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Friday 7 June 2024 – Afternoon

A Level Further Mathematics B (MEI)

Y431/01 Mechanics Minor

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space, use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 A car of mass 1500 kg travels along a horizontal straight road. There are no resistances to the car's motion. The power developed by the car as it increases its speed from $20\,\text{m s}^{-1}$ to $30\,\text{m s}^{-1}$ over t seconds is a constant 5000 W.

(a) Determine the value of t. **[3]**

(b) Find the acceleration of the car when its speed is $25\,\text{m s}^{-1}$. **[2]**

2 **(a)** State the dimensions of force. **[1]**

Use the following metric-imperial conversion factors for the rest of this question.

- 1 kg = 2.2 lb (pounds)
- 1 m = 39.4 in (inches)

A unit of force used in the imperial system is the pound-force (lbf). 1 lbf is defined as the gravitational force exerted on 1 lb on the surface of the Earth.

(b) Show that 1 lbf is approximately equal to 4.45 N. **[1]**

The pascal (Pa) is a unit of pressure equivalent to 1 Newton per square metre. Pressure can also be measured in pound-force per square inch (psi).

A diver, at a depth of 40 m, experiences a typical pressure of 5×10^5 Pa.

(c) Determine whether this is greater or less than the pressure in a bicycle tyre of 80 psi. **[3]**

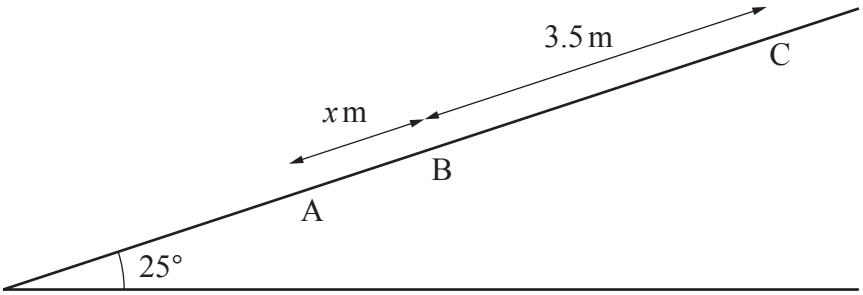
In various physical contexts, energy density is the amount of energy stored in a given region of space per unit volume.

(d) Show that energy density and pressure are dimensionally equivalent. **[3]**

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3 The diagram shows the three points A, B and C that lie along a line of greatest slope on a rough plane which is inclined at an angle of 25° to the horizontal.



A block of mass 6 kg is placed at B and is projected up the plane towards C with an initial speed of u m s^{-1}. The block travels 3.5 m before coming instantaneously to rest at C, before sliding back down the plane. When the block is sliding back down the plane it attains its initial speed at A, which lies x m down the plane from B.

It is given that the work done against resistance throughout the motion is 4 joules per metre.

(a) Use an energy method to determine the following.

(i) The value of u **[3]**

(ii) The value of x **[3]**

A student claims that half of the energy lost due to resistances is accounted for by friction between the block and the plane, and the other half by air resistance.

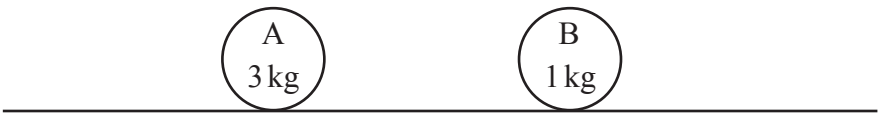
(b) Assuming that the student's claim is correct, determine the coefficient of friction between the block and the plane. **[3]**

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4 **Fig. 4.1** shows two spheres, A and B, on a smooth horizontal surface. Their masses are 3 kg and 1 kg respectively.

Fig. 4.1



Initially, sphere A travels at a speed of $1\,\text{m s}^{-1}$ in a straight line towards B, which is at rest. The spheres collide and the coefficient of restitution between A and B is e.

(a) Show that, after the collision, A has a speed of $\frac{1}{4}(3-e)\,\text{m s}^{-1}$, and find an expression for the speed of B in terms of e. **[4]**

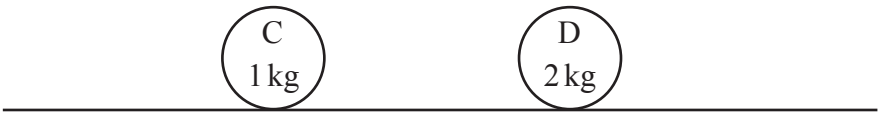
During the collision, the kinetic energy of the system decreases by 21%.

(b) Determine the value of e. **[3]**

(c) State why in part **(a)** it was necessary to assume that A and B have equal radii. **[1]**

Fig. 4.2 shows two spheres, C and D, of equal radii on a smooth horizontal surface. Their masses are 1 kg and 2 kg respectively.

Fig. 4.2



Spheres C and D travel towards each other along the same straight line, C with a speed of $u\,\text{m s}^{-1}$ and D with a speed of $1\,\text{m s}^{-1}$. The spheres collide and during the collision C exerts an impulse on D of magnitude $\frac{2}{3}(u+1)\,\text{N s}$.

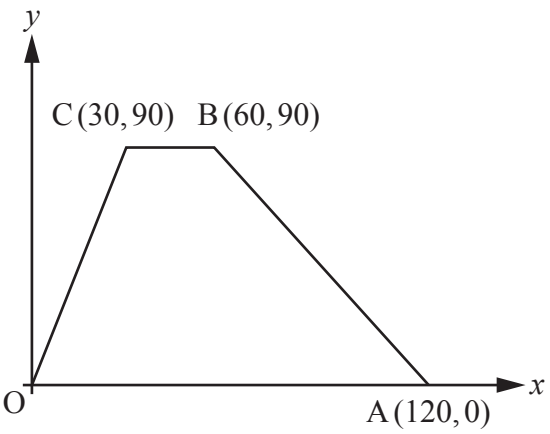
(d) Show that C and D have the same velocity after the collision. **[4]**

(e) Determine the fraction of kinetic energy lost due to the collision between C and D as $u \to \infty$. **[3]**

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5 A uniform lamina OABC is in the shape of a trapezium where O is the origin of the coordinate system in which the points A, B and C have coordinates (120, 0), (60, 90) and (30, 90) respectively (see diagram). The units of the axes are centimetres.



The centre of mass of the lamina lies at $(\overline{x}, \overline{y})$.

(a) Show that $\overline{x} = 54$ and determine the value of $\overline{y}$. **[5]**

The lamina is placed horizontally so that it rests on three supports, whose points of contact are at B, C and D, where D lies on the edge OA and has coordinates $(d, 0)$.

(b) Determine the range of values of d for the lamina to rest in equilibrium. **[3]**

It is now given that $d = 63$, and that the lamina has a weight of 100 N.

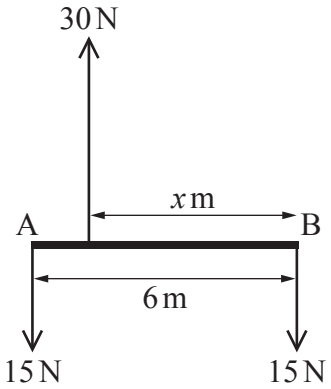
(c) Determine the forces exerted on the lamina by each of the supports at B, C and D. **[4]**

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6 **Fig. 6.1** shows three forces of magnitude 15 N, 15 N and 30 N acting on a rigid beam AB of length 6 m. One of the forces of magnitude 15 N acts at A, and the other force of magnitude 15 N acts at B. The force of magnitude 30 N acts at distance of x m from B. All three forces act in a direction perpendicular to the beam as shown in **Fig. 6.1**. The beam and the three forces all lie in the same horizontal plane. The three forces form a couple of magnitude 42 N m in the clockwise direction.

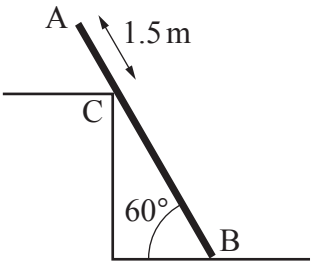
Fig. 6.1



(a) Determine the value of x. **[2]**

Fig. 6.2 shows the same beam, without the three forces from **Fig. 6.1**, resting in limiting equilibrium against a step. The point of contact, C, between the beam and the edge of the step lies 1.5 m from A. The other end of the beam rests on a horizontal floor. The contacts between the beam and both the step and the floor are rough.

Fig. 6.2



It is given that the beam is non-uniform, and that its centre of mass lies $\sqrt{3}$ m from B.

(b) Draw a diagram to show all the forces acting on the beam. **[2]**

The coefficient of friction between the beam and the step and the coefficient of friction between the beam and the floor are the same, and are denoted by μ.

(c) (i) Show that $\mu^2 - 6\mu + 1 = 0$. **[5]**

(ii) Hence determine the value of μ. **[2]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) Gain in KE $= \frac{1}{2} \times 1500 \times (30^2 - 20^2)$

$= 750 \times 500 = 375\,000$ J

Power constant, no resistances, so work done = gain in KE

$5000t = 375\,000$

$t = 75$

1 (b) Driving force $F = \frac{P}{v} = \frac{5000}{25} = 200$ N

N2L: $200 = 1500a$

$a = \frac{200}{1500} = \frac{2}{15} = 0.133\ m\ s^{-2}$

2 (a) $[\text{force}] = MLT^{-2}$

2 (b) 1 lb $= \frac{1}{2.2}$ kg

1 lbf $= \frac{1}{2.2} \times 9.8 = 4.4545...$ N

≈ 4.45 N as required

2 (c) 1 N $= \frac{1}{4.4545}$ lbf, $1\ m^2 = 39.4^2\ in^2$

5×10^5 Pa $= \frac{5 \times 10^5}{4.4545 \times 39.4^2}$ psi

Question number

$= 72.3$ psi

72.3 psi < 80 psi, so the pressure on the diver is less than the pressure in the bicycle tyre.

2 (d) $[\text{pressure}] = \frac{[\text{force}]}{[\text{area}]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

$[\text{energy}] = [\frac{1}{2}mv^2] = M(LT^{-1})^2 = ML^2T^{-2}$

$[\text{energy density}] = \frac{[\text{energy}]}{[\text{volume}]} = \frac{ML^2T^{-2}}{L^3}$

$= ML^{-1}T^{-2}$

Same dimensions as pressure, so they are dimensionally equivalent.

3 (a)(i) B to C: initial KE = gain in GPE + work done against resistance

$\frac{1}{2} \times 6u^2 = 6 \times 9.8 \times 3.5 \sin 25° + 4 \times 3.5$

$3u^2 = 86.975... + 14 = 100.975...$

$u^2 = 33.658...$

$u = 5.80$

3 (a)(ii) C to A: distance $(3.5 + x)$ m down the plane, KE at A = KE at B

loss in GPE − work against resistance = KE at A

$6 \times 9.8(3.5 + x) \sin 25° - 4(3.5 + x) = \frac{1}{2} \times 6 \times 5.8016^2$

$(3.5 + x)(24.850... - 4) = 100.975...$

$3.5 + x = \frac{100.975...}{20.850...} = 4.8429...$

Question number

$x = 1.34$

3 (b)

Resistance is 4 N in total, so friction $F = 2$ N

Perpendicular to plane: $R = 6g \cos 25° = 53.291...$ N

$F = \mu R \Rightarrow \mu = \dfrac{2}{6 \times 9.8 \cos 25°}$

$\mu = 0.0375$

4 (a)

Let velocities after collision be v_A and v_B in direction AB.

COLM: $3(1) + 1(0) = 3v_A + v_B \Rightarrow 3v_A + v_B = 3$

NEL: $\dfrac{v_B - v_A}{1 - 0} = e \Rightarrow v_B = v_A + e$

$3v_A + v_A + e = 3$

$4v_A = 3 - e \Rightarrow v_A = \frac{1}{4}(3 - e)$

So A has speed $\frac{1}{4}(3 - e)$ m s^{-1} as required

$v_B = \frac{1}{4}(3 - e) + e = \frac{3}{4}(1 + e)$

B has speed $\frac{3}{4}(1 + e)$ m s^{-1}

4 (b)

Initial KE $= \frac{1}{2} \times 3 \times 1^2 = 1.5$ J

Final KE $= 0.79 \times 1.5 = 1.185$ J

$\dfrac{1}{2} \times 3 \times \dfrac{(3 - e)^2}{16} + \dfrac{1}{2} \times 1 \times \dfrac{9(1 + e)^2}{16} = 1.185$

$3(3 - e)^2 + 9(1 + e)^2 = 37.92$

Question number

$27 - 18e + 3e^2 + 9 + 18e + 9e^2 = 37.92$

$12e^2 = 1.92 \Rightarrow e^2 = 0.16$

$e = 0.4$

4 (c) If the radii were different, the line of centres would not be along the direction of motion, so the impact would not be direct (the spheres would not move along the same line after).

4 (d) Take direction C to D as positive; let velocities after be v_C, v_D.

Impulse on D: $\frac{2}{3}(u + 1) = 2v_D - 2(-1)$

$2v_D = \frac{2}{3}u + \frac{2}{3} - 2 \Rightarrow v_D = \frac{1}{3}u - \frac{2}{3}$

Impulse on C is $-\frac{2}{3}(u + 1)$:

$-\frac{2}{3}(u + 1) = 1 \times v_C - 1 \times u$

$v_C = u - \frac{2}{3}u - \frac{2}{3} = \frac{1}{3}u - \frac{2}{3}$

$v_C = v_D = \frac{1}{3}(u - 2)$, so C and D have the same velocity after the collision as required

4 (e) KE before $= \frac{1}{2} \times 1 \times u^2 + \frac{1}{2} \times 2 \times 1^2 = \frac{1}{2}u^2 + 1$

KE after $= \frac{1}{2} \times 3 \times (\frac{u - 2}{3})^2 = \frac{(u - 2)^2}{6}$

Question number

Fraction remaining $= \frac{(u-2)^2}{6} \div \frac{u^2+2}{2} = \frac{u^2-4u+4}{3u^2+6}$

Dividing top and bottom by u^2:

Fraction remaining $= (1 - 4/u + 4/u^2) \div (3 + 6/u^2)$

As $u \to \infty$, $u^{-1} \to 0$ and $u^{-2} \to 0$, so fraction remaining $\to \frac{1}{3}$

So fraction of KE lost $\to 1 - \frac{1}{3} = \frac{2}{3}$

5 (a) Split into: triangle O(0,0), (30,0), C; rectangle $30 \leqslant x \leqslant 60$;

triangle (60,0), A, B

Areas: $\frac{1}{2} \times 30 \times 90 = 1350$, $30 \times 90 = 2700$, $\frac{1}{2} \times 60 \times 90 = 2700$

Total area $= \frac{1}{2}(30 + 120) \times 90 = 6750$

Centres of mass: (20, 30), (45, 45), (80, 30)

$$6750\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 1350\begin{pmatrix}20\\30\end{pmatrix} + 2700\begin{pmatrix}45\\45\end{pmatrix} + 2700\begin{pmatrix}80\\30\end{pmatrix}$$

$6750\bar{x} = 27\,000 + 121\,500 + 216\,000 = 364\,500$

$\bar{x} = \frac{364\,500}{6750} = 54$ as required

$6750\bar{y} = 40\,500 + 121\,500 + 81\,000 = 243\,000$

$\bar{y} = \frac{243\,000}{6750} = 36$

5 (b) For equilibrium G(54, 36) must lie within triangle BCD;

extreme cases are G on BD or G on CD.

Question number

G on BD: $\frac{36-0}{54-d} = \frac{90-36}{60-54} = 9$

$36 = 486 - 9d \Rightarrow d = 50$

G on CD: $\frac{36-0}{54-d} = \frac{90-36}{30-54} = -\frac{9}{4}$

$144 = -486 + 9d \Rightarrow d = 70$

So $50 \leq d \leq 70$

5 (c) Let the support forces be R_B, R_C, R_D (N), all vertical.

Vertically: $R_B + R_C + R_D = 100$

Moments about line BC ($y = 90$):

$90R_D = 100 \times (90 - 36) = 5400 \Rightarrow R_D = 60$

Moments about the line $x = 30$ (through C):

$30R_B + 33R_D = 100 \times 24$

$30R_B = 2400 - 1980 = 420 \Rightarrow R_B = 14$

$R_C = 100 - 60 - 14 = 26$

Forces: at B 14 N, at C 26 N, at D 60 N (all vertically upwards)

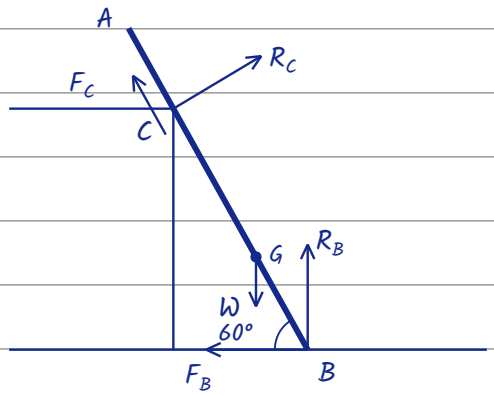
6 (a) Moments about B (clockwise = couple):

$30x - 15 \times 6 = 42$

$30x = 132 \Rightarrow x = 4.4$

Question number

6 (b)



6 (c)(i) $BC = 6 - 1.5 = 4.5$ m; R_C is perpendicular to the beam.

Moments about B: $R_C \times 4.5 = W \times \sqrt{3} \cos 60°$

$4.5R_C = \frac{\sqrt{3}}{2}W \Rightarrow R_C = \frac{\sqrt{3}}{9}W$

Horizontally: $R_C \cos 30° = F_C \cos 60° + F_B$

Vertically: $R_B + R_C \sin 30° + F_C \sin 60° = W$

Limiting at both contacts: $F_C = \mu R_C$, $F_B = \mu R_B$

Horizontally: $\frac{\sqrt{3}}{2}R_C = \frac{1}{2}\mu R_C + \mu R_B$

$\Rightarrow \mu R_B = \frac{1}{2}R_C(\sqrt{3} - \mu)$

Vertically $\times \mu$: $\mu R_B = \mu W - \frac{1}{2}\mu R_C - \frac{\sqrt{3}}{2}\mu^2 R_C$

So $\frac{1}{2}R_C(\sqrt{3} - \mu) = \mu W - \frac{1}{2}\mu R_C(1 + \sqrt{3}\mu)$

Substitute $R_C = \frac{\sqrt{3}}{9}W$ and multiply by $\frac{18}{W}$:

$\sqrt{3}(\sqrt{3} - \mu) = 18\mu - \sqrt{3}\mu(1 + \sqrt{3}\mu)$

Question number

$3 - \sqrt{3}\mu = 18\mu - \sqrt{3}\mu - 3\mu^2$

$3\mu^2 - 18\mu + 3 = 0$

$\mu^2 - 6\mu + 1 = 0$ as required

6 (c)(ii) $\mu = \frac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm 2\sqrt{2}$

$\mu = 0.172$ or $\mu = 5.83$

From vertical: $R_B = W - \frac{1}{2}R_C(1 + \sqrt{3}\mu) = W\left(1 - \frac{\sqrt{3} + 3\mu}{18}\right)$

If $\mu = 3 + 2\sqrt{2}$: $\frac{\sqrt{3} + 3\mu}{18} = 1.07 > 1$, so $R_B < 0$,

which is impossible, so reject this root.

$\mu = 3 - 2\sqrt{2} = 0.172$