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Friday 20 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 A binary operation, $\circ$, is defined on real numbers a and b by $a \circ b = 3ab$.

(a) Prove that $\circ$ is associative over the real numbers $\mathbb{R}$. **[2]**

(b) Determine the value of e, the identity element for $\circ$. **[2]**

(c) Explain why $\mathbb{R}$ is closed under $\circ$. **[1]**

(d) Explain why $\mathbb{R}$ under $\circ$ does **not** form a group. Justify your answer. **[2]**

2 **(a)** Determine the general solution of the recurrence relation $25u_{n+2} - 35u_{n+1} + 12u_n = c, n \geqslant 0$, where c is a constant. **[5]**

The manager of a company is using the recurrence relation in part **(a)** to model the number of containers held by the company using the sequence $u_0, u_1, u_2, \ldots$ which satisfies this recurrence relation.

The quantity u_0 represents the number of containers held by the company at the **start** of January 2000. The quantity u_n, for $n \geqslant 1$, represents the number of containers held by the company at the **end** of month n, where the first month is January 2000. If necessary, the quantity u_n is rounded to the nearest integer.

The company holds 20 containers at the start of January 2000 and 70 containers at the end of January 2000.

(b) In an initial model the manager uses $c = -10$.

(i) Determine the particular solution for u_n when $c = -10$. **[3]**

(ii) The model predicts a single peak value for u_n.

Find, by direct calculation, the largest number of containers held by the company according to the model. **[2]**

(iii) Show that the model in part **(b)(i)** breaks down in the 19th month. **[3]**

(c) In a second model the manager uses $c = 10$ instead of $c = -10$.

Show that the second model predicts that eventually the number of containers held by the company will be a constant k, where k is to be determined. **[2]**

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3 M is the group $(\{0, 1, 2, 3, 4, 5, 6, 7, 8\}, +_9)$ where $+_9$ denotes the binary operation of addition modulo 9.

(a) Write down the identity element of M. Justify your answer. **[1]**

(b) By considering the order of M, explain why there can be **no** subgroup of M of order 5. **[2]**

You are given that M is a cyclic group.

(c) Find **all** possible generators of M. **[2]**

You are given that there is only one proper, non-trivial subgroup of M, denoted by $(H, +_9)$.

(d) (i) Write down H. **[1]**

(ii) Show that there can be **no** other proper, non-trivial subgroups of M. **[2]**

4 A surface, S, is defined in 3-D by $z = \mathrm{f}(x, y)$ where $\mathrm{f}(x, y) = x^3 - 12xy^2 + 96y^2 + 30$.

You are given that the point (0, 0, 30) on S is a stationary point.

(a) Determine the coordinates of any other stationary points on S. **[6]**

(b) By sketching the section $z = \mathrm{f}(x, 0)$ on the coordinate axes provided in the Printed Answer Booklet, determine the nature of the stationary point (0, 0, 30) on S. **[2]**

The normal to S at the point where $x = 3$ and $y = 2$ passes through the point $(a, 482, 295)$ where a is a constant.

(c) Determine the value of a. **[5]**

The contour given by $z = 542$ comprises a straight line and a curve. The curved portion of the contour is denoted by C. You are given that there are **two** points on C where the x coordinate is 8.

(d) Determine the values of the y coordinates at these **two** points on C. **[3]**

Turn over for question 5

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5 In this question you must show detailed reasoning.

The matrix **A** is given by $\begin{pmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 1 \end{pmatrix}$ where a and b are constants with $a^2 + b^2 = 1$ and $a \neq \pm 1$.

(a) (i) By finding the characteristic equation of **A** in terms of a, show that, for any value of a, 1 is always one of the eigenvalues of **A**. **[3]**

(ii) By showing that the other eigenvalues of **A** are $a \pm \sqrt{a^2 - 1}$, prove that the other eigenvalues of **A** are **not** 1. **[3]**

(b) Hence show that **either** some of the eigenvalues of **A** must be non-real **or** some of the elements of **A** must be non-real. **[3]**

You are now given the following information.

- a and b are both real.
- **A** represents a rotation by an acute angle in 3-D space.
- **e** is a real eigenvector of **A** associated with the eigenvalue 1.

(c) (i) Explain the significance of **e**, and the fact that its associated eigenvalue is 1, in relation to the transformation that **A** represents. **[2]**

(ii) Explain **geometrically** why the other eigenvectors of **A** can **not** be real. **[1]**

(iii) Find a vector equation for the axis of rotation of the transformation that **A** represents. **[2]**

END OF QUESTION PAPER

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Y435/01 Extra Pure

Printed Answer Booklet

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AB

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Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

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1(a)

$(a \bigcirc b) \bigcirc c = (3ab) \bigcirc c = 3(3ab)c = 9abc$

$a \bigcirc (b \bigcirc c) = a \bigcirc (3bc) = 3a(3bc) = 9abc$

So $(a \bigcirc b) \bigcirc c = a \bigcirc (b \bigcirc c)$ for all real a, b, c

$\therefore \bigcirc$ is associative over $\mathbb{R}$.

1(b)

Need $a \bigcirc e = a$ for all $a \in \mathbb{R}$:

$3ae = a \Rightarrow 3e = 1$

$e = 1/3$ (and $e \bigcirc a = 3(1/3)a = a$ also)

1(c)

If a and b are real numbers then $3ab$ is a product of real numbers, so $a \bigcirc b = 3ab$ is also a real number.

1(d)

For $0 \in \mathbb{R}$: $0 \bigcirc b = 3 \times 0 \times b = 0$ for every real b, so $0 \bigcirc b$ can never equal the identity $e = 1/3$.

So 0 has no inverse under $\bigcirc$.

The inverse axiom fails (not every element of $\mathbb{R}$ has an inverse), so $\mathbb{R}$ under $\bigcirc$ does not form a group.

2(a)

Auxiliary equation: $25r^2 - 35r + 12 = 0$

$(5r - 3)(5r - 4) = 0 \Rightarrow r = 3/5$ or $r = 4/5$

Complementary function: $u_n = A(3/5)^n + B(4/5)^n$

Particular solution: try $u_n = \lambda$ (constant)

$25\lambda - 35\lambda + 12\lambda = c \Rightarrow 2\lambda = c \Rightarrow \lambda = c/2$

General solution:

$u_n = A(3/5)^n + B(4/5)^n + c/2$

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2(a) **(continued)**

2(b)(i)

$c = -10$: $u_n = A(3/5)^n + B(4/5)^n - 5$

$u_0 = 20$: $A + B - 5 = 20 \Rightarrow A + B = 25$

$u_1 = 70$: $(3/5)A + (4/5)B - 5 = 70 \Rightarrow 3A + 4B = 375$

$3A + 4B - 3(A + B) = 375 - 75 \Rightarrow B = 300$

$A = 25 - 300 = -275$

$u_n = 300(4/5)^n - 275(3/5)^n - 5$

2(b)(ii)

$u_2 = 300(0.64) - 275(0.36) - 5 = 192 - 99 - 5 = 88$

$u_3 = 300(0.512) - 275(0.216) - 5 = 89.2$

$u_4 = 300(0.4096) - 275(0.1296) - 5 = 82.24$ (decreasing)

Largest number of containers = 89 (end of month 3)

2(b)(iii)

$u_{18} = 300(0.8)^{18} - 275(0.6)^{18} - 5 = 0.376... \approx 0$

$u_{19} = 300(0.8)^{19} - 275(0.6)^{19} - 5 = -0.693... \approx -1$

So the model predicts a negative number of containers at the end of the 19th month, which is impossible, so the model breaks down in the 19th month.

2(c)

$c = 10$: $u_n = A(3/5)^n + B(4/5)^n + 5$ $(A = -265, B = 280)$

As $n \to \infty$, $(3/5)^n \to 0$ and $(4/5)^n \to 0$

so $u_n \to 5$. Eventually the number of containers is constant: $k = 5$

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3(a)	Identity is 0, since $0 +_9 a = a +_9 0 = a$ for all $a \in M$.
3(b)	The order of M is 9.
	By Lagrange's theorem, the order of any subgroup of M must
	be a factor of the order of M, i.e. 1, 3 or 9.
	5 is not a factor of 9, so M has no subgroup of order 5.
3(c)	A generator must have order 9.
	Generators: 1, 2, 4, 5, 7, 8
	(3 and 6 have order 3, 0 has order 1)
3(d)(i)	$H = \{0, 3, 6\}$
3(d)(ii)	By Lagrange's theorem a subgroup of M has order 1, 3 or 9.
	Proper means order $\neq 9$, non-trivial means order $\neq 1$,
	so any proper, non-trivial subgroup has order 3.
	The order of each element must divide the order of the
	subgroup, so every element in it has order 1 or 3.
	But 1, 2, 4, 5, 7, 8 all have order 9, so they cannot be in it.
	So the subgroup can only contain 0, 3 and 6, and it has
	3 elements, so it must be $\{0, 3, 6\} = H$.
	$\therefore$ there are no other proper, non-trivial subgroups of M.

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4(a)

$\partial z/\partial x = 3x^2 - 12y^2$

$\partial z/\partial y = -24xy + 192y = -24y(x - 8)$

Stationary points where $\partial z/\partial x = 0$ and $\partial z/\partial y = 0$

$\partial z/\partial y = 0 \Rightarrow y = 0$ or $x = 8$

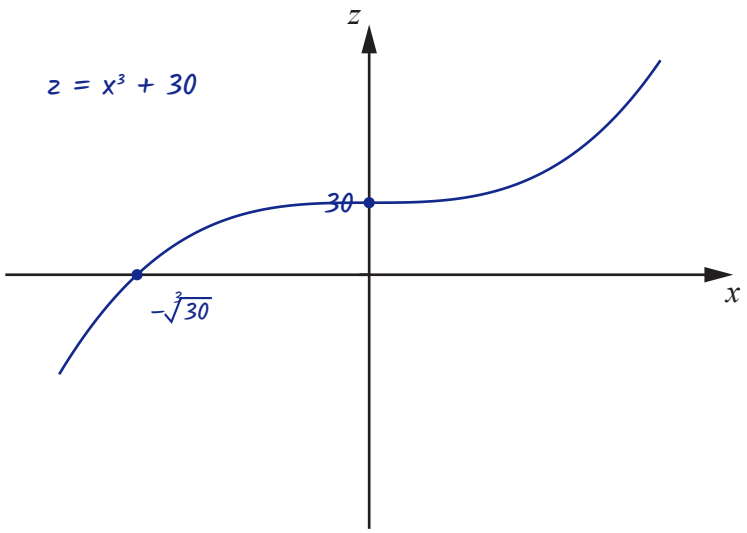
$y = 0$: $3x^2 = 0 \Rightarrow x = 0$, giving $(0, 0, 30)$ (already known)

$x = 8$: $3(64) - 12y^2 = 0 \Rightarrow 12y^2 = 192 \Rightarrow y^2 = 16$, $y = \pm 4$

$z = 8^3 - 12(8)(16) + 96(16) + 30 = 512 - 1536 + 1536 + 30 = 542$

Other stationary points: $(8, 4, 542)$ and $(8, -4, 542)$

4(b)



Section $z = x^3 + 30$ has a stationary point of inflection at $x = 0$:

z decreases one side of $(0, 0, 30)$ and increases the other side,

so it is neither a max nor a min: $(0, 0, 30)$ is a saddle point.

4(c)

At $x = 3$, $y = 2$:

$z = 27 - 12(3)(4) + 96(4) + 30 = 27 - 144 + 384 + 30 = 297$

$\partial z/\partial x = 3(9) - 12(4) = 27 - 48 = -21$

$\partial z/\partial y = -24(3)(2) + 192(2) = -144 + 384 = 240$

Normal direction: $n = (\partial z/\partial x, \partial z/\partial y, -1) = (-21, 240, -1)$

Normal: $r = (3, 2, 297) + \lambda(-21, 240, -1)$

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4(c) **(continued)**

Passes through (a, 482, 295):

y: $2 + 240\lambda = 482 \Rightarrow \lambda = 2$

z: $297 - \lambda = 297 - 2 = 295$ ✓ (consistent)

x: $a = 3 - 21\lambda = 3 - 42$

$a = -39$

4(d)

$z = 542$: $x^3 - 12xy^2 + 96y^2 + 30 = 542$

$96y^2 - 12xy^2 = 512 - x^3$

$12y^2(8 - x) = 512 - x^3$

$512 - x^3 = 8^3 - x^3 = (8 - x)(64 + 8x + x^2)$

So $12y^2(8 - x) = (8 - x)(64 + 8x + x^2)$

$(8 - x)(12y^2 - x^2 - 8x - 64) = 0$

The straight line is $x = 8$, and the curve C is

$12y^2 = x^2 + 8x + 64$

On C with $x = 8$: $12y^2 = 64 + 64 + 64 = 192$

$y^2 = 16$

$y = 4$ or $y = -4$

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5(a)(i)

$A - \lambda I$ has rows $(a - \lambda, b, 0)$, $(-b, a - \lambda, 0)$, $(0, 0, 1 - \lambda)$

Expanding $\det(A - \lambda I)$ along the third row:

$\det(A - \lambda I) = (1 - \lambda)[(a - \lambda)^2 - (b)(-b)]$

$= (1 - \lambda)[(a - \lambda)^2 + b^2]$

$= (1 - \lambda)(\lambda^2 - 2a\lambda + a^2 + b^2)$

$= (1 - \lambda)(\lambda^2 - 2a\lambda + 1)$ since $a^2 + b^2 = 1$

Characteristic equation: $(1 - \lambda)(\lambda^2 - 2a\lambda + 1) = 0$

i.e. $\lambda^3 - (1 + 2a)\lambda^2 + (1 + 2a)\lambda - 1 = 0$

Substituting $\lambda = 1$:

$1 - (1 + 2a) + (1 + 2a) - 1 = 0$ for every value of a

(equivalently the factor $(1 - \lambda)$ gives $\lambda = 1$),

so 1 is always an eigenvalue of A.

5(a)(ii)

Other eigenvalues satisfy $\lambda^2 - 2a\lambda + 1 = 0$

$\lambda = [2a \pm \sqrt{(2a)^2 - 4(1)(1)}] / 2 = [2a \pm \sqrt{4a^2 - 4}] / 2$

$\lambda = [2a \pm 2\sqrt{a^2 - 1}] / 2$

$\lambda = a \pm \sqrt{a^2 - 1}$ as required

Suppose one of these equals 1: $a \pm \sqrt{a^2 - 1} = 1$

$\pm \sqrt{a^2 - 1} = 1 - a$

$a^2 - 1 = (1 - a)^2 = 1 - 2a + a^2$

$2a = 2 \Rightarrow a = 1$

But $a \neq \pm 1$ (given), so this is impossible.

So the other eigenvalues of A are not 1.

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5(b)	Case 1: a is non-real. Then A has non-real elements.
	Case 2: a is real with $a^2 > 1$. Then $b^2 = 1 - a^2 < 0$,
	so $b = \pm\sqrt{1 - a^2}$ is non-real, so A has non-real elements.
	Case 3: a is real with $a^2 < 1$. Then $a^2 - 1 < 0$, so
	$\sqrt{a^2 - 1}$ is non-real and $\lambda = a \pm \sqrt{a^2 - 1}$ is non-real,
	so two of the eigenvalues of A are non-real.
	($a^2 = 1$ is not possible since $a \neq \pm 1$.)
	These cover all possible values of a, so either some
	eigenvalues of A are non-real or some elements of A are non-real.
5(c)(i)	$Ae = 1e = e$, so e is unchanged by the transformation:
	e is invariant, so it lies along the axis of rotation
	(the direction of e is the axis that A rotates about).
	The eigenvalue 1 means the magnitude of e is unchanged too:
	the axis of rotation is a line of invariant points, since a
	rotation leaves every point on its axis fixed.
5(c)(ii)	In a rotation through an acute angle the only direction left
	unchanged is the axis of rotation; every other real vector is
	turned to a different direction. A real eigenvector maps to a
	multiple of itself (an invariant direction), so there can be no
	other real eigenvectors.
5(c)(iii)	$(A - 1)e = 0$ with $e = (x, y, z)$:
	$(a - 1)x + by = 0$ and $-bx + (a - 1)y = 0$
	$\det = (a - 1)^2 + b^2 = 2 - 2a \neq 0$ $(a \neq 1)$, so $x = y = 0$
	$e = (0, 0, 1)$
	Axis of rotation: $r = \lambda(0, 0, 1)$

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EXTRA ANSWER SPACE

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