OCR
Oxford Cambridge and RSA

GCE

Further Mathematics B MEI

Y434/01: Numerical methods

A Level

Mark Scheme for June 2023

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS/A Level Further Mathematics B (MEI)

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it

easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

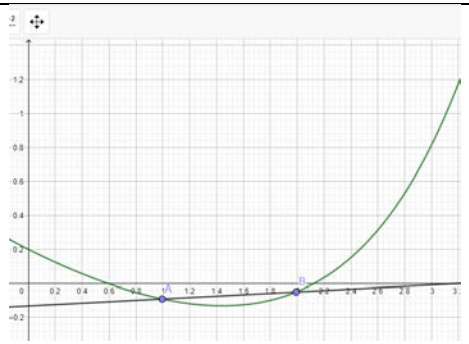
Question			Answer	Marks	AO	Guidance
1	(a)		$1.15 - 0.85$ or $1.05 - 0.95$ $0.1 < x_2 - x_1 < 0.3$ **oe isw**	**M1** **A1**	**1.1a** **1.1**	both values attempted, one must be correct; allow non-strict inequality **or** eg 0.1 to 0.3 **oe** allow **SC1** for correct answer unsupported
				[2]		
1	(b)		$2.8 - 2.3$ *or* $2.7 - 2.4$ $0.3 < y_2 - y_1 < 0.5$ **oe isw**	**M1** **A1**	**1.1a** **1.1**	both values attempted, one must be correct allow non-strict inequality **or** eg 0.3 to 0.5 **oe** allow **SC1** for correct answer unsupported
				[2]		
1	(c)		*their* $\frac{0.5}{0.1}$ **and** *their* $\frac{0.3}{0.3}$ $1 < m < 5$ **cao isw**	**M1** **A1**	**3.1a** **3.2a**	
				[2]		
1	(d)		the **denominator** involves the **subtraction** of **nearly equal** quantities **oe**	**B1**	**2.4**	must refer to **denominator** or to **division by difference** between two **nearly equal** numbers
				[1]		
2	(a)		because the values of ***t*** are not evenly spaced **oe**	**B1**	**1.2**	do not allow eg because the data are not evenly spaced eg because the *x* values are not evenly spaced
				[1]		

Question			Answer	Marks	AO	Guidance
2	(b)		$\frac{(t-0)(t-3)}{(5-0)(5-3)} \times 27.0 + \frac{(t-0)(t-5)}{(3-0)(3-5)} \times 29.4 + \frac{(t-3)(t-5)}{(0-3)(0-5)} \times 34.5$	M1 A1	3.3 1.1	allow sign errors in substitution of *x*-values and one numerical error all substitutions correct
			$0.1t^2 - 2t + 34.5$	A1	1.1	two terms correct
			$P_2(t) = 0.1t^2 - 2t + 34.5$ **or** $P = 0.1t^2 - 2t + 34.5$	A1	1.1	all correct; **A0** if other variable used or *P* omitted; do not allow f(*t*)
				[4]		
2	(c)		*t* = 6 gives *P* = 26.1 ≈ 26.0 so good fit	B1	3.4	if **B0B0** allow **SC1** for 26.1 and 24.5 seen
			t =10 gives *P* =24.5 ≈ 24.4 so good fit	B1	3.4	may see one comment for both comparisons
				[2]		
2	(d)		model predicts tyre pressures will start to increase (after *t* = 10)	B1	3.5b	allow eg as $t \to \infty, P \to \infty$
				[1]		
3	(a)		$\frac{2.318559-2.094395}{2.5-2}$ **oe soi**	M1	1.1	
			0.448328 **isw** or 0.44833 **isw**	A1	1.1	allow **B2** for correct answer unsupported
				[2]		
3	(b)		$\frac{2.318559-1.682137}{2.5-1.5}$ **oe soi**	M1	1.1	
			0.636422 **isw** or 0.63642 **isw**	A1	1.1	allow **B2** for correct answer unsupported
				[2]		

Question			Answer	Marks	AO	Guidance
3	(c)		chord joining point on curve where $x = 1.5$ to point on curve where $x = 2.5$ **and** tangent to the curve at $x = 2$ drawn; [and gradient of chord identified as an approximation to the gradient of the tangent]	**B1**	**1.1**	
				[1]		
3	(d)		central difference method is (usually) a **second order** method whereas forward difference is (usually) a **first order** method, so answer to part (b) probably more accurate *or* central difference method uses ***x*-values** on **both** sides of 2 **oe** whereas forward difference method uses a step in the positive *x*-direction only **oe**	**B1**	**2.2b**	must mention both methods must mention both methods
				[1]		
4	(a)		$x^{\frac{1}{x}}$ seen correct limits identified in integral $\int_{1.25}^{1.75} x^{\frac{1}{x}}\,\mathrm{d}x$	**B1** **B1** **B1**	**1.1** **1.1** **1.1**	not necessarily as an integral all correct if **B1B0B0** allow **B1 SCB1** for $\int_a^b x^{\frac{1}{x}}\,\mathrm{d}x$, where *a* and *b* are numerical values **symmetrical** about 1.5
				[3]		

Question			Answer	Marks	AO	Guidance
4	(b)		the curve is **concave down** so the midpoint rule will give an **over-estimate**	**B1**	**2.4**	**or** the curve is increasing **and** the gradient of the curve is decreasing so the midpoint rule will give an over-estimate
				[1]		
5	(a)		the **gradient** of g(x) at the **root** is less than –1 **oe** so the iteration will not converge	**B1**	**2.4**	must refer to gradient at root or gradient of g(x) at intersection of [y =] x and [y =] g(x); allow eg won't converge because $\lvert g'(1.8)\rvert > 1$
				[1]		
5	(b)		2 1.81651017847 1.81159652649 1.81159008685 1.81159008298 1.811590	**B1** **B1** **B1**	**1.1** **2.1** **2.2a**	1.816510 ... **and** 1.811596 ... **oe** seen 1.8115900 ... **and** 1.8115900 ... **oe** seen dependent on award of first two marks if **B1B0B0** allow **B1 SCB2** for 1.811590 seen twice and 6 dp accuracy confirmed by change of sign
				[3]		
5	(c)		ratio of differences appears to converge to a constant [0.0006] (which suggests) first order convergence	**B1** **B1**	**2.2a** **2.2b**	ignore comments on speed of convergence **B0** if spoiled by incorrect reasoning
				[2]		
6	(a)	(i)	$\frac{(3.14^2+2)-(\pi^2+2)}{(\pi^2+2)}$ or $\frac{(3.14^2+2)}{(\pi^2+2)} - 1$ **oe** **awrt** –0.000 843 **cao**	**M1** **A1**	**1.1** **1.1**	allow omission of 2 in numerator; allow bracket error; may be embedded in modulus **NB** -8.43×10^{-4} allow **B2** for correct answer unsupported

Question			Answer	Marks	AO	Guidance
				[2]		
6	**(a)**	**(ii)**	it would be different as the order of operations is different	**B1**	**2.4**	
				[1]		
6	**(b)**	**(i)**	1.00001×10^{-12}	**B1**	**1.1**	or 0.000 000 000 001 000 01
				[1]		
6	**(b)**	**(ii)**	**not zero** because the value stored in B2 and B3 is not stored as 1, even though they are displayed as 1	**B1**	**2.4**	do not allow just eg the spreadsheet stores values to greater accuracy than it displays
			not equal to the [values in A2 and A3] because the values in B2 and B3 are different to each other (even though they are displayed as 1), so different values are used to compute the values in C2 and C3	**B1**	**2.4**	do not allow just eg the spreadsheet stores values to greater accuracy than it displays
				[2]		
7	**(a)**		use of $f(2 + 0.03) \approx f(2) + 0.03f'(2)$ **soi**	**M1**	**1.2**	may see eg $\pm 2.8 \times 0.03$ may be implied by 5.916
			(error ≈) –0.084 **cao**	**A1**	**1.1**	mark the final answer allow **SC1** for correct final answer unsupported
				[2]		

Question			Answer	Marks	AO	Guidance
7	(b)	(i)	ratio of differences is decreasing (which suggests) convergence is faster than first order; allow (which suggests) convergence is higher than first order	**B1** **B1**	**2.2a** **2.2b**	ignore superfluous comments do not allow if spoiled by incorrect reasoning; do not allow (which suggests) second order convergence do not allow greater than first order convergence if **B0B0** allow **SC1** for ratio of differences not converging to a constant so convergence not first order
				[2]		
7	(b)	(ii)	2.182419066 since convergence is faster than first order / generally second order	**B1** **B1**	**2.2b** **2.4**	allow 2.18241907 or 2.1824191 **or** allow **B1** for 2.182419 and **B1** for eg since x_4 and x_5 agree to this precision **oe**; allow last two estimates or values in B6 and B7
				[2]		
8	(a)			**B1**	**1.1**	chord from curve at $x = 1$ and through the curve at $x = 2$ which joins the x-axis to the right of the root **isw**
				[1]		
8	(b)		= L3	**B1**	**1.1**	
				[1]		

Question			Answer	Marks	AO	Guidance
8	(c)		= (J3*M3 – K3*L3)/(M3 – K3) or = (K3*L3 – J3*M3)/(K3 – M3) **oe**	**M1**	**1.1**	use of secant formula; allow spreadsheet syntax errors but not error in formula **NB or** = (J3*K4 – K3*L3)/(K4 – K3) or = (K3*L3 – J3*K4)/(K3 – K4) **or** (J3*K4 – K3*J4)/(K4 – K3) or = (K3*J4 – J3*K4)/(K3 – K4)
				A1	**1.1**	all correct
				[2]		
8	(d)		2.12664	**B1**	**2.2a**	
				[1]		
8	(e)		f(2.126635) = **awrt** –0.00007 f(2.126645) = **awrt** –0.00007 **no sign change** so **not** correct to 5 dp	**M1** **A1**	**2.1** **2.2a**	both calculated and one correct to 1 sf **NB** –0.000070377215 and –0.000066109812 **M1FT** 2.13042 **awrt** 0.002 in each case **NB** 0.0015487 and 0.001553 both values correct to 1 sf **A1 FT awrt** 0.002 and 0.002 so **no sign change** so **not** correct to 5 dp
				[2]		

Question			Answer	Marks	AO	Guidance
8	(f)		$1 < a \leq 1.74$	B1	3.1a	
				[1]		
9	(a)		$\frac{4\times0.66617652-0.52764369}{3}$ **oe** or $\frac{4\times0.72534275-0.66617652}{3}$ **oe**	M1	1.1	
			$[S_2 =]$ 0.712354 to 0.71235413 and $[S_4 =]$ 0.7450648 to 0.745065	A1	1.1	allow **A1** if not attributed, but **A0** if wrongly attributed **NB** 0.745064826667 allow **SC1** for both answers correct unsupported
				[2]		
9	(b)		0.7 because the two Simpson's estimates only agree to this precision	B1	2.2b	**or** 0.8 because the approximations are increasing and the difference between S_4 and S_2 is large enough to suggest value closer to 0.8 than 0.7
				[1]		
9	(c)		ratio of differences seems to be converging [to 0.4]	B1	2.2b	
			$0.25 < r < 0.5$ hence order of (convergence of) method is between first and second order (in this case)	B1	2.2b	Do not allow eg order of convergence is between 1 and 2
				[2]		

Question			Answer	Marks	AO	Guidance
9	(d)		$0.7655404 + 0.001618 \times \frac{0.4}{1-0.4}$	**M1**	**3.1a**	extrapolation with values of T_n , associated difference and *r* deduced from table; allow for partial extrapolation
				A1	**1.1**	allow 0.4 to 0.401 for *r*; allow for partial extrapolation **NB** 0.7661876 with *r* = 0.4 **NB** 0.7661891371 with *r* = 0.40095
			0.766619 to 0.766624	**A1**	**1.1**	**NB** 0.766619066667 and 0.76662334316 from extrapolation to infinity with 0.4 and 0.40095 respectively
			0.767 or 0.7666 since extrapolation much more accurate **oe**	**A1**	**3.2a**	allow 0.77 is secure by comparison of T_{64} with extrapolated value **oe** if **M0,** allow **SC2** for extrapolation with *r* = 0.25 to obtain 0.766079…
				[4]		

APPENDIX

Exemplar responses for Q2(b)

Response	Mark

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