - b^2)/b)\sin(t)(\cos^2(t) - 1)$

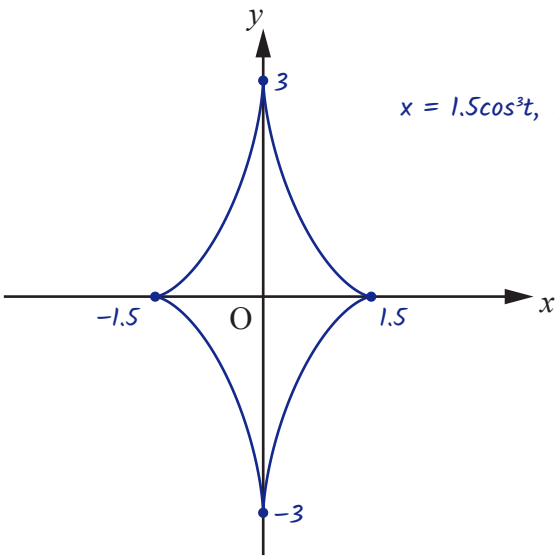
$= ((a^2 - b^2)/b)\sin(t)(-\sin^2(t))$

$y(t) = ((b^2 - a^2)/b)\sin^3(t)$ as required

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1(f)



1(g)

$(ax)^{2/3} + (by)^{2/3}$

$= ((a^2 - b^2)\cos^3 t)^{2/3} + ((b^2 - a^2)\sin^3 t)^{2/3}$

$= (a^2 - b^2)^{2/3}\cos^2 t + (b^2 - a^2)^{2/3}\sin^2 t$

But $(b^2 - a^2)^{2/3} = ((-1)(a^2 - b^2))^{2/3} = (a^2 - b^2)^{2/3}$

$= (a^2 - b^2)^{2/3}(\cos^2 t + \sin^2 t)$

$= (a^2 - b^2)^{2/3}$

So a cartesian equation of the envelope is

$(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

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2(a)(i)	$7^{10} = 282475249$, so $7^{10} \equiv 249 \pmod{1000}$
2(a)(ii)	13 is prime, and 2 is co-prime to 13 (hcf(2, 13) = 1).
	So Fermat with $p = 13$, $x = 2$ gives $2^{13-1} = 2^{12} \equiv 1 \pmod{13}$.
2(a)(iii)	$2^{12q+1} = (2^{12})^q \times 2$
	$\equiv 1^q \times 2 \pmod{13}$ using $2^{12} \equiv 1 \pmod{13}$
	$\equiv 1 \times 2 \pmod{13}$
	$2^{12q+1} \equiv 2 \pmod{13}$
2(b)(i)	$354 = 2 \times 3 \times 59$, $27 = 3^3$, so $(354, 27) = 3$

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2(b)(ii)

```
Program (Python):
import math
def phi(n):
    t = 0
    for m in range(1, n + 1):
        if math.gcd(m, n) == 1:
            t = t + 1
    return t

print(phi(1000))
```

(range(1, n + 1) runs m = 1, 2, ..., n; math.gcd(m, n) is the highest common factor (m, n); t counts the m with (m, n) = 1.)

Check: phi(12) returns 4, as in the question.

2(b)(iii)

$\varphi(1000) = 400$

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2(b)(iv)	$1000 = 2^3 \times 5^3$ and 7 is prime, so $(7, 1000) = 1$:
	7 and 1000 are co-prime.
	By Euler's theorem, with $\varphi(1000) = 400$:
	$7^{400} \equiv 1 \pmod{1000}$
	$7^{400r + 10} = (7^{400})^r \times 7^{10}$
	$\equiv 1^r \times 7^{10} \pmod{1000}$
	$\equiv 7^{10} \pmod{1000}$
	$\equiv 249 \pmod{1000}$ from part (a)(i)
2(b)(v)	$2010 = 400 \times 5 + 10$, so take $r = 5$ in part (b)(iv):
	$7^{2010} \equiv 249 \pmod{1000}$
	So the last three digits of 7^{2010} are 249.
	The tens digit of 7^{2010} is 4.

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2(c)

$p \geqslant 3$ is prime, so p is odd: $p \equiv 1$ or $p \equiv 3 \pmod 4$.

$1^2 = 1 \equiv 1 \pmod 4$ and $3^2 = 9 \equiv 1 \pmod 4$

So $p^2 \equiv 1 \pmod 4$.

Left hand side: $p^{2x} = (p^2)^x \equiv 1^x = 1 \pmod 4$

so $p^{2x} + 1 \equiv 2 \pmod 4$.

Right hand side: $y \geqslant 1$, so $2^{2y} = 4^y \equiv 0 \pmod 4$.

If $p^{2x} + 1 = 2^{2y}$ then the two sides would be congruent

mod 4, giving $2 \equiv 0 \pmod 4$, which is false.

So the two sides can never be equal for positive integers x, y.

Hence there are no integer solutions to $p^{2x} + 1 = 2^{2y}$.

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3(a)(i)

$a = 0$, so $(**)$ is $dy/dx = x - y$.

$y = 2e^{-x} + x - 1 \quad \Rightarrow \quad dy/dx = -2e^{-x} + 1$

$x - y = x - (2e^{-x} + x - 1)$

$= 1 - 2e^{-x}$

$= dy/dx$, so y satisfies $(**)$.

When $x = 0$: $y = 2e^0 + 0 - 1 = 2 - 1 = 1$

So $y = 2e^{-x} + x - 1$ is the particular solution with $y = 1$ when $x = 0$.

3(a)(ii)

At the minimum, $dy/dx = 0$:

$1 - 2e^{-x} = 0$

$e^{-x} = 1/2 \quad \Rightarrow \quad e^x = 2$

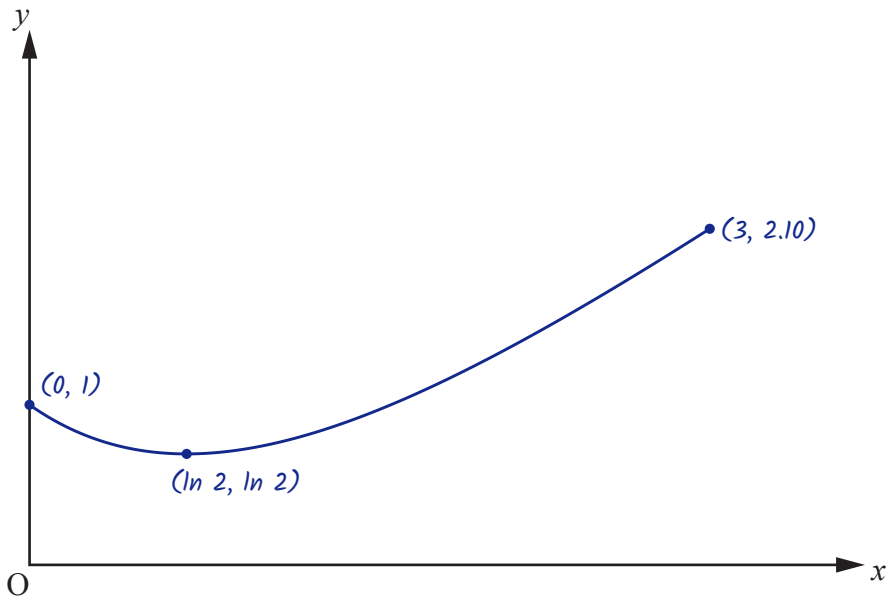
$m = \ln 2$

($d^2y/dx^2 = 2e^{-x} > 0$, so this is a minimum; $n = 2 \times \frac{1}{2} + \ln 2 - 1 = \ln 2$)

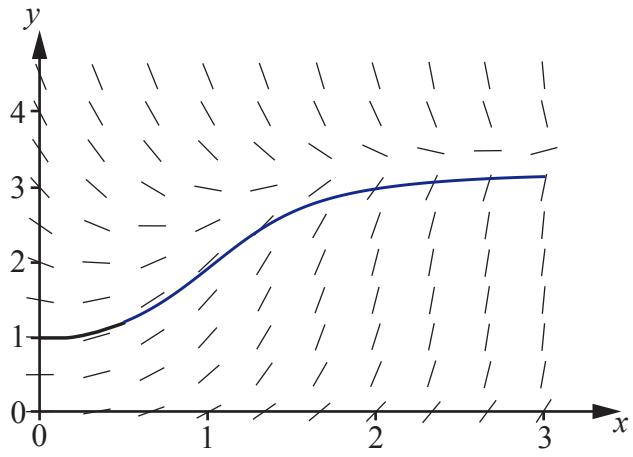
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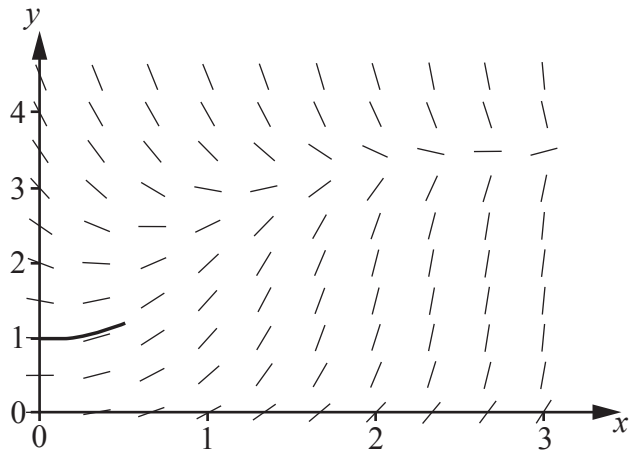
3(a)(iii)



3(b)



A spare copy of the tangent field is provided below.



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3(c)(i)

Spreadsheet layout (h in cell B1, a in cell D1 so both can be varied):

A1: h B1: 0.05 C1: a D1: 1

Row 3 headings: A: x_n B: y_n C: k_1 D: k_2 E: k_3 F: k_4

A4: 0 (x_0) B4: 4 (y_0)

C4 (k_1): =$B$1*(A4−B4+$D$1*EXP(A4)*SIN(B4))

D4 (k_2): =$B$1*((A4+$B$1/2)−(B4+C4/2)
+$D$1*EXP(A4+$B$1/2)*SIN(B4+C4/2))

E4 (k_3): =$B$1*((A4+$B$1/2)−(B4+D4/2)
+$D$1*EXP(A4+$B$1/2)*SIN(B4+D4/2))

F4 (k_4): =$B$1*((A4+$B$1)−(B4+E4)
+$D$1*EXP(A4+$B$1)*SIN(B4+E4))

A5 (x_{n+1}): =A4+$B$1

B5 (y_{n+1}): =B4+(C4+2*D4+2*E4+F4)/6

Fill C4:F4 down, and fill A5:B5 down, for as many rows as needed.

(Here $k_1 = hf(x_n, y_n)$ with $f(x, y) = x - y + ae^x \sin(y)$.)

3(c)(ii)

With a = 1, h = 0.05: at x = 1 (row 24) y = 2.43112..., so y(1) ≈ 2.43

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3(c)(iii)

$a = 0.5$. With $h = 0.005$, values of y near the minimum:

x	y
1.020	2.1790556
1.025	2.1789769
1.030	2.1789524
1.035	2.1789815
1.040	2.1790640

y decreases up to $x = 1.030$ and increases after it, so

$1.025 < r < 1.035$ ($h = 0.01$ gives the same smallest y at $x = 1.03$).

$r = 1.03$ (2 d.p.)

3(c)(iv)

$h = 0.05$, varying a in cell D1 and reading y at $x = 1$:

a	$y(1)$	$y(1)$ to 2 d.p.
1.00	2.43112	2.43
1.05	2.45160	2.45
1.10	2.47131	2.47
1.15	2.49028	2.49
1.20	2.50854	2.51

$y(1) = 2.47$ to 2 d.p. when $a = 1.10$, so $a = 1.1$ (1 d.p.)

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EXTRA ANSWER SPACE

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