OCR
Oxford Cambridge and RSA

Thursday 22 June 2023 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

9972798312

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2023 [A/508/5598]
DC (PQ/JG) 326282/6

OCR is an exempt Charity

stemup.co.uk

1 You are given that $(x_1, y_1) = (0.9, 2.3)$ and $(x_2, y_2) = (1.1, 2.7)$.

The values of x_1 and x_2 have been **rounded** to **1** decimal place.

(a) Determine the range of possible values of $x_2 - x_1$. **[2]**

The values of y_1 and y_2 have been **chopped** to **1** decimal place.

(b) Determine the range of possible values of $y_2 - y_1$. **[2]**

You are given that $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

(c) Determine the range of possible values of m. **[2]**

(d) Explain why your answer to part **(c)** is much larger than your answer to part **(a)** and your answer to part **(b)**. **[1]**

2 A car tyre has a slow puncture. Initially the tyre is inflated to a pressure of 34.5 psi. The pressure is checked after 3 days and then again after 5 days. The time t in days and the pressure, P psi, are shown in the table below. You are given that the pressure in a car tyre is measured in pounds per square inch (psi).

t	0	3	5
P	34.5	29.4	27.0

The owner of the car believes the relationship between P and t may be modelled by a polynomial.

(a) Explain why it is not possible to use Newton's forward difference interpolation method for these data. **[1]**

(b) Use Lagrange's form of the interpolating polynomial to find an interpolating polynomial of degree 2 for these data. **[4]**

The car owner uses the polynomial found in part **(b)** to model the relationship between P and t.

Subsequently it is found that when $t = 6$, $P = 26.0$ and when $t = 10$, $P = 24.4$.

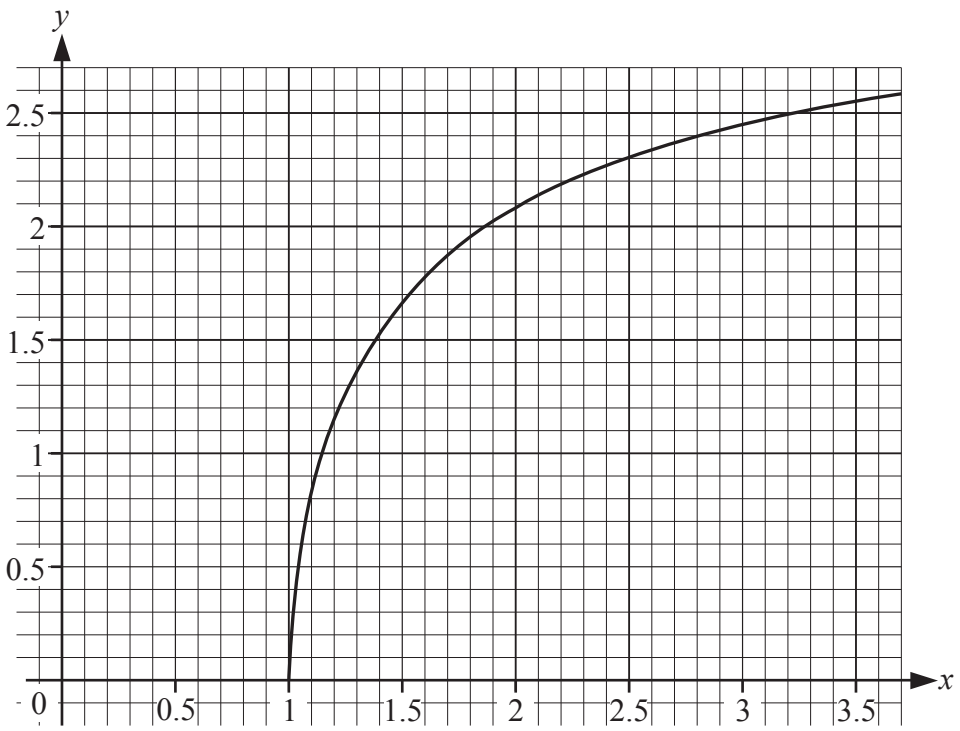
(c) Determine whether the owner's model is a good fit for these data. **[2]**

(d) Explain why the model would not be suitable in the long term. **[1]**

© OCR 2023

stemup.co.uk

3 The diagram shows the graph of $y = f(x)$ for values of x from 1 to 3.5.



The table shows some values of x and the associated values of y.

x	1.5	2	2.5
y	1.682137	2.094395	2.318559

(a) Use the forward difference method to calculate an approximation to $\frac{dy}{dx}$ at $x = 2$. **[2]**

(b) Use the central difference method to calculate an approximation to $\frac{dy}{dx}$ at $x = 2$. **[2]**

(c) On the copy of the diagram in the Printed Answer Booklet, show how the central difference method gives the approximation to $\frac{dy}{dx}$ at $x = 2$ which was found in part **(b)**. **[1]**

(d) Explain whether your answer to part **(a)** or your answer to part **(b)** is likely to give a better approximation to $\frac{dy}{dx}$ at $x = 2$. **[1]**

© OCR 2023

stemup.co.uk

4 A spreadsheet is used to approximate $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$ using the midpoint rule with 1 strip.

The output is shown in the table below.

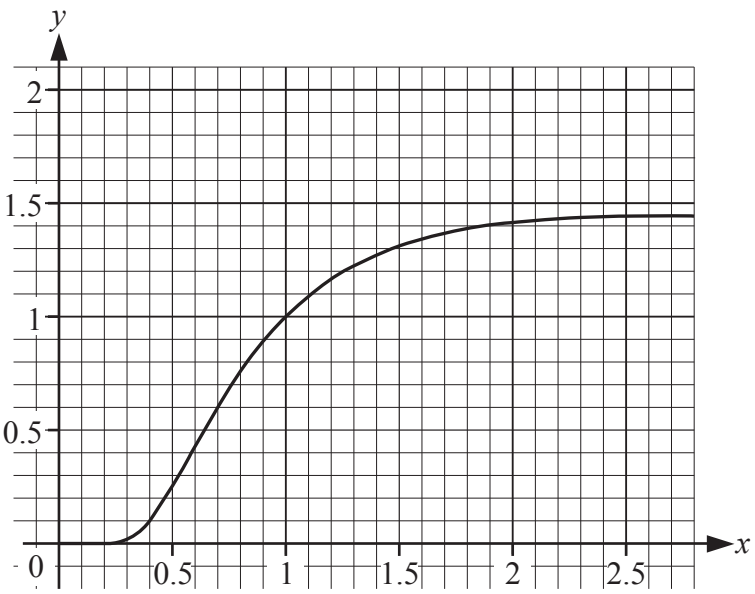
	B	C	D
3	x	$\mathrm{f}(x)$	M_1
4	1.5	1.3103707	0.65518535

The formula in cell C4 is `=B4^(1/B4)` .

The formula in cell D4 is `=0.5*C4` .

(a) Write the integral in standard mathematical notation. **[3]**

A graph of $y = \mathrm{f}(x)$ is included in the diagram below.



(b) Explain whether 0.65518535 is an over-estimate or an under-estimate of $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$. **[1]**

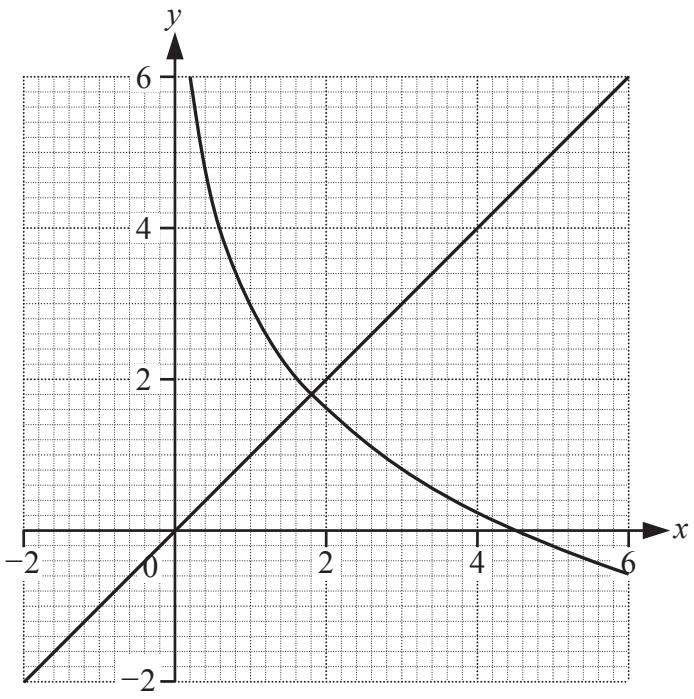
© OCR 2023

stemup.co.uk

5 The equation $3-2\ln x-x=0$ has a root near $x=1.8$.

A student proposes to use the iterative formula $x_{n+1}=\mathrm{g}(x_n)=3-2\ln x_n$ to find this root.

The diagram shows the graphs of $y=x$ and $y=\mathrm{g}(x)$ for values of x from -2 to 6.



(a) With reference to the graph, explain why it might not be possible to use the student's iterative formula to find the root near $x=1.8$. **[1]**

(b) Use the relaxed iteration $x_{n+1}=\lambda \mathrm{g}(x_n)+(1-\lambda)x_n$, with $\lambda=0.475$ and $x_0=2$, to determine the root correct to **6** decimal places. **[3]**

A student uses the same relaxed iteration with the same starting value. Some analysis of the iterates is carried out using a spreadsheet, which is shown in the table below.

r	difference	ratio
0		
1	−0.1834898	
2	−0.0049137	0.02678
3	−6.44E-06	0.00131
4	−3.862E-09	0.0006
5	−2.313E-12	0.0006

(c) Explain what the analysis tells you about the order of convergence of this sequence of approximations. **[2]**

stemup.co.uk

6 **(a)** **(i)** Calculate the relative error when π is **chopped** to **2** decimal places in approximating $\pi^2 + 2$. **[2]**

(ii) **Without** doing any calculation, explain whether the relative error would be the same when π is **chopped** to **2** decimal places when approximating $(\pi + 2)^2$. **[1]**

The table shows some spreadsheet output. The values of x in column A are exact.

	A	B	C
1	x	10^x	$\log_{10} 10^x$
2	1E-12	1	1.00001E-12
3	1E-11	1	9.99998E-12

The formula in cell B2 is `=10^A2` .

This has been copied down to cell B3.

The formula in cell C2 is `=LOG(B2)` .

This formula has been copied down to cell C3.

(b) **(i)** Write the value displayed in cell C2 in standard mathematical notation. **[1]**

(ii) Explain why the values in cells C2 and C3 are neither zero nor the same as the values in cells A2 and A3 respectively. **[2]**

© OCR 2023

stemup.co.uk

7 The value of a function, $y = \mathrm{f}(x)$, and its gradient function, $\frac{\mathrm{d}y}{\mathrm{d}x}$, when $x = 2$, is given in **Table 7.1**.

Table 7.1

x	$\mathrm{f}(x)$	$\frac{\mathrm{d}y}{\mathrm{d}x}$
2	6	-2.8

(a) Determine the approximate value of the error when $\mathrm{f}(2)$ is used to estimate $\mathrm{f}(2.03)$. **[2]**

The Newton-Raphson method is used to find a sequence of approximations to a root, α, of the equation $\mathrm{f}(x) = 0$. The spreadsheet output showing the iterates, together with some further analysis, is shown in **Table 7.2**.

Table 7.2

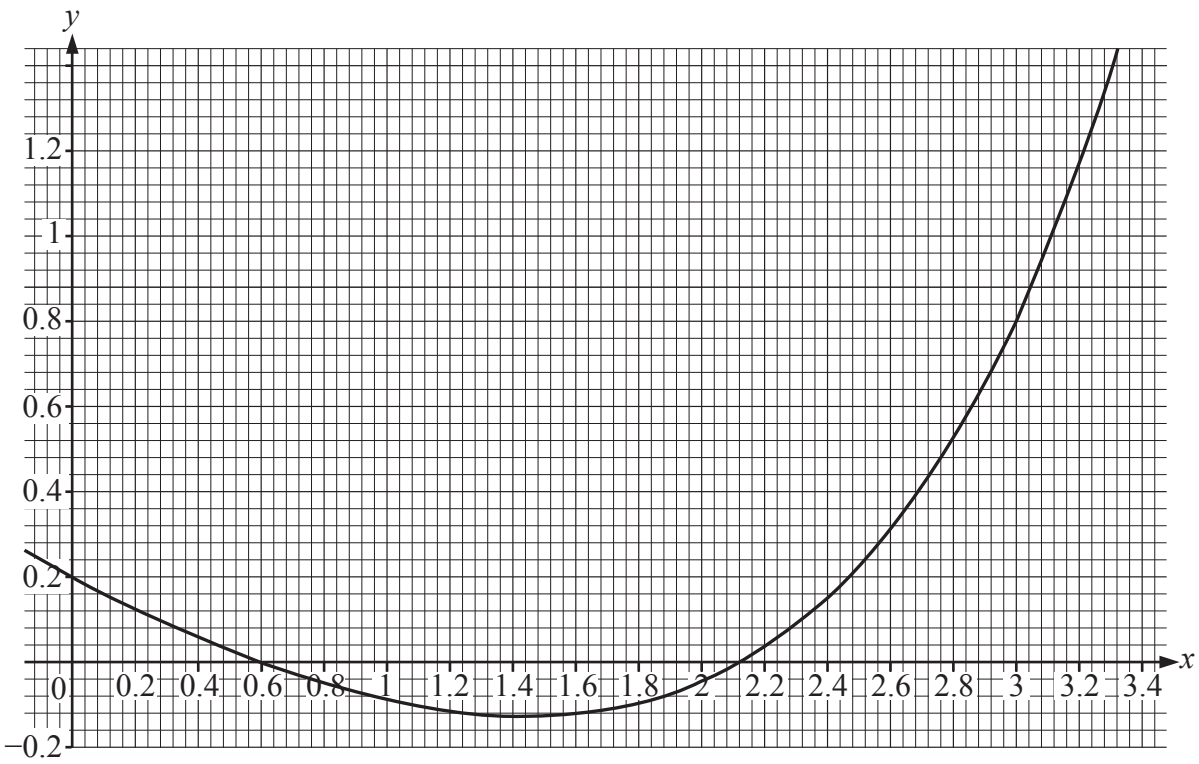
	A	B	C	D
1	r	x_r	difference	ratio
2	0	12		
3	1	−13.1165572	−25.1165572	
4	2	1.76283279	14.87939004	−0.5924136
5	3	2.18052157	0.41768878	0.02807163
6	4	2.182419024	0.001897454	0.00454275
7	5	2.182419066	4.13985E-08	2.1818E-05

(b) (i) Explain what the values in column D tell you about the order of convergence of this sequence of approximations. **[2]**

(ii) **Without** doing any further calculation, state the value of α as accurately as you can, justifying the precision quoted. **[2]**

stemup.co.uk

8 The graph of $y = 0.2\cosh x - 0.4x$ for values of x from 0 to 3.32 is shown on the graph below.



The equation $0.2\cosh x - 0.4x = 0$ has two roots, α and β where $\alpha < \beta$, in the interval $0 < x < 3$. The secant method with $x_0 = 1$ and $x_1 = 2$ is to be used to find β.

(a) On the copy of the graph in the Printed Answer Booklet, show how the secant method works with these two values of x to obtain an improved approximation to β. **[1]**

© OCR 2023

stemup.co.uk

The spreadsheet output in the table below shows the result of applying the secant method with $x_0 = 1$ and $x_1 = 2$.

	I	J	K	L	M
2	r	x_r	$f(x_r)$	x_{r+1}	$f(x_{r+1})$
3	0	1	−0.0914	2	−0.0476
4	1	2	−0.0476	3.08529	0.95784
5	2	3.08529	0.95784	2.05134	−0.0298
6	3	2.05134	−0.0298	2.08259	−0.0181
7	4	2.08259	−0.0181	2.13042	0.00155
8	5	2.13042	0.00155	2.12664	−7E-05

(b) Write down a suitable cell formula for cell J4. **[1]**

(c) Write down a suitable cell formula for cell L4. **[2]**

(d) Write down the most accurate approximation to β which is displayed in the table. **[1]**

(e) Determine whether your answer to part **(d)** is correct to **5** decimal places. You should **not** calculate any more iterates. **[2]**

(f) It is decided to use the secant method with starting values $x_0 = 1$ and $x_1 = a$, where $a > 1$, to find α. State a suitable value for a. **[1]**

stemup.co.uk

9 The trapezium rule is used to calculate 3 approximations to $\int_0^1 \sqrt[3]{\sinh(x)}\,\mathrm{d}x$ with 1, 2 and 4 strips respectively. The results are shown in **Table 9.1**.

Table 9.1

n	T_n
1	0.52764369
2	0.66617652
4	0.72534275

(a) **Use these results** to determine **two** approximations to $\int_0^1 \sqrt[3]{\sinh(x)}\,\mathrm{d}x$ using Simpson's rule. **[2]**

(b) Use your answers to part **(a)** to state the value of $\int_0^1 \sqrt[3]{\sinh(x)}\,\mathrm{d}x$ as accurately as you can, justifying the precision quoted. **[1]**

Table 9.2 shows some further approximations found using the trapezium rule, together with some analysis of these approximations.

Table 9.2

n	T_n	difference	ratio
1	0.5276437		
2	0.6661765	0.138533	
4	0.7253427	0.059166	0.42709
8	0.7498821	0.024539	0.41475
16	0.7598858	0.010004	0.40766
32	0.7639221	0.004036	0.40348
64	0.7655404	0.001618	0.40095

(c) Explain what can be deduced about the order of the method in this case. **[2]**

(d) Use extrapolation to obtain the value of $\int_0^1 \sqrt[3]{\sinh(x)}\,\mathrm{d}x$ as accurately as you can, justifying the precision quoted. **[4]**

END OF QUESTION PAPER

© OCR 2023

stemup.co.uk

BLANK PAGE

© OCR 2023

stemup.co.uk

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2023

stemup.co.uk

Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) $1.05 \leq x_2 < 1.15$ and $0.85 \leq x_1 < 0.95$

Largest: $1.15 - 0.85 = 0.3$

Smallest: $1.05 - 0.95 = 0.1$

$\therefore 0.1 < x_2 - x_1 < 0.3$

1 (b) $2.7 \leq y_2 < 2.8$ and $2.3 \leq y_1 < 2.4$

Largest: $2.8 - 2.3 = 0.5$

Smallest: $2.7 - 2.4 = 0.3$

$\therefore 0.3 < y_2 - y_1 < 0.5$

1 (c) Largest $m = \dfrac{\text{largest numerator}}{\text{smallest denominator}} = \dfrac{0.5}{0.1} = 5$

Smallest $m = \dfrac{\text{smallest numerator}}{\text{largest denominator}} = \dfrac{0.3}{0.3} = 1$

$\therefore 1 < m < 5$

1 (d) The denominator $x_2 - x_1$ is a subtraction of two nearly equal quantities, so it has a large relative error, and dividing by this small, uncertain number makes the range of m much larger.

2 (a) The values of t (0, 3, 5) are not equally spaced, and Newton's forward difference method needs equal steps in t.

Question number

2 (b)

$$P = 34.5 \times \frac{(t-3)(t-5)}{(0-3)(0-5)} + 29.4 \times \frac{(t-0)(t-5)}{(3-0)(3-5)}$$

$$+ 27.0 \times \frac{(t-0)(t-3)}{(5-0)(5-3)}$$

$$= 2.3(t^2 - 8t + 15) - 4.9(t^2 - 5t) + 2.7(t^2 - 3t)$$

$$= (2.3 - 4.9 + 2.7)t^2 + (-18.4 + 24.5 - 8.1)t + 34.5$$

$$P = 0.1t^2 - 2t + 34.5$$

2 (c)

$t = 6$: $P = 0.1(36) - 12 + 34.5 = 26.1$ (actual 26.0)

$t = 10$: $P = 0.1(100) - 20 + 34.5 = 24.5$ (actual 24.4)

Both predictions are within 0.1 psi of the measured values, so the model is a good fit for these data.

2 (d)

The quadratic has a minimum at $t = 10$, so after that the model predicts the pressure increases ($P \to \infty$ as $t \to \infty$), which cannot happen with a puncture.

3 (a)

$$\frac{dy}{dx} \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{2.318559 - 2.094395}{0.5}$$

$$= 0.448328$$

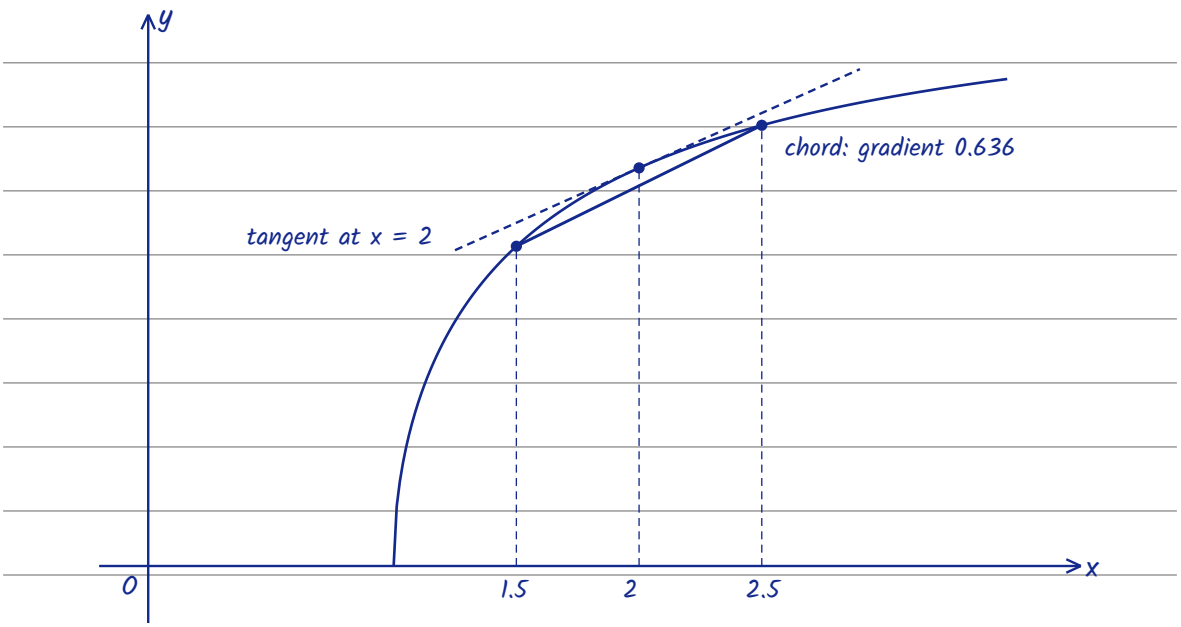
3 (b)

$$\frac{dy}{dx} \approx \frac{f(2.5) - f(1.5)}{2.5 - 1.5} = \frac{2.318559 - 1.682137}{1}$$

$$= 0.636422$$

Question number

3 (c)



The gradient of the chord from $x = 1.5$ to $x = 2.5$ approximates the gradient of the tangent to the curve at $x = 2$.

3 (d) Part (b) is likely to be better: the central difference method is second order whereas the forward difference method is only first order, and it uses points on both sides of $x = 2$ rather than only to the right.

4 (a) $f(x) = x^{1/x}$, midpoint 1.5 and strip width $h = 0.5$

so $a = 1.5 - 0.25 = 1.25$ and $b = 1.5 + 0.25 = 1.75$

$$\int_{1.25}^{1.75} x^{1/x}\,dx$$

4 (b) The curve is concave down (gradient decreasing) for $1.25 \leqslant x \leqslant 1.75$, so the midpoint rule gives an over-estimate.

5 (a) At the intersection of $y = x$ and $y = g(x)$ the gradient of g is steeper than -1 ($|g'(1.8)| = 2/1.8 > 1$), so the iteration

Question number	
	will not converge to the root.
5 (b)	$x_{n+1} = 0.475(3 - 2 \ln x_n) + 0.525x_n$, $x_0 = 2$
	$x_1 = 1.81651018$
	$x_2 = 1.81159653$
	$x_3 = 1.81159009$
	$x_4 = 1.81159008$
	x_3 and x_4 agree to 6 d.p., so root = 1.811590 (6 d.p.)
5 (c)	The ratio of differences appears to be converging to a
	constant (about 0.0006), which suggests first order
	convergence (with a very small ratio, so it is fast).
6 (a)(i)	π chopped to 2 d.p. = 3.14
	$3.14^2 + 2 = 11.8596$, $\pi^2 + 2 = 11.869604...$
	Relative error = $\frac{11.8596 - 11.869604...}{11.869604...}$
	$= -0.000843$ (3 s.f.)
6 (a)(ii)	No, it would be different: the order of operations is
	different (2 is added before squaring), so the error in π
	propagates differently.
6 (b)(i)	1.00001×10^{-12}
6 (b)(ii)	They are not zero because B2 and B3 are not stored as
	exactly 1; they are only displayed as 1 (log 1 would be 0).
	They are not equal to A2 and A3 because the stored values

Question number

of 10^x have rounding errors (and B2 and B3 differ from each other), so slightly different values are used to compute C2 and C3.

7 (a) $f(2 + 0.03) \approx f(2) + 0.03\, f'(2)$

$= 6 + 0.03 \times (-2.8) = 5.916$

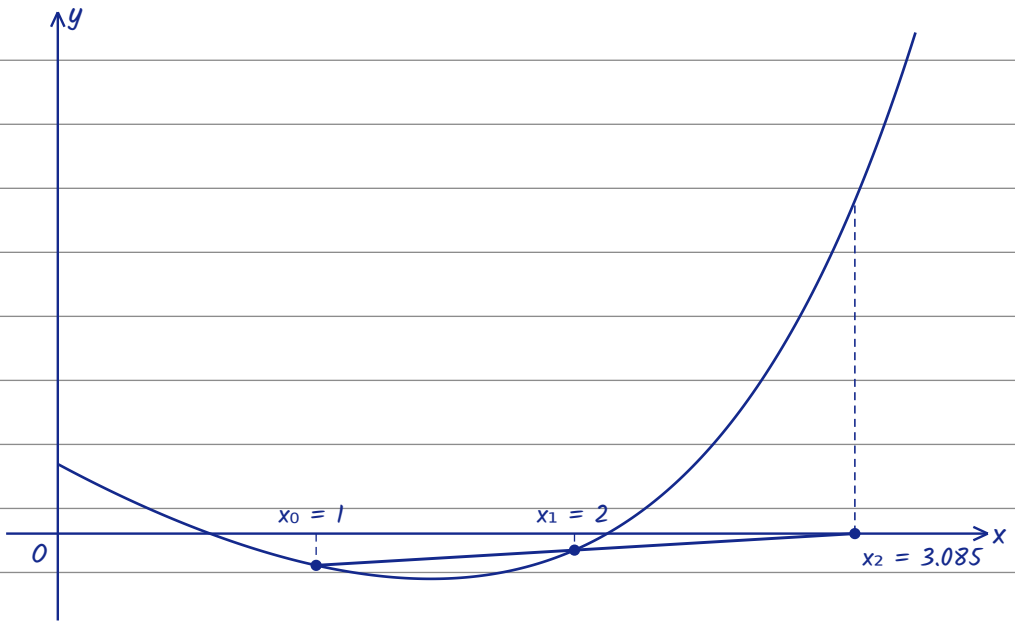
error $\approx 0.03 \times (-2.8) = -0.084$

7 (b)(i) The ratio of differences is decreasing (towards 0), so it is not converging to a constant: the convergence is faster than first order.

7 (b)(ii) $\alpha = 2.182419066$

Since convergence is faster than first order, the last difference (4×10^{-8}) means x_5 is accurate to all the figures shown.

8 (a)



The chord through the points where $x = 1$ and $x = 2$ is extended

Question number	
	to meet the x-axis at $x_2 \approx 3.085$, the next approximation.
8 (b)	= L3
8 (c)	= (J3*M3 − K3*L3)/(M3 − K3)
8 (d)	$\beta \approx 2.12664$
8 (e)	$f(x) = 0.2 \cosh x - 0.4x$ $f(2.126635) = -0.0000704$ $f(2.126645) = -0.0000661$ No change of sign, so β is not in [2.126635, 2.126645]: 2.12664 is not correct to 5 decimal places.
8 (f)	$a = 1.1$ (any value with $1 < a \leq 1.74$ works) $f(1.1) = -0.106 < f(1) = -0.091$, so the chord slopes down and meets the x-axis to the left of $x = 1$, near α.
9 (a)	$S_2 = \frac{4T_2 - T_1}{3} = \frac{4 \times 0.66617652 - 0.52764369}{3} = 0.71235413$ $S_4 = \frac{4T_4 - T_2}{3} = \frac{4 \times 0.72534275 - 0.66617652}{3} = 0.74506483$
9 (b)	0.7, since S_2 and S_4 only agree to 1 d.p.
9 (c)	The ratio of differences seems to be converging to about 0.4. Since $0.25 < 0.4 < 0.5$, the order of the method in this case is between first order and second order.

Question number

9 (d)

Using $r = 0.4$ and the last difference 0.001618:

$$T_\infty \approx 0.7655404 + 0.001618 \times \frac{0.4}{1 - 0.4}$$

$$= 0.7655404 + 0.0010787 = 0.766619$$

Extrapolation is much more accurate than any of the trapezium values, so this is secure to 3 d.p.:

$$\int_0^1 \sqrt[3]{(\sinh x)}\, dx = 0.767 \text{ (3 d.p.)}$$