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Wednesday 18 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 The table shows some values of x and the associated values of $f(x)$.

x	1.9	2.0	2.1
$f(x)$	0.216 92	0.2	0.184 84

(a) Determine an estimate of $f'(2.0)$ using the forward difference method. **[2]**

(b) Determine an estimate of $f'(2.0)$ using the central difference method. **[2]**

2 The numbers p and q are approximated by

$P = 323$ and $Q = 162$.

P has been found by **rounding** p to the nearest whole number.

Q has been found by **chopping** q to the nearest whole number.

(a) (i) Find the maximum possible relative error in using P to approximate p. **[1]**

(ii) Find the maximum possible relative error in using Q to approximate q. **[1]**

(b) Determine the range of possible values of $R = \frac{200}{p-2q}$. **[3]**

(c) Explain why your answer to part **(b)** is so large. **[1]**

3 Approximations to $\int_{0.5}^{1.3} \sqrt{1+x^3}\,dx$ using the midpoint rule, the trapezium rule and Simpson's rule with $n = 1$ and $n = 2$ are shown in the table. The table is incomplete.

n	M_n	T_n	S_{2n}
1			1.081 111
2	1.074 256		

(a) Complete the copy of the table in the **Printed Answer Booklet**. Give your answers to **6** decimal places. **[4]**

(b) **Without doing any further calculations**, state the value of $\int_{0.5}^{1.3} \sqrt{1+x^3}\,dx$ as accurately as possible. You must justify the precision quoted. **[1]**

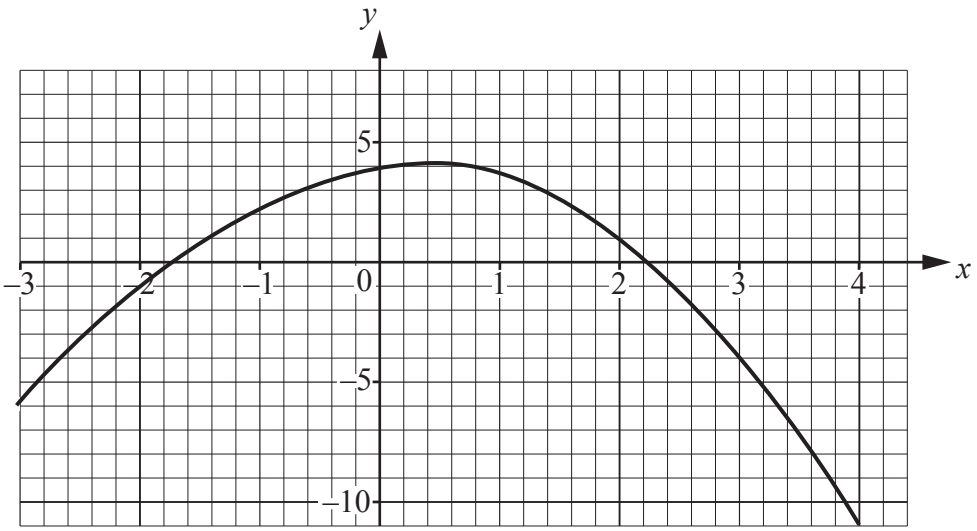
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4 The Newton-Raphson method is to be used to find the positive root of the equation

$\tanh x - x^2 + 4 = 0$.

The diagram shows part of the graph of $y = \tanh x - x^2 + 4$.



(a) On the copy of the diagram in the **Printed Answer Booklet**, show how the Newton-Raphson method works to find x_1 using the starting value $x_0 = 1$. **[1]**

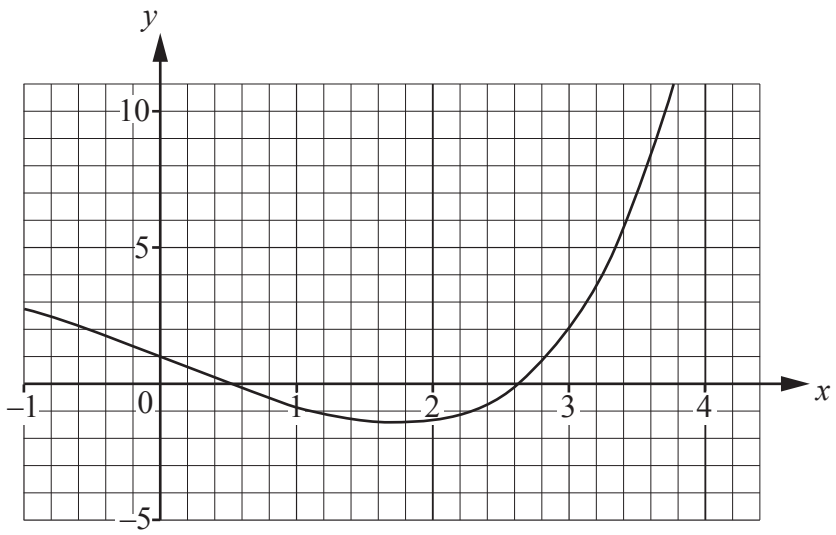
(b) Use the Newton-Raphson method using the starting value $x_0 = 1$ to determine the values of x_1 and x_2 correct to **8** decimal places. **[4]**

(c) Continue the iteration to determine the value of the positive root of the equation $\tanh x - x^2 + 4 = 0$ correct to **7** decimal places. **[2]**

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5 The diagram shows part of the graph of $y = \sinh x - 3x + 1$.



The largest positive root, α, of the equation $\sinh x - 3x + 1 = 0$ is to be found using the secant method with $x_0 = 2$ and $x_1 = 3$.

Table 5.1 shows the associated spreadsheet output.

Table 5.1

	A	B	C	D	E
2	r	x_r	$f(x_r)$	x_{r+1}	$f(x_{r+1})$
3	0	2	–1.37314	3	2.01787
4	1	3	2.017875	2.404935	–0.7211
5	2	2.404935	–0.72109	2.561598	–0.2451
6	3	2.561598	–0.24513	2.642285	0.06018
7	4	2.642285	0.060175	2.626382	–0.0035
8	5	2.626382	–0.00348	2.627252	–5E-05
9	6	2.627252	–4.5E-05	2.627264	3.5E-08

(a) Write down a suitable formula for cell D4. **[2]**

(b) State the most accurate approximation to α in the spreadsheet output. **[1]**

(c) Determine whether this approximation is accurate to **6** decimal places. **[2]**

(d) Explain what would happen if the initial entry in cell D3 were to be changed from 3 to 1. **[1]**

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Table 5.2 shows some further analysis of the sequence of approximations to α.

Table 5.2

A	B	C	D	E
r	x_r	$x_{r+1} - x_r$	$\frac{x_{r+2} - x_{r+1}}{x_{r+1} - x_r}$	$\frac{x_{r+2} - x_{r+1}}{(x_{r+1} - x_r)^2}$
1	3	–0.5951	–0.2633	0.44242
2	2.404935	0.15666	0.51504	3.28756
3	2.561598	0.08069	–0.1971	–2.4427
4	2.642285	–0.0159	–0.0547	3.44193
5	2.626382	0.00087	0.01315	15.1113
6	2.627252	1.1E-05		
7	2.627264			

(e) Explain what may be inferred from the entries in column D about the order of convergence of this sequence of approximations to α. **[2]**

(f) Explain what may be inferred from the entries in column E about the order of convergence of this sequence of approximations to α. **[2]**

6 The midpoint rule is used to find a sequence of approximations to $\int_0^1 \sqrt{\sin x}\,\mathrm{d}x$.

The associated spreadsheet output, together with some further analysis, is shown in the table.

n	M_n	difference	ratio
1	0.6924056		
2	0.6615057	–0.0308999	
4	0.6498274	–0.0116782	0.37794
8	0.6454773	–0.0043501	0.3725
16	0.6438811	–0.0015962	0.36692
32	0.643302	–0.0005791	0.36281
64	0.6430936	–0.0002085	0.35996
128	0.6430189	–7.463E-05	0.35801

(a) Explain what the entries in the ratio column tell you about the order of the midpoint rule in this case. **[2]**

(b) Use the information in the table to determine the most accurate estimate of $\int_0^1 \sqrt{\sin x}\,\mathrm{d}x$ possible. You must justify the precision quoted. **[5]**

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7 The equation $8\ln(x+2)-3x+1=0$ has a positive root, γ, and a negative root, δ.

It is proposed that the iterative formula

$x_{n+1}=\mathrm{g}(x_n)$, where $\mathrm{g}(x)=8\ln(x+2)-2x+1$,

should be used to find γ.

It is found that $\mathrm{g}'(\gamma)\approx -0.977$.

(a) Explain what this tells you about the speed of convergence of the sequence of approximations to γ which would be generated by $x_{n+1}=\mathrm{g}(x_n)$. **[2]**

The relaxed iteration

$x_{n+1}=(1-\lambda)x_n+\lambda\mathrm{g}(x_n)$ with $\lambda=0.506$

is used to find a sequence of approximations to γ.

The spreadsheet output is shown in the table, together with the values of $\mathrm{f}(x_r)=8\ln(x_r+2)-3x_r+1$ for $r=0,1,2,3,4$.

	E	F	G
1	r	x_r	$\mathrm{f}(x_r)$
2	0	5	1.567281192
3	1	5.7930443	0.046719768
4	2	5.8166845	3.04288E-05
5	3	5.8166999	–4.07401E-09
6	4	5.8166999	5.47118E-13

(b) State the value of γ as accurately as you can, justifying the precision quoted. **[1]**

(c) Explain why the values displayed in cells F5 and F6 are the same as each other, but the values in cells G5 and G6 are different to each other. **[2]**

The negative root of the equation $8\ln(x+2)-3x+1=0$, δ, is close to –1.

It is found that $\mathrm{g}'(\delta)\approx 13.9$.

(d) Explain why the iterative formula

$x_{n+1}=\mathrm{g}(x_n)$ with $x_0=-1$

may **not** be used to find δ. **[1]**

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(e) **(i)** Use the relaxed iteration

$x_{n+1} = (1-\lambda)x_n + \lambda g(x_n)$ with $\lambda = 0.506$ and $x_0 = -1$

to find x_1, x_2, x_3 and x_4, giving your answers correct to **3** decimal places. **[2]**

(ii) State what would happen if the iteration in part **(e)(i)** were to be continued. **[1]**

(f) Use the relaxed iteration

$x_{n+1} = (1-\lambda)x_n + \lambda g(x_n)$ with $\lambda = -0.078$ and $x_0 = -1$

to determine the value of δ correct to **7** decimal places. **[2]**

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8 Following an injury an athlete returns to training in preparation for a regional competition. At the end of each week the athlete's trainer records the time, t seconds, the athlete takes to run 100 metres.

Table 8.1 shows some of the times recorded at the end of week W.

Table 8.1

W	0	2	3
t	18.00	17.68	16.83

The trainer uses the information in the table to construct a polynomial of degree 2 to model these data.

(a) Use Lagrange's interpolating polynomial of degree 2 to obtain the trainer's model. **[3]**

Subsequently it is found that when $W = 4$, $t = 15.44$.

(b) Determine whether the model is a good fit for these values. **[1]**

The trainer decides to use the three most recent data points, when $W = 2$, 3 and 4, to construct a new model using Newton's forward difference interpolation formula.

Table 8.2 shows the difference table for these data.

Table 8.2

W	t		
2	17.68		
		−0.85	
3	16.83		−0.54
		−1.39	
4	15.44		

(c) The trainer obtains the interpolation formula

$t = -0.27W^2 + BW + C$, where B and C are constants.

- Determine the value of B.
- Determine the value of C. **[3]**

(d) Explain why the new model is a refinement of the model found in part **(a)**. **[1]**

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At the regional competition, the athlete needs to run 100 metres in less than 11 seconds in order to qualify for a national competition. The regional competition takes place when $W = 6$.

(e) Determine whether, according to the new model, the athlete will qualify for the national competition. **[1]**

(f) Explain whether it would be appropriate to use the new model to predict the athlete's times for $W \geqslant 7$. **[1]**

END OF QUESTION PAPER

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Printed Answer Booklet

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AB

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Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) StemUp Model Solution

Last name

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1(a)	$f(2.0) \approx (f(2.1) - f(2.0)) / 0.1$
	$= (0.18484 - 0.2) / 0.1$
	$= -0.01516 / 0.1$
	$f(2.0) \approx -0.1516$
1(b)	$f(2.0) \approx (f(2.1) - f(1.9)) / (2.1 - 1.9)$
	$= (0.18484 - 0.21692) / 0.2$
	$= -0.03208 / 0.2$
	$f(2.0) \approx -0.1604$
2(a)(i)	$322.5 \leqslant p < 323.5$, so the maximum error is 0.5
	Max relative error $= 0.5 / 322.5$
	$= 0.00155$ (3 s.f.) $\quad (= 1/645)$
2(a)(ii)	$162 \leqslant q < 163$, so $-1 < Q - q \leqslant 0$
	Max relative error $= (162 - 163) / 163$
	$= -1/163 = -0.00613$ (3 s.f.)

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2(b)

$322.5 \leq p < 323.5$ and $162 \leq q < 163$

Largest value of $p - 2q$: $323.5 - 2 \times 162 = -0.5$

Smallest value of $p - 2q$: $322.5 - 2 \times 163 = -3.5$

so $-3.5 < p - 2q < -0.5$ (always negative)

$R = 200 / (p - 2q)$

$p - 2q = -0.5 \Rightarrow R = 200 / (-0.5) = -400$

$p - 2q = -3.5 \Rightarrow R = 200 / (-3.5) = -57.142...$

$\therefore -400 < R < -57.1$ (i.e. $-400 < R < -400/7$)

2(c)

$p \approx 2q$, so the denominator $p - 2q$ is the difference of two nearly equal numbers: it is very small and has a large relative error. Dividing by a very small number is unstable, so a small change in p or q gives a very large change in R.

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3(a)

n	M_n	T_n	S_{2n}
1	1.051932	1.139470	1.081111
2	1.074256	1.095701	1.081404

$h = 1.3 - 0.5 = 0.8$, $f(x) = \sqrt{1 + x^3}$

$M_1 = 0.8 \times f(0.9) = 0.8 \times 1.3149144 = 1.051932$

$T_1 = 0.4 \times (f(0.5) + f(1.3)) = 0.4 \times (1.0606602 + 1.7880157)$

$T_1 = 1.139470$

$T_2 = (T_1 + M_1)/2 = (1.1394703 + 1.0519316)/2 = 1.095701$

$S_4 = (2M_2 + T_2)/3 = (2 \times 1.0742560 + 1.0957009)/3 = 1.081404$

(check: $S_2 = (2M_1 + T_1)/3 = 1.081111$ as given)

3(b)

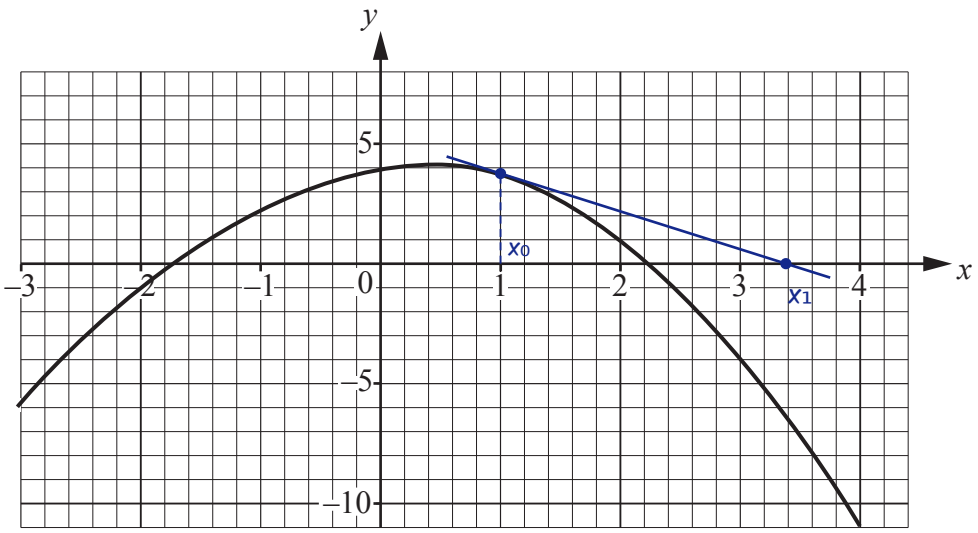
$S_2 = 1.081111$ and $S_4 = 1.081404$ agree to 3 d.p., and

Simpson's rule converges fast (error ÷ about 16 when n doubles),

so the 4th d.p. is secure: $\int \approx 1.0814$

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4(a)



4(b)

$f(x) = \tanh x - x^2 + 4, \quad f'(x) = \operatorname{sech}^2 x - 2x$

$x_{n+1} = x_n - (\tanh x_n - x_n^2 + 4) / (\operatorname{sech}^2 x_n - 2x_n)$

$x_1 = 1 - (\tanh 1 - 1 + 4) / (\operatorname{sech}^2 1 - 2)$

$x_1 = 1 - 3.7615942 / (-1.5800257)$

$x_1 = 3.38071713 \quad (8 \text{ d.p.})$

$x_2 = 3.38071713 - f(3.38071713) / f'(3.38071713)$

$x_2 = 3.38071713 - (-6.4315608) / (-6.7568147)$

$x_2 = 2.42885430 \quad (8 \text{ d.p.})$

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4(c)	$x_3 = 2.23935179$
	$x_4 = 2.23097667$
	$x_5 = 2.23096010$
	$x_6 = 2.23096010$
	x_5 and x_6 agree to 8 d.p., so the positive root is
	$x = 2.2309601$ (7 d.p.)
5(a)	D4: = (B3*E3 − D3*C3) / (E3 − C3)
	($x_2 = (x_0 f(x_1) - x_1 f(x_0)) / (f(x_1) - f(x_0))$, the secant method)
5(b)	$\alpha \approx 2.627264$
5(c)	$f(x) = \sinh x - 3x + 1$
	$f(2.6272635) = -3.73 \times 10^{-7} < 0$
	$f(2.6272645) = 3.58 \times 10^{-6} > 0$
	Change of sign (f continuous), so $2.6272635 < \alpha < 2.6272645$
	∴ 2.627264 is accurate to 6 decimal places.

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5(d)

With $x_0 = 2$ and $x_1 = 1$ the secant method would converge to a different root (the smaller positive root, about 0.511), not to α.

5(e)

The ratios $(x_{r+2} - x_{r+1}) / (x_{r+1} - x_r)$ are decreasing in magnitude (towards 0) rather than settling to a constant, so the order of convergence is higher than first order (faster than first order convergence).

5(f)

The ratios $(x_{r+2} - x_{r+1}) / (x_{r+1} - x_r)^2$ are increasing in magnitude (3.29, −2.44, 3.44, 15.1) so they do not settle to a constant: the order of convergence is lower than second order (slower than second order convergence).

6(a)

The ratios of differences are converging, to about 0.35. A ratio of 0.5 would mean first order and a ratio of 0.25 would mean second order. Since $0.25 < 0.35... < 0.5$, the midpoint rule is behaving with order between first and second order in this case.

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6(b)	Ratio of differences $r \approx 0.358$ (still changing very slowly)
	Last difference $d = -7.463 \times 10^{-5}$; later differences $\approx dr, dr^2, \ldots$
	Limit $\approx M_{128} + d(r + r^2 + r^3 + \ldots) = M_{128} + d \times r/(1 - r)$
	$= 0.6430189 + (-7.463 \times 10^{-5}) \times 0.358/(1 - 0.358)$
	$= 0.6430189 - 0.0000416$
	$= 0.6429773$
	Using $r = 0.355$ or $r = 0.36$ instead gives 0.6429778 or
	0.6429769, which all agree to 5 d.p.
	The extrapolated value is an improvement on M_{128}, and
	5 d.p. is secure, so
	$\int_0^1 \sqrt{\sin x}\, dx \approx 0.64298$ (5 d.p.)
7(a)	$\lvert g'(\gamma)\rvert < 1$ so the iteration converges, but -0.977 is very
	close to -1, so the convergence will be very slow
	(oscillating either side of γ, since $g'(\gamma) < 0$).
7(b)	$\gamma = 5.8166999$ (7 d.p.), since x_3 and x_4 (cells F5 and F6)
	agree to 7 decimal places.
7(c)	F5 and F6 are only displayed to 7 d.p.: the stored values are
	different but both round to 5.8166999. The spreadsheet works out
	G5 and G6 from the full stored values in F5 and F6, which are
	different, so the values of $f(x_3)$ and $f(x_4)$ are different.

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7(d)	$g'(\delta) \approx 13.9$ and $\lvert 13.9 \rvert > 1$, so the iteration diverges
	away from δ: it cannot converge to δ.
7(e)(i)	$g(x) = 8\ln(x + 2) - 2x + 1$
	$x_{n+1} = 0.494x_n + 0.506g(x_n)$
	$x_0 = -1$: $g(-1) = 8\ln 1 + 2 + 1 = 3$
	$x_1 = 0.494 \times (-1) + 0.506 \times 3 = 1.024$
	$x_2 = 0.494 \times 1.024 + 0.506 \times 7.8046437 = 4.4550057$
	$x_3 = 0.494 \times 4.4550057 + 0.506 \times 7.0088359 = 5.7472438$
	$x_4 = 0.494 \times 5.7472438 + 0.506 \times 5.8842096 = 5.8165485$
	$x_1 =$ 1.024
	$x_2 =$ 4.455
	$x_3 =$ 5.747
	$x_4 =$ 5.817
7(e)(ii)	The iteration would converge to $\gamma = 5.8166999\ldots$,
	the positive root, not to δ.
7(f)	$x_{n+1} = 1.078x_n - 0.078g(x_n)$, $x_0 = -1$
	$x_1 = -1.312$
	$x_2 = -1.46365294$
	$x_3 = -1.49541206$
	$x_4 = -1.49651428$
	$x_5 = -1.49650987$
	$x_6 = -1.49650989$, $x_7 = -1.49650989$
	x_6 and x_7 agree to 8 d.p., so
	$\delta = -1.4965099$ (7 d.p.)

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8(a)

$t = 18 \times (w - 2)(w - 3) / ((0 - 2)(0 - 3))$

$\quad + 17.68 \times w(w - 3) / ((2 - 0)(2 - 3))$

$\quad + 16.83 \times w(w - 2) / ((3 - 0)(3 - 2))$

$t = 3(w - 2)(w - 3) - 8.84w(w - 3) + 5.61w(w - 2)$

$t = 3(w^2 - 5w + 6) - 8.84(w^2 - 3w) + 5.61(w^2 - 2w)$

$t = 3w^2 - 15w + 18 - 8.84w^2 + 26.52w + 5.61w^2 - 11.22w$

$t = -0.23w^2 + 0.3w + 18$

8(b)

$w = 4$: $t = -0.23 \times 16 + 0.3 \times 4 + 18 = 15.52$

15.52 is very close to 15.44 (only 0.08 out),

so the model is a good fit.

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8(c)

Newton's forward difference formula, $W_0 = 2$, $h = 1$:

$t = 17.68 + (W - 2)(-0.85) + (W - 2)(W - 3)(-0.54) / 2!$

$t = 17.68 - 0.85W + 1.7 - 0.27(W^2 - 5W + 6)$

$t = 17.68 - 0.85W + 1.7 - 0.27W^2 + 1.35W - 1.62$

$t = -0.27W^2 + 0.5W + 17.76$

so $B = 0.5$ and $C = 17.76$

$B =$ 0.5

$C =$ 17.76

8(d)

The new model uses the three most recent data values ($W = 2$, 3 and 4), including the new value at $W = 4$, so it fits the latest data and is better for predicting.

8(e)

$W = 6$: $t = -0.27 \times 36 + 0.5 \times 6 + 17.76 = 11.04$

$11.04 > 11$, so the athlete will not qualify.

8(f)

No. The quadratic keeps decreasing: $W = 7$ gives $t = 8.03$ s, faster than the world record, and eventually t becomes negative.

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EXTRA ANSWER SPACE

If you need extra space use this lined page. You must write the question numbers clearly in the margin.

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