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Friday 6 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Time allowed: 2 hours 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Section A (33 marks)

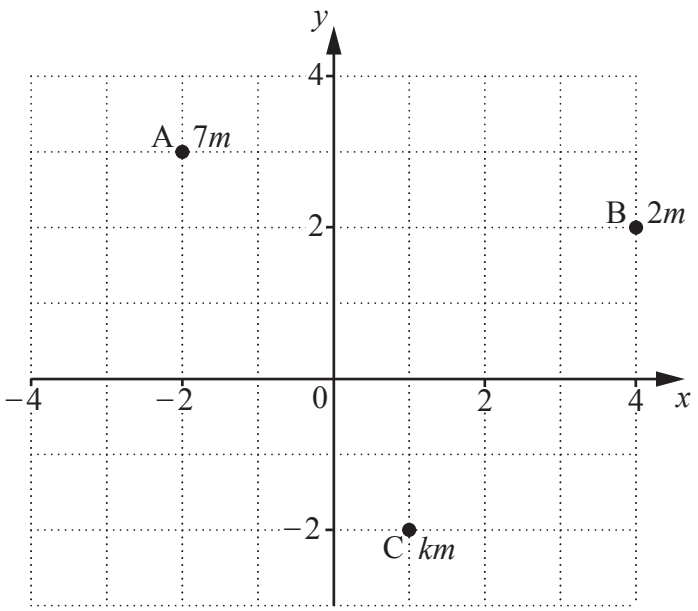
1 A small box of mass 3 kg is pulled along a horizontal floor by a constant force of magnitude 35 N. The force acts at an angle of θ above the horizontal.

The total resistance to the motion of the box has magnitude 25 N.

The box starts from rest at a point O, and passes through a point 4 m from O with speed $1.5\,\text{m s}^{-1}$.

Use an energy method to determine the value of θ. **[3]**

2



The diagram shows a system of three particles of masses $7m$, $2m$ and km situated in the x-y plane at the points A(−2, 3), B(4, 2) and C(1, −2) respectively.

The centre of mass of the three particles is at the point with coordinates $(0, d)$.

(a) Find the value of k. **[2]**

(b) Find the value of d. **[2]**

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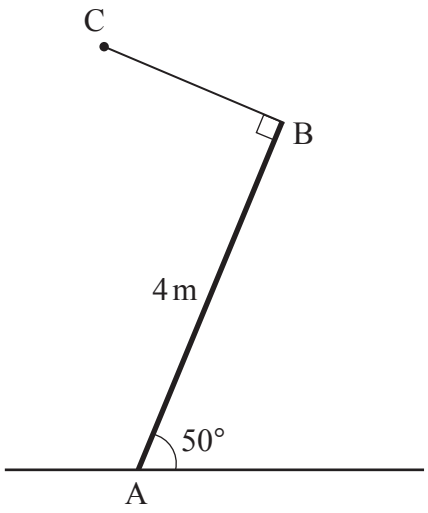
3 At time t seconds, the total force acting on a particle P of mass 4 kg is $\begin{pmatrix} 32\cos 2t \\ -8\sin t \\ 0 \end{pmatrix}$ N.

When $t = 0$ the velocity of P is $\begin{pmatrix} 0 \\ 5 \\ 6 \end{pmatrix}$ m s^{-1}.

(a) Find, in column vector form, the acceleration of P at time t seconds. **[1]**

(b) Determine the speed of P when $t = \frac{1}{3}\pi$. **[5]**

4



A uniform rod AB of mass 3 kg and length 4 m rests with one end A on rough horizontal ground. A light inextensible string has one end attached to the rod at B and the other end attached to a fixed point C.

The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 50° to the ground, as shown in the diagram.

(a) Complete the diagram in the **Printed Answer Booklet** to show all the forces acting on the rod. **[1]**

(b) By taking moments about A, find the tension in the string. **[2]**

(c) Determine the magnitude of the total contact force between the rod and the ground. **[4]**

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5 **In this question you may assume that if r and s are any physical quantities then $\left[\frac{\mathrm{d}r}{\mathrm{d}s}\right] = \left[\frac{r}{s}\right]$.**

You may also assume that all given numerical constants are dimensionless.

(a) Find the dimensions of power. **[1]**

A car of mass m moves horizontally in a straight line. When the car is a distance x from a point O, it is moving away from O with speed v.

The power developed by the car is P. The maximum possible speed of the car during its motion is U.

The differential equation for the motion of the car is given by

$$m^{\alpha} v^{\beta} \frac{\mathrm{d}v}{\mathrm{d}x} = P^{\gamma}\left(1 - \frac{v^{\delta}}{U^{3}}\right).$$

(b) Explain why $\delta = 3$. **[1]**

(c) Show that the dimensions of $\frac{\mathrm{d}v}{\mathrm{d}x}$ are T^{-1}. **[1]**

(d) Use dimensional analysis to determine the values of α, β and γ. **[3]**

Once the car attains a speed of $\frac{1}{2}U$, the power developed by the car is immediately reduced to zero.

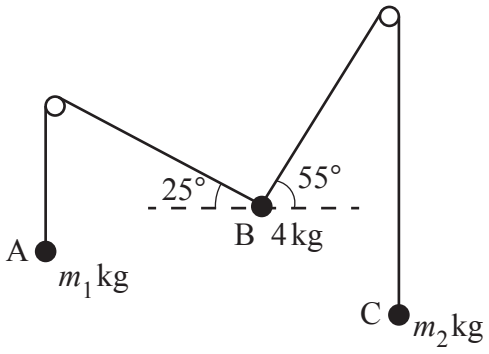
A student calculates that the time taken, t, for the speed of the car to reduce from $\frac{1}{2}U$ to $\frac{1}{4}U$ is given by $t = \frac{2mU^{2}}{P}$.

(e) Determine whether this expression for t is dimensionally consistent. **[2]**

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6



One end of a light inextensible string is attached to a particle A of mass m_1 kg. The other end of the string is attached to a second particle B of mass 4 kg.

A second light inextensible string is attached to B. The other end of this second string is attached to a third particle C of mass m_2 kg. Both strings are taut and pass over small smooth fixed pulleys.

The diagram shows the three particles A, B and C, and the two strings. The diagram also shows the angles that the non-vertical portions of the strings make with the horizontal.

The system is in equilibrium with all three particles at rest in the same vertical plane.

(a) Draw a closed figure to represent the three forces acting on B. **[1]**

(b) Hence, or otherwise, find the following.

- The value of m_1
- The value of m_2 **[4]**

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Section B (87 marks)

7 Two uniform smooth spheres, A and B, have the same radius. The mass of A is 4 kg, and the mass of B is 6 kg.

The spheres are travelling in the same direction in a straight line on a smooth horizontal surface, A with speed $6\,\text{m s}^{-1}$ and B with speed $3\,\text{m s}^{-1}$.

Sphere A collides directly with B and, immediately after the collision, the speed of B is $5\,\text{m s}^{-1}$.

(a) Determine the exact value of the coefficient of restitution between A and B. **[3]**

Sphere B subsequently collides directly with a uniform smooth stationary sphere C. The mass of C is 2 kg, and C has the same radius as A and B. The coefficient of restitution between B and C is 0.6.

(b) Show that there will be **no** further collisions between any of the three spheres. **[4]**

(c) Show that the impulse of B on C is equal in magnitude to the impulse of A on B. **[2]**

(d) Determine the total kinetic energy lost due to the **two** collisions. **[3]**

8 A box of mass m kg rests on a horizontal platform. The platform moves vertically with simple harmonic motion of period T seconds and amplitude a m. At time t seconds, the upward displacement of the platform from its mean position is x m.

(a) Express $\ddot{x}$ in terms of and π, T and x. **[2]**

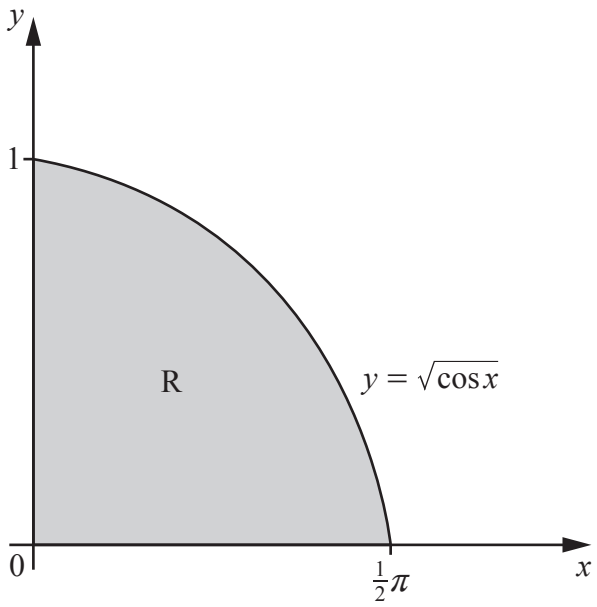
(b) Show that the box will maintain contact with the platform throughout its motion if $T > 2\pi\sqrt{\frac{a}{g}}$. **[4]**

(c) Given that $T = 4\pi\sqrt{\frac{a}{g}}$, determine the ratio of the magnitudes of the greatest and least forces exerted by the platform on the box. Give your answer in the form $p : q$ where p and q are integers to be found. **[2]**

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9



The diagram shows the shaded region R bounded by the curve $y = \sqrt{\cos x}$, the x-axis and the y-axis.

The region R is rotated through 2π radians about the x-axis to form a uniform solid of revolution S.

(a) Determine the exact x-coordinate of the centre of mass of S. **[6]**

The uniform solid S is placed with its plane face on an inclined plane. The inclined plane makes an angle θ with the horizontal.

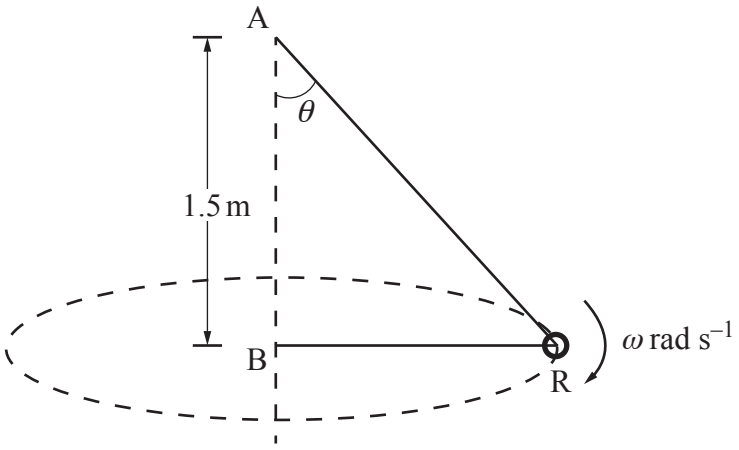
(b) Given that the inclined plane is sufficiently rough to prevent S from sliding and that S is on the point of toppling when $\theta = \alpha$, find the angle α. **[2]**

(c) Given instead that S is on the point of sliding down the plane when $\theta = \beta$ and the coefficient of friction between S and the plane is 0.4, find the angle β. **[2]**

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10



One end of a light inextensible string is attached to a fixed point A. The other end of the string is attached to a fixed point B, vertically below A, where AB = 1.5 m.

A small smooth ring R of mass 2 kg is threaded on the string. The ring R moves in a horizontal circle with centre B.

The upper part of the string makes an angle θ with the downward vertical and R moves with constant angular speed ω rad s^{-1} (see diagram).

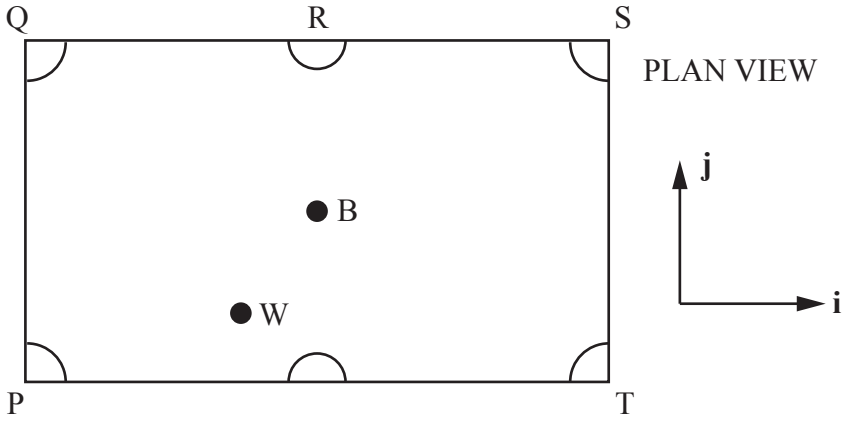
(a) By finding an expression for ω^2 in terms of g and $\sin\theta$, show that $\omega > 2\sqrt{\frac{1}{3}g}$. **[6]**

(b) Given that $\omega = \sqrt{2g}$, determine, in terms of g, the exact tension in the string. **[3]**

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11



A student is playing snooker. The snooker table, PQST, is rectangular and the unit vectors, **i** and **j**, are parallel to PT and PQ respectively (see diagram).

The surface of the table is horizontal and smooth. The balls on the table are all uniform smooth spheres with the same radius and mass. The coefficient of restitution between the balls is 0.6. The blue ball, B, lies at rest at the centre of the table.

The student strikes the white ball, W, so that it collides obliquely with B. Immediately before the collision with B, W is travelling with velocity $(6\mathbf{i} + 8\mathbf{j})\,\text{m s}^{-1}$. As a result of the collision, B travels directly towards R, where R is the mid-point of QS.

(a) Explain why, at the moment of the collision, the line of centres of B and W must be parallel to **j**. **[1]**

(b) Determine, correct to the nearest degree, the angle through which the direction of motion of W is deflected by the collision. **[7]**

12 One end of a light elastic string of natural length a m and modulus of elasticity $4mg$ N is attached to a fixed point O. The other end of the string is attached to a particle P of mass m kg. The particle is projected vertically downwards from O with speed $\sqrt{4ga}\,\text{m s}^{-1}$.

(a) When P is at a distance of h m below O, where $h > a$, it has speed, $v\,\text{m s}^{-1}$.

By using the principle of conservation of energy, show that $v = \sqrt{10gh - \frac{4gh^2}{a}}$. **[4]**

(b) Determine, in terms of a, the greatest distance below O attained by P. **[2]**

(c) Determine, in terms of g and a, the maximum speed of P during its downward motion. **[4]**

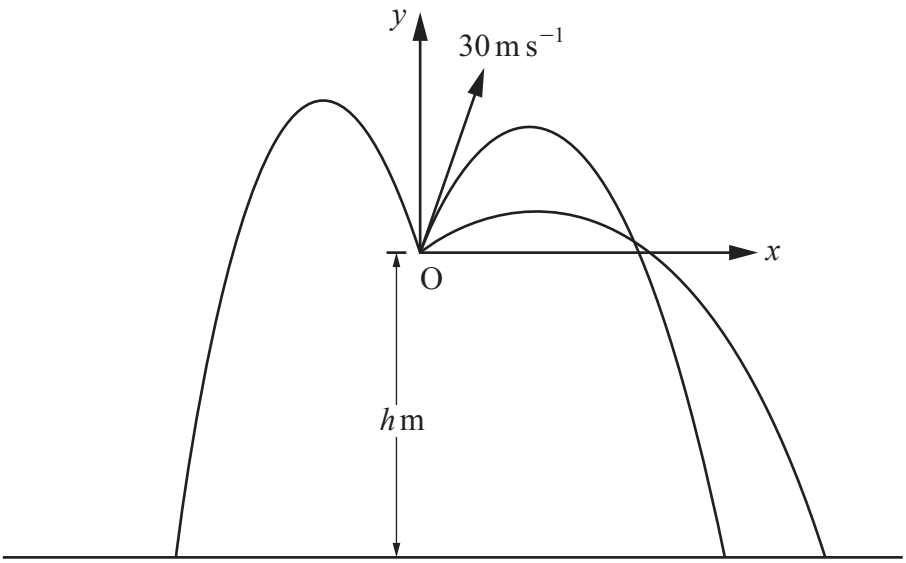
(d) Find, in terms of a, the greatest height above O attained by P. **[2]**

(e) Explain why the greatest distance below O attained by P may **not** in practice be given by your answer to part **(b)**. **[1]**

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13



In this question you should take the acceleration due to gravity to be 10 m s^{-2}.

A machine can launch balls with speed 30 m s^{-1} from a height h m above horizontal ground. The point of projection O is the origin for a horizontal x-axis and a vertical y-axis which define the vertical plane in which the balls move (see diagram). You should assume that the balls, as they leave the machine, can be modelled as particles moving freely under gravity.

At time $t = 0$, a ball B is projected at an angle θ above the horizontal in the x-y plane. Before B hits the ground, the horizontal and vertically upwards displacements of B from O at time t seconds are denoted by x m and y m respectively.

You are given that B hits the ground at the point $(X, -h)$.

(a) By expressing x and y in terms of t, show that

$$X^2 \tan^2\theta - 180X\tan\theta + X^2 - 180h = 0.$$ **[5]**

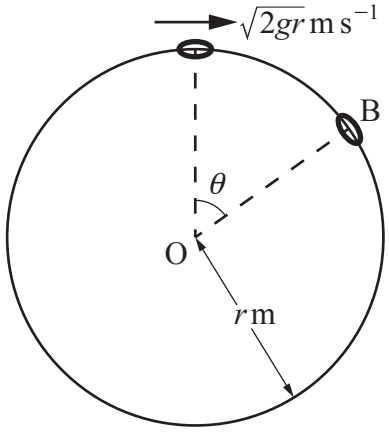
(b) Determine an expression, as $\tan\theta$ varies, for the maximum value of X^2 at the instant when B hits the ground. Give your answer in the form $X^2 = p + qh$, where p and q are integers to be determined. **[3]**

(c) Hence find the equation of the curve that bounds the region in the x-y plane through which B can pass. Give your answer in the form $y = k_1x^2 + k_2$, where k_1 and k_2 are constants to be determined. **[2]**

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14



A smooth thin wire is formed into a circle of radius r m and centre O and is fixed in a vertical plane. A small bead B of mass m kg is threaded on the wire.

The bead is initially at the highest point of the wire and is set in motion along the wire with speed $\sqrt{2gr}\,\text{m s}^{-1}$ (see diagram).

(a) Show that, when the radius from O to B is inclined at an angle θ to the upward vertical, the downward vertical component of the acceleration of B is

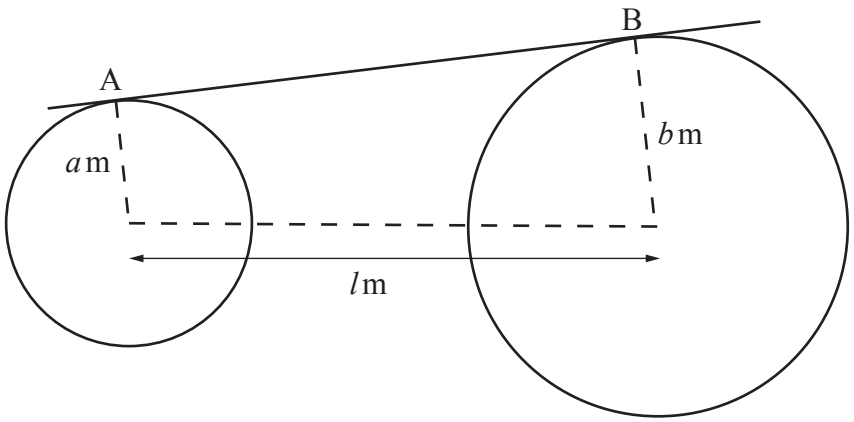
$(1 + 4\cos\theta - 3\cos^2\theta)g$. **[5]**

(b) Determine, in terms of m and g, the magnitude of the force of the bead on the wire when the downward vertical component of the acceleration is a maximum. **[6]**

Turn over for question 15

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15



The diagram shows two fixed rough circular cylinders of radii a m and b m, where $b > a$. The axes of the cylinders are parallel and in the same horizontal plane at a distance l m apart. A heavy non-uniform straight rod rests in limiting equilibrium on the cylinders at A and B. The centre of mass of the rod lies between A and B but is not at the mid-point of AB. The coefficient of friction between the rod and each cylinder is μ.

Determine, in terms of a, b and l, an expression for μ. **[6]**

END OF QUESTION PAPER

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Friday 6 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Printed Answer Booklet

Time allowed: 2 hours 15 minutes

AB

You must have:
- Question Paper Y421/01 (inside this document)
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) StemUp Model Solution

Last name

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ADVICE
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Section A (33 marks)

1

Work-energy principle from O to the point 4 m from O:

Work done by 35 N force = $35 \cos\theta \times 4 = 140 \cos\theta$ J

Work done against resistance = $25 \times 4 = 100$ J

Gain in KE = $\frac{1}{2} \times 3 \times 1.5^2 = 3.375$ J

$140 \cos\theta - 100 = 3.375$

$\cos\theta = 103.375 \div 140 = 0.73839...$

$\theta = 42.4°$

2(a)

Moments about the y-axis (x-coordinates), $\bar{x} = 0$:

$7m \times (-2) + 2m \times 4 + km \times 1 = (7m + 2m + km) \times 0$

$-14m + 8m + km = 0$

$k = 6$

2(b)

Total mass = $7m + 2m + 6m = 15m$

Moments about the x-axis (y-coordinates):

$7m \times 3 + 2m \times 2 + 6m \times (-2) = 15m \times d$

$21m + 4m - 12m = 15md$

$13m = 15md$

$d = \frac{13}{15}$

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3(a)

$$a = F \div 4 = \begin{pmatrix} 8\cos 2t \\ -2\sin t \\ 0 \end{pmatrix} \text{ m s}^{-2}$$

3(b)

$v = \int a \, dt$

$$v = \begin{pmatrix} 4\sin 2t \\ 2\cos t \\ 0 \end{pmatrix} + c$$

When $t = 0$, $v = 5j + 6k$:

$$\begin{pmatrix} 0 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + c \quad \Rightarrow \quad c = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix}$$

$$v = \begin{pmatrix} 4\sin 2t \\ 2\cos t + 3 \\ 6 \end{pmatrix}$$

When $t = \pi/3$:

$$v = \begin{pmatrix} 4\sin(2\pi/3) \\ 2\cos(\pi/3) + 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 2\sqrt{3} \\ 4 \\ 6 \end{pmatrix}$$

$|v| = \sqrt{(2\sqrt{3})^2 + 4^2 + 6^2}$

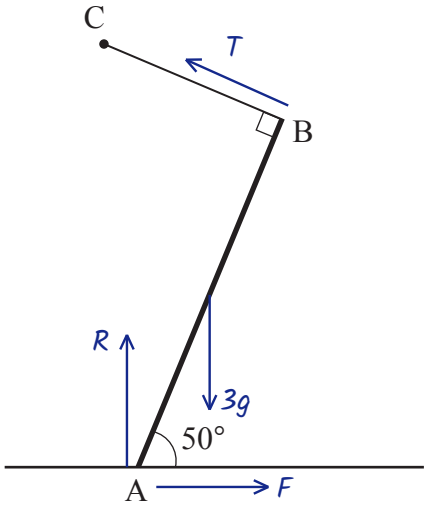
$= \sqrt{12 + 16 + 36} = \sqrt{64}$

Speed $= 8 \text{ m s}^{-1}$

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4(a)



4(b)

Moments about A (weight acts at the midpoint, 2 m from A):

$T \times 4 = 3g \times 2 \cos 50°$

$T = 1.5g \cos 50° = 9.4489...$

$T = 9.45$ N (3 s.f.)

4(c)

Let R be the normal reaction and F the friction at A.

The string is perpendicular to the rod, so T acts at 50° to the vertical.

Resolve vertically: $R + T \cos 50° = 3g$

$R = 29.4 - 9.4489... \times \cos 50° = 23.326...$

Resolve horizontally: $F = T \sin 50°$

$F = 9.4489... \times \sin 50° = 7.2383...$

Total contact force $= \sqrt{R^2 + F^2}$

$= \sqrt{23.326...^2 + 7.2383...^2}$

$= 24.4$ N (3 s.f.)

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5(a)

Power = force × velocity

$[P] = MLT^{-2} \times LT^{-1} = ML^2T^{-3}$

5(b)

1 is dimensionless, so $(1 - v^{\delta}/U^3)$ must be dimensionless.

So $[v^{\delta}] = [U^3]$. v and U are both speeds:

$(LT^{-1})^{\delta} = (LT^{-1})^3$, so $\delta = 3$

5(c)

$[dv/dx] = [v] \div [x]$

$= LT^{-1} \div L$

$= T^{-1}$ as required

5(d)

The bracket is dimensionless, so $[m^{\alpha}v^{\beta}dv/dx] = [P^{\gamma}]$

$M^{\alpha}(LT^{-1})^{\beta}T^{-1} = (ML^2T^{-3})^{\gamma}$

$M^{\alpha}L^{\beta}T^{-\beta-1} = M^{\gamma}L^{2\gamma}T^{-3\gamma}$

M: $\alpha = \gamma$

L: $\beta = 2\gamma$

T: $-\beta - 1 = -3\gamma \Rightarrow -2\gamma - 1 = -3\gamma \Rightarrow \gamma = 1$

$\alpha = 1, \ \beta = 2, \ \gamma = 1$

5(e)

$[2mU^2/P] = M(LT^{-1})^2 \div ML^2T^{-3}$

$= ML^2T^{-2} \div ML^2T^{-3}$

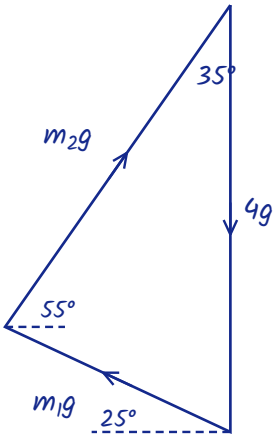
$= T$

This has the dimensions of time, so yes, the expression for t is dimensionally consistent.

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6(a)



6(b)

A and C are at rest and the pulleys are smooth, so the tensions

are m_1g (string to A) and m_2g (string to C).

Angles in the triangle of forces:

opposite m_1g: $90° - 55° = 35°$

opposite m_2g: $90° - 25° = 65°$

opposite 4g: $180° - 35° - 65° = 80°$

Sine rule:

$m_1g \div \sin 35° = 4g \div \sin 80°$

$m_1 = 4 \sin 35° \div \sin 80° = 2.3296...$

$m_2g \div \sin 65° = 4g \div \sin 80°$

$m_2 = 4 \sin 65° \div \sin 80° = 3.6811...$

Check horizontally: $m_1 \cos 25° = 2.111 = m_2 \cos 55°$

$m_1 = 2.33$

$m_2 = 3.68$

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Section B (87 marks)

7(a)

CLM: $4 \times 6 + 6 \times 3 = 4u_A + 6 \times 5$

$42 = 4u_A + 30 \Rightarrow u_A = 3 \text{ m s}^{-1}$

NEL: $5 - 3 = e(6 - 3)$

$e = \frac{2}{3}$

7(b)

B (6 kg, 5 m s^{-1}) hits C (2 kg, at rest); velocities after v_B, v_C.

CLM: $6 \times 5 + 0 = 6v_B + 2v_C \Rightarrow 3v_B + v_C = 15$

NEL: $v_C - v_B = 0.6 \times (5 - 0) = 3$

Subtracting: $4v_B = 12 \Rightarrow v_B = 3$, $v_C = 6$

After both collisions all move in the same direction with

A: 3 m s^{-1}, B: 3 m s^{-1}, C: 6 m s^{-1}

$v_A = v_B$, so A never catches up with B, and $v_C > v_B$,

so B never catches up with C.

Hence there are no further collisions.

7(c)

Impulse of A on B = change in momentum of B = $6(5 - 3) = 12$ N s

Impulse of B on C = change in momentum of C = $2(6 - 0) = 12$ N s

Both have magnitude 12 N s, so they are equal in magnitude.

7(d)

KE before = $\frac{1}{2} \times 4 \times 6^2 + \frac{1}{2} \times 6 \times 3^2 = 72 + 27 = 99$ J

KE after = $\frac{1}{2} \times 4 \times 3^2 + \frac{1}{2} \times 6 \times 3^2 + \frac{1}{2} \times 2 \times 6^2$

$= 18 + 27 + 36 = 81$ J

Total KE lost = $99 - 81 = 18$ J

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8(a)

SHM with period T: $\omega = 2\pi/T$

$\ddot{x} = -\omega^2 x$

$\ddot{x} = -(4\pi^2/T^2)x$

8(b)

Let R be the normal contact force of the platform on the box.

N2L for the box (upwards positive): $R - mg = m\ddot{x}$

$R = mg - (4\pi^2 m/T^2)x$

R is least when x is greatest, i.e. at the top where $x = a$:

$R_{min} = mg - 4\pi^2 ma/T^2$

Contact is maintained throughout if $R_{min} > 0$:

$mg > 4\pi^2 ma/T^2$

$T^2 > 4\pi^2 a/g$

$T > 2\pi\sqrt{a/g}$ as required

8(c)

$T = 4\pi\sqrt{a/g} \Rightarrow T^2 = 16\pi^2 a/g \Rightarrow 4\pi^2/T^2 = g/(4a)$

$R = mg - m \times (g/4a) \times x = mg - mgx/(4a)$

Greatest when $x = -a$: $R_{max} = mg + \frac{1}{4}mg = 5mg/4$

Least when $x = a$: $R_{min} = mg - \frac{1}{4}mg = 3mg/4$

$R_{max} : R_{min} = 5mg/4 : 3mg/4$

$= 5 : 3$

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9(a)

$V = \pi\int_0^{\pi/2} y^2\,dx = \pi\int_0^{\pi/2} \cos x\,dx$

$= \pi[\sin x]_0^{\pi/2} = \pi(1 - 0) = \pi$

$V\bar{x} = \pi\int_0^{\pi/2} xy^2\,dx = \pi\int_0^{\pi/2} x\cos x\,dx$

By parts: $u = x$, $dv/dx = \cos x \Rightarrow du/dx = 1$, $v = \sin x$

$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x$

$V\bar{x} = \pi[x\sin x + \cos x]_0^{\pi/2}$

$= \pi[(\frac{1}{2}\pi \times 1 + 0) - (0 + 1)]$

$= \pi(\frac{1}{2}\pi - 1)$

$\bar{x} = \pi(\frac{1}{2}\pi - 1) \div \pi$

$\bar{x} = \frac{1}{2}\pi - 1$

9(b)

The plane face is a circle of radius 1 ($y = 1$ at $x = 0$), and G is $\frac{1}{2}\pi - 1$ from the face. On the point of toppling, G is vertically above the lowest point of the face: $\tan\alpha = 1 \div (\frac{1}{2}\pi - 1)$

$\tan\alpha = 1.7519...$

$\alpha = 60.3°$

9(c)

On the point of sliding: $R = mg\cos\beta$ and $F = \mu R = mg\sin\beta$

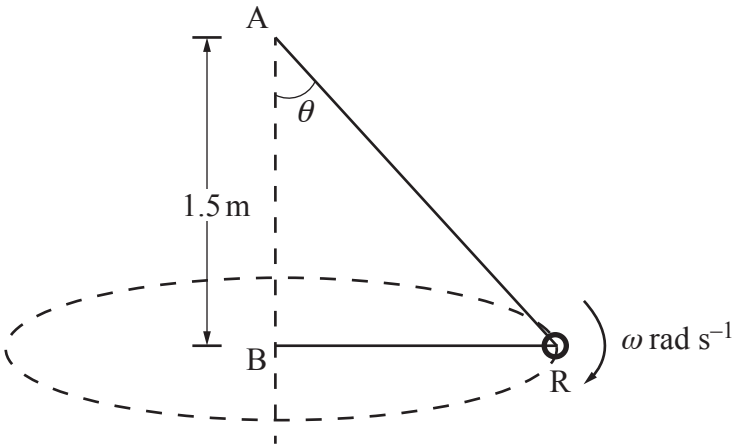
$\tan\beta = \mu = 0.4$

$\beta = 21.8°$

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10(a)



The ring is smooth, so the tension T is the same throughout the string.

Radius of the circle: $r = BR = 1.5 \tan \theta$

Resolve vertically: $T \cos \theta = 2g$

N2L horizontally (towards B): $T \sin \theta + T = 2r\omega^2$

$T(1 + \sin \theta) = 2(1.5 \tan \theta)\omega^2 = 3\omega^2 \tan \theta$

Substitute $T = 2g/\cos \theta$:

$(2g/\cos \theta)(1 + \sin \theta) = 3\omega^2 \sin \theta/\cos \theta$

$2g(1 + \sin \theta) = 3\omega^2 \sin \theta$

$\omega^2 = 2g(1 + \sin \theta)/(3 \sin \theta)$

$\omega^2 = (2g/3)(1 + 1/\sin \theta)$

Since $0 < \theta < 90°$, $0 < \sin \theta < 1$, so $1/\sin \theta > 1$

$\omega^2 > (2g/3)(1 + 1) = 4g/3$

$\omega > \sqrt{4g/3} = 2\sqrt{g/3}$ as required

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10(b)	$\omega^2 = 2g$: $(2g/3)(1 + 1/\sin\theta) = 2g$
	$1 + 1/\sin\theta = 3 \Rightarrow \sin\theta = ½ \Rightarrow \theta = 30°$
	$T = 2g \div \cos\theta = 2g \div \cos 30°$
	$= 2g \div (\sqrt{3}/2)$
	$T = 4g/\sqrt{3}$ N $(= 4\sqrt{3}g/3$ N$)$

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11(a)

The balls are smooth, so the impulse on B acts along the line of centres. B was at rest, so B moves along the line of centres.

B moves directly towards R, i.e. parallel to PQ, so the line of centres must be parallel to j.

11(b)

Perpendicular to the line of centres (i): no impulse, so W keeps 6.

Along the line of centres (j): W has 8 before, B has 0.

After: W has j-component w, B has velocity bj (equal masses m).

CLM (j): $8m = mw + mb \Rightarrow w + b = 8$

NEL: $b - w = 0.6 \times (8 - 0) = 4.8$

Adding: $2b = 12.8 \Rightarrow b = 6.4, \quad w = 1.6$

Velocity of W after the collision = $(6\mathbf{i} + 1.6\mathbf{j})$ m s^{-1}

Angle of W with i before: $\tan^{-1}(8/6) = 53.130...°$

Angle of W with i after: $\tan^{-1}(1.6/6) = 14.931...°$

Deflection = $53.130...° - 14.931...° = 38.198...°$

Angle of deflection = 38° (nearest degree)

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12(a)

Take O as the zero level of GPE. At depth $h > a$, extension $= h - a$.

At O: KE $= \frac{1}{2}m(\sqrt{4ga})^2 = 2mga$, GPE $= 0$, EPE $= 0$

At depth h: KE $= \frac{1}{2}mv^2$, GPE $= -mgh$, EPE $= 4mg(h-a)^2/(2a)$

Conservation of energy:

$2mga = \frac{1}{2}mv^2 - mgh + 2mg(h-a)^2/a$

$v^2 = 4ga + 2gh - 4g(h^2 - 2ah + a^2)/a$

$v^2 = 4ga + 2gh - 4gh^2/a + 8gh - 4ga$

$v^2 = 10gh - 4gh^2/a$

$v = \sqrt{10gh - 4gh^2/a}$ as required

12(b)

Greatest distance when $v = 0$: $10gh - 4gh^2/a = 0$

$2gh(5 - 2h/a) = 0$, $h \neq 0 \Rightarrow h = 5a/2$

Greatest distance below O $= 5a/2$ m

12(c)

Speed is greatest when the acceleration is zero (tension $= mg$):

$4mg(h-a)/a = mg \Rightarrow h - a = \frac{1}{4}a \Rightarrow h = 5a/4$

$v^2 = 10g \times (5a/4) - (4g/a) \times (25a^2/16)$

$v^2 = 25ga/2 - 25ga/4 = 25ga/4$

Maximum speed $= (5/2)\sqrt{ga}$ m s^{-1}

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12(d)

Energy is conserved, so P passes up through O with speed $\sqrt{4ga}$.

If the string stayed slack: $\frac{1}{2}m(4ga) = mgH \Rightarrow H = 2a > a$, so the string is taut again at the highest point, height X above O:

$2mga = mgX + 4mg(X - a)^2/(2a)$

$2a^2 = aX + 2(X - a)^2 \Rightarrow 2X^2 - 3aX = 0$

$X = 3a/2 \quad (X \neq 0)$

Greatest height above O = $3a/2$ m

12(e)

The model assumes Hooke's law holds throughout the motion. The extension (up to 1.5a) may exceed the elastic limit of the string, and air resistance (energy lost) would also reduce the distance.

13(a)

$x = 30 \cos\theta \times t$

$y = 30 \sin\theta \times t - \frac{1}{2} \times 10 \times t^2 = 30t \sin\theta - 5t^2$

$t = x/(30 \cos\theta)$

$y = 30 \sin\theta \times x/(30 \cos\theta) - 5(x/(30 \cos\theta))^2$

$y = x \tan\theta - x^2/(180 \cos^2\theta)$ and $1/\cos^2\theta = 1 + \tan^2\theta$

$y = x \tan\theta - x^2(1 + \tan^2\theta)/180$

At the ground, $x = X$ and $y = -h$:

$-h = X \tan\theta - X^2(1 + \tan^2\theta)/180$

$\times 180$: $-180h = 180X \tan\theta - X^2 - X^2\tan^2\theta$

$X^2\tan^2\theta - 180X \tan\theta + X^2 - 180h = 0$ as required

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13(b)

This is a quadratic in tan θ. For real values of tan θ,

discriminant ≥ 0: $(-180X)^2 - 4X^2(X^2 - 180h) \geq 0$

$32400X^2 - 4X^4 + 720hX^2 \geq 0$

$\div 4X^2$: $8100 - X^2 + 180h \geq 0$

$X^2 \leq 8100 + 180h$

Maximum value: $X^2 = 8100 + 180h$ $(p = 8100, q = 180)$

13(c)

Any point (x, y) the ball passes through can be treated as a landing point with $X = x$ and $h = -y$. The bounding curve is where X^2 takes its maximum value:

$x^2 = 8100 + 180(-y)$

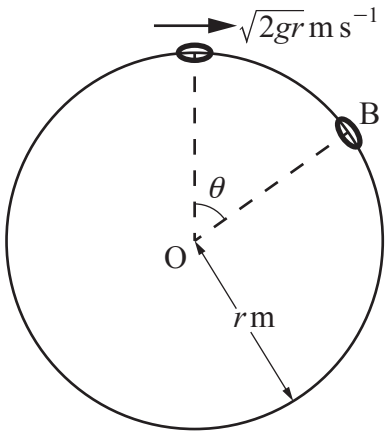
$y = -(1/180)x^2 + 45$

$(k_1 = -1/180, \ k_2 = 45)$

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14(a)



Energy from the top to angle θ (drop in height $r - r\cos\theta$):

$\frac{1}{2}mv^2 = \frac{1}{2}m(\sqrt{2gr})^2 + mgr(1 - \cos\theta)$

$v^2 = 2gr + 2gr - 2gr\cos\theta = 4gr - 2gr\cos\theta$

Radial acceleration v^2/r acts towards O; its downward component is $(v^2/r)\cos\theta$.

Tangentially (wire smooth, normal force radial): $m\,dv/dt = mg\sin\theta$

$dv/dt = g\sin\theta$; its downward component is $g\sin\theta \times \sin\theta$.

Downward acceleration $= (v^2/r)\cos\theta + g\sin^2\theta$

$= (4g - 2g\cos\theta)\cos\theta + g(1 - \cos^2\theta)$

$= 4g\cos\theta - 2g\cos^2\theta + g - g\cos^2\theta$

$= g(1 + 4\cos\theta - 3\cos^2\theta)$ as required

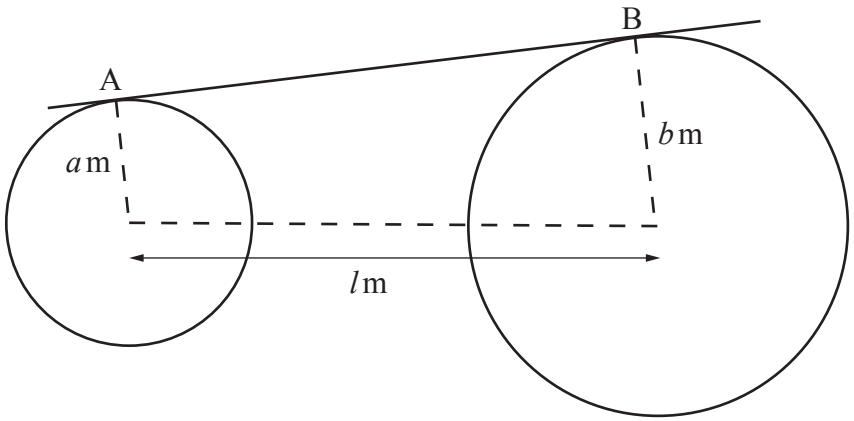
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14(b)	Let $a_v = g(1 + 4\cos\theta - 3\cos^2\theta)$
	$da_v/d\theta = g(-4\sin\theta + 6\cos\theta\sin\theta) = 2g\sin\theta(3\cos\theta - 2)$
	$da_v/d\theta = 0 \Rightarrow \sin\theta = 0$ or $\cos\theta = 2/3$
	$\theta = 0$ gives $a_v = 2g$; $\theta = 180°$ gives $a_v = -6g$
	$\cos\theta = 2/3$ gives $a_v = g(1 + 8/3 - 4/3) = 7g/3$, the maximum
	At $\cos\theta = 2/3$: $v^2 = 4gr - 2gr \times 2/3 = 8gr/3$
	Let R be the force of the wire on the bead, towards O.
	N2L radially (towards O): $mg\cos\theta + R = mv^2/r$
	$2mg/3 + R = 8mg/3$
	$R = 2mg$
	By Newton's third law, the force of the bead on the wire
	has magnitude 2mg N.

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15



The radii to A and B are both perpendicular to the rod, so

$AB^2 + (b - a)^2 = l^2 \Rightarrow AB = \sqrt{l^2 - (b - a)^2}$

Let θ be the angle of the rod to the horizontal (line of centres):

$\sin\theta = (b - a)/l \Rightarrow \tan\theta = (b - a)/\sqrt{l^2 - (b - a)^2}$

The rod is about to slip down along its length, so the friction at A and B acts up the rod and is limiting at both:

$F_A = \mu R_A$, $F_B = \mu R_B$. Let the weight be W.

Resolve perpendicular to the rod: $R_A + R_B = W\cos\theta$

Resolve parallel to the rod: $F_A + F_B = W\sin\theta$

$\mu R_A + \mu R_B = W\sin\theta$

$\mu W\cos\theta = W\sin\theta$

$\mu = \tan\theta$

$\mu = (b - a)/\sqrt{l^2 - (b - a)^2}$

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EXTRA ANSWER SPACE

If you need extra space use these lined pages. You must write the question numbers clearly in the margin.

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