OCR
Oxford Cambridge and RSA

GCE

Further Mathematics B MEI

Y421/01: Mechanics major

A Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING

RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: RM A*ssessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the RM Assessor messaging system, or by email.

5. **Crossed Out Responses**
   Where a candidate has crossed out a response and provided a clear alternative then the crossed out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed out response where legible.

   Rubric Error Responses – Optional Questions
   Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

Multiple Choice Question Responses
When a multiple choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate).

Contradictory Responses
When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only **one mark per response**)
Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

Short Answer Questions (requiring a more developed response, worth **two or more marks**)
If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space.)

Longer Answer Questions (requiring a developed response)
Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there then add a tick to confirm that the work has been seen.

7. Award No Response (NR) if:

- there is nothing written in the answer space.

Award Zero '0' if:

- anything is written in the answer space and is not worthy of credit (this includes text and symbols).

Team Leaders must confirm the correct use of the NR button with their markers before live marking commences and should check this when reviewing scripts.

8. The RM Assessor **comments box** is used by your team leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**
If you have any questions or comments for your team leader, use the phone, the RM Assessor messaging system, or e-mail.

9. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

10. For answers marked by levels of response:
a. **To determine the level** – start at the highest level and work down until you reach the level that matches the answer
b. **To determine the mark within the level**, consider the following:

Descriptor	**Award mark**
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1			$15D = 27000$	M1	1.1	Use of $P = Dv$ with $P = 27000$ or 27
			$D = 1800\,(\text{N})$ or $\frac{27000}{15}$	A1	1.1	Or implied by later working
			$D - 600 = 800a$	M1	3.3	N2L – correct number of terms with their D (or just D) – condone sign errors but must be correct mass of 800 (so dimensionally consistent)
			$a = 1.5\,(\text{m s}^{-2})$	A1	1.1	
				[4]		
2			$0.5(6) + 2(2) = 0.5(0.2) + 2v$	M1	3.3	Use of CLM – correct number of terms – allow sign errors and a slip in one value only – allow mgv for **M** marks only
			$v = 3.45\,(\text{m s}^{-1})$	A1	1.1	
			$0.5(6) + 2(2) = 0.5(-0.2) + 2v$	M1	3.1b	Use of CLM (again) – correct number of terms. When compared to first application of CLM must be the same total momentum before collision but different sign of 0.2 in expression for total momentum after the collision
			$v = 3.55\,(\text{m s}^{-1})$	A1	1.1	
				[4]		

Question			Answer	Marks	AO	Guidance
3	(a)		$T\cos\theta = 0.2g$	M1	1.1	Resolving vertically – correct number of terms and T resolved. **M0** if 0.2 used as the weight. Allow with either T or 15
			$\cos\theta = \frac{49}{375} \Rightarrow \theta = 82^\circ$	A1	1.1	For reference: 82.491881… (no indication of why this is the greatest value is required) allow awrt 82 and awrt 1.44 (radians) - condone $\theta \leqslant 82$ but not < 82 Ignore incorrect use of inequality symbols if 82 given as answer
				[2]		
	(b)		$r = 0.75\sin\theta$	B1	1.1	Allow a value of 0.74 (or better) to imply this mark
			$T\sin\theta = 0.2(0.75\sin\theta)\omega^2$	M1	3.3	Applying N2L horizontally – correct number of terms and T resolved (allow with either T or 15). Allow with r for radius and $a = r\omega^2$ or $a = \frac{v^2}{r}$ but not just a
			$\omega = 10$	A1	1.1	Condone awrt 10 from correct working (might see 10.018… from using r = 0.74 and $\theta = 82^\circ$) condone $\omega \leqslant 10$ but not $\omega < 10$ Ignore incorrect use of inequality symbols if 10 given as answer
				[3]		

Question			Answer	Marks	AO	Guidance
4	(a)		Horizontal component of **F** is 0	**B1**	**1.1**	This mark can be implied if direction is stated as vertical (oe)
			e.g. $2\cos\theta + 4\cos\theta - 3\cos\theta - 5\cos\theta \left(= -2\cos\theta\right)$ e.g. $\sqrt{1^2 + 1^2 - 2(1)(1)\cos(2(90-\theta))}$	**M1**	**3.3**	Resolving vertically (oe) (at least four terms required) where θ is the angle between AB and the upward vertical – condone if trig. terms are not consistent (but all four must have been resolved)
			$\cos\theta = \frac{3}{5} \Rightarrow$ vertical component of **F** is 1.2	**A1**	**1.1**	Sight of ± 1.2
			Magnitude of **F** is 1.2 Direction of **F** is in the same direction as AC	**A1**	**2.2a**	Magnitude must be positive – allow 'vertically upwards' or 'upwards' only (oe) for direction ('vertically' only though is **A0**, but may be clarified with a diagram)
				[4]		
	(b)		e.g. moments about A: $G = 6\left(3\sin\theta\right) + 6\left(4\sin\theta\right)$ or $3(4\sin\theta) + 4(4\cos\theta) + 3(6\sin\theta)$ or $3(3\sin\theta) + 4(3\cos\theta) + 3(4\sin\theta) + 4(4\cos\theta)$ or e.g. moments about O: $3(3\sin\theta) + 3(2\sin\theta) + 4(4\cos\theta) + 4(5\cos\theta)$ or $4(4+2)\cos\theta + 4(3+5)\cos\theta$	**B1**	**3.1b**	Taking moments (resolved force $\times$ distance (oe)) with correct terms for their chosen point (note that if their answer to **(a)** is correct then taking moments about D or B, their **F** must be included too). A correct answer implies this mark
			$G = 42\sin\theta = 42\left(\frac{4}{5}\right) = 33.6$	**B1**	**1.1**	No direction required but must be positive (seen anywhere in their solution so ISW if 33.6 is then stated as –33.6)
				[2]		

Question			Answer	Marks	AO	Guidance
5	(a)		$[F]=\text{MLT}^{-2}$ or $[v]=\text{LT}^{-1}$	B1	1.2	For either the dimensions of force or velocity correct (possibly seen in an expression for the dimensions of k)
			$[k]=\dfrac{\text{MLT}^{-2}}{\text{LT}^{-1}}$	M1	1.1	Use of $[F]=[k]\times[v]$ with at least one of [F] or [v] correct
			$[k]=\text{MT}^{-1}$	A1	2.2a	
				[3]		
	(b)		e.g. $[T]=\left[k^2m^\alpha u^\beta F^\gamma\right]$	M1*	2.1	For correctly equating the dimensions of T (or either of the other two terms on the rhs) to the dimensions of $\frac{1}{3}k^2m^\alpha u^\beta F^\gamma$ - this mark can be implied by setting up an equation in M, L and T – if k is missing or assumed to be dimensionless then no marks in this part. **Must** be powers of k, m, u and F and no other (incorrect) terms
			$\text{T}=(\text{MT}^{-1})^2\text{M}^\alpha(\text{LT}^{-1})^\beta(\text{MLT}^{-2})^\gamma$	M1dep*	1.1	Setting up an equation in M, L and T using their [k] and at least one of [F] or [v] correct
			$\beta+2\gamma=-3$ $\alpha+\gamma=-2$ $\beta+\gamma=0$	M1	1.1	Setting up three equations in α,β and γ from their MLT (dependent on both previous **M** marks) – allow one slip from their MLT equation
			$\alpha=1,\beta=3,\gamma=-3$	A1	2.2a	
				[4]		

Question			Answer	Marks	AO	Guidance
6	(a)		$\mathbf{v} = (4t-12)\mathbf{i} + (3t^2+6t-8)\mathbf{j}$	B1	1.1	Condone lack of brackets and accept column vector form
				[1]		
	(b)		$1(3t^2+6t-8) = -4(4t-12)$	M1*	3.1b	Setting up a quadratic equation in t only – allow sign errors (including on the 1 and the 4) and the 1 and the –4 on the wrong side (or multiples of 1 and –4)
			$3t^2+22t-56=0 \Rightarrow t = ...$	M1dep*	1.1	Re-arrange **and** solves their three term quadratic equation in t to obtain a positive value of t
			$t = 2$	A1	1.1	Ignore $t = -\frac{28}{3}$ unless used
			$\lvert\mathbf{v}\rvert = \sqrt{(4t-12)^2 + (3t^2+6t-8)^2}$ with their positive value of t	M1	3.4	Dependent on both previous **M** marks – if working not shown then numerical components of **v** must be consistent with their positive value of t
			$\lvert\mathbf{v}\rvert = 16.5\,(\text{m s}^{-1})$	A1	1.1	awrt 16.5 (allow exact $4\sqrt{17}$) – for reference: 16.4924225…
				[5]		
	(c)		$\mathbf{a} = 4\mathbf{i} + (6t+6)\mathbf{j}$	B1ft	1.1	Follow their **v** from **(a)**
			$4^2 + (6t+6)^2 = 20.2^2$	M1	1.1	Setting up an equation in t using 20.2 and their **a**
			$t = 2.3$	A1	1.1	Positive value of t only
				[3]		

Question			Answer	Marks	AO	Guidance
7	(a)		$R = 2g\cos 30$	M1*	3.3	Resolving perpendicular to the plane for A – correct number of terms – allow sin/cos confusion (**M0** if 2 used for the weight)
			$F = \frac{1}{2\sqrt{3}}(2g\cos 30)$ $(=\frac{1}{2}g)$	M1dep*	3.4	Use of $F = \mu R$ with correct μ and their R
			Work done against friction is $\frac{1}{2}gd$	A1	1.1	Must be in terms of g and d
				[3]		
7	(b)		PE (lost by) B: $4gd$	B1	1.1	Allow $\pm 4gd$ or $\pm 39.2d$
			PE (gained by) A: $2g(d\sin 30)$ $(=gd)$	B1	1.1	Allow $\pm gd$ or $\pm 9.8d$
			KE (gained by) A and B: $\frac{1}{2}(2)(1.75)^2 + \frac{1}{2}(4)(1.75)^2$	B1	1.1	KE (gained) for either A **or** B (so **B1** for either ± 3.0625 or ± 6.125 or ± 9.1875)
			$\frac{1}{2}(2)(1.75)^2 + \frac{1}{2}(4)(1.75)^2 = (4gd - gd) - \frac{1}{2}gd$	M1	3.4	Work energy principle – condone sign errors and slips but must be the correct number of terms (so must be considering KE for A and B and three terms in d) and dimensionally correct
			$d = 0.375$	A1	2.2a	
				[5]		
7	(c)		• Consider the dimensions of the pulley or block(s) • Consider the weight/mass of the rope • More accurate value of g • Friction at the pulley • Elastic rope	B1	3.5c	Must be suggesting an improvement so **B0** for 'do not modelled blocks as particles' (oe) **B0** for 'include air resistance/resistance/wind' or 'friction/resistance should be proportional to speed or speed squared'
				[1]		

Question			Answer	Marks	AO	Guidance
8	(a)		Area below the curve $=\int_0^4 \sqrt{3x+4}\,\mathrm{d}x = \frac{112}{9}$	B1	1.1	**BC** or $\frac{128}{9}-\frac{16}{9}$ (possibly embedded in other calculations)
			$A\bar{y} = \frac{1}{2}\int_0^4 3x+4\,\mathrm{d}x = 20$	B1	1.1	**BC -** y-coordinate of centre of mass of the lamina below the curve is $\frac{45}{28}$ which implies **B1B1** Might be seen in the evaluation of $A\bar{y} = \frac{1}{2}\int_0^4 (3x+4)\mathrm{d}x - \frac{1}{2}\int_k^4 16\left(\frac{x-k}{4-k}\right)^2 \mathrm{d}x$ Possibly embedded in other calculations
			Centre of mass of triangle is at a distance of $\frac{4}{3}$ from the x-axis	B1	1.2	Used in a moment calculation (so not just stated) - possibly implied in later working If done by integration then this mark is awarded for correctly evaluating $\frac{1}{2}\int_k^4 16\left(\frac{x-k}{4-k}\right)^2 \mathrm{d}x$ as $\frac{8}{3}(4-k)$ (or unsimplified)
				M1	2.1	Table of values idea – correct number of terms (dimensionally consistent) – so of the form $\pm k_1 \pm \mathrm{f}(k) = (\pm k_2 \pm \mathrm{g}(k))\bar{y}$ where f(k) and g(k) are equivalent to linear functions of k
			$20-\frac{1}{2}(4)(4-k)\left(\frac{4}{3}\right) = \left(\frac{112}{9}-\frac{1}{2}(4)(4-k)\right)\bar{y}$	B1	1.1	For either the correct lhs or rhs seen (e.g. $\frac{40}{9}+2k$ or $\frac{28}{3}+\frac{8k}{3}$ or unsimplified equivalent expressions seen anywhere in their solution so do not need to be seen in a complete equation for $\bar{y}$)
			$\bar{y} = \frac{42+12k}{20+9k}$	A1	2.2a	a = 42, b = 12, c = 20, d = 9 (or correct integer multiples e.g. a = 84, b = 24, c = 40 and d = 18 etc.)
				[6]		

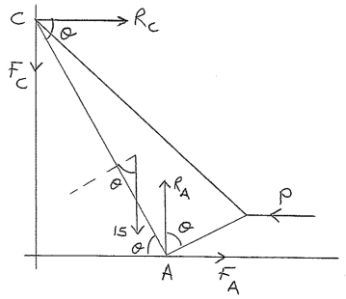
Question			Answer	Marks	AO	Guidance
	(b)		$\frac{42+12k}{20+9k}=\frac{3}{2}\Rightarrow k=...$	**M1**	**3.1a**	Sets their expression for $\bar{y}$ equal to 1.5 (which when fractions are cleared must be a linear equation in k) and attempts to solve for k
			$k=8$ but $0 < k < 4$ so $\bar{y}$ cannot be 1.5	**A1**	**2.3**	$k = 8$ plus reason (so some mention of $k < 4$)
				[2]		

Question			Answer	Marks	AO	Guidance
9	(a)		$\ddot{x} = -g\sin\alpha, \quad \ddot{y} = -g\cos\alpha$	B1	1.1	Possibly implied by later working
			$y = (20\sin\theta)t + 0.5(\ddot{y})t^2$	M1*	3.3	Applying $s = ut + 0.5at^2$ perpendicular to the plane – allow sin/cos confusion with the component of 20 (but not in terms of α)
			$20\left(\frac{3}{5}\right)t - \frac{1}{2}g\left(\frac{12}{13}\right)t^2 = 0 \Rightarrow t = ...$	M1dep*	3.4	Sets y = 0 and solves for t (acceleration must have been a component of either $\sin\alpha$ or $\cos\alpha$)
			$t = 2.6$	A1	1.1	Allow $\frac{26}{g}$ (provided g not later used as 9.8). Could be implied by correct values for the velocity components (provided g = 10) or correct value of OA
			$\dot{x} = 20\cos\theta + (-g\sin\alpha)t = 20\left(\frac{4}{5}\right) - 10\left(\frac{5}{13}\right)t$	M1	3.3	Applying $v = u + at$ (oe) parallel to the plane – allow sin/cos confusion (but correct use of theta/alpha for those components) – allow with their t or just t but must have substituted trigonometric values
			$\dot{y} = 20\sin\theta + (-g\cos\alpha)t = 20\left(\frac{3}{5}\right) - 10\left(\frac{12}{13}\right)t$	M1	3.3	Applying $v = u + at$ (oe) perpendicular to the plane – allow sin/cos confusion (but correct use of theta/alpha for those components) – allow with their t or just t but must have substituted trigonometric values
			$\dot{x} = 6, \dot{y} = -12$	A1	2.2a	Allow 12
			$\text{OA} = (20\cos\theta)t + \frac{1}{2}(-g\sin\alpha)t^2 = 20\left(\frac{4}{5}\right)t - 5\left(\frac{5}{13}\right)t^2$	M1	3.4	Applying $s = ut + 0.5at^2$ parallel to the plane – allow with their t or just t (or equivalent e.g. $\dot{x}^2 = (20\cos\theta)^2 + 2(-g\sin\alpha)(\text{OA})$) but must have substituted trigonometric values but allow sign errors
			OA = 28.6 (m)	A1	2.2a	**If g missing from acceleration terms can score first two M marks only** **If using g = 9.8 then first and last A marks cannot be awarded (so max. 7/9 – look out for t = 2.653… and OA = 29.183…)**
				[9]		

Question			Answer	Marks	AO	Guidance
	(b)		Maximum value of $\beta \Rightarrow e = 1$	**B1**	**3.1b**	Indication that the greatest angle occurs when the collision is elastic (award if using the values for $\dot{x}, \dot{y}$ found in part **(a)**)
			$\beta = \tan^{-1}\left(\frac{\lvert\dot{y}\rvert}{\dot{x}}\right) + \tan^{-1}\left(\frac{5}{12}\right)$	**M1**	**3.4**	Correct method for finding β - all values substituted
			$\beta = 86^\circ$	**A1**	**3.2a**	For reference: 86.05481… - allow awrt 86 (but not from incorrect working) or awrt 1.50 (radians)
				[3]		

Question			Answer	Marks	AO	Guidance
10	(a)		Initially, PE = $mg(r + r\sin 30)$ $(=1.5mgr)$	B1	1.1	Or if reference level is through O then for $mgr\sin 30$
			At angle θ, PE = $mg(r - r\cos\theta)$	B1	1.1	Or if reference level is through O then for $-mgr\cos\theta$ Or **B2** for $mgr(\sin 30 + \cos\theta)$
			$\frac{1}{2}mu^2 + mg(r + r\sin 30) = mg(r - r\cos\theta) + \frac{1}{2}mv^2$	M1*	3.3	Use of conservation of energy between initial position and when P is at B (at least 2 KE and 2 PE terms)
			$v^2 = u^2 + gr + 2gr\cos\theta$	A1	1.1	Correct expression for the speed or speed-squared when P is at B (so must have been re-arranged or implied by later working) – correct expression for v or v^2 implies the first four marks
			$R - mg\cos\theta = \frac{mv^2}{r}$	M1*	3.3	N2L radially with correct number of terms and weight resolved (but allow sin/cos confusion and sign errors) – acceleration must be either $\frac{v^2}{r}$ or $r\omega^2$
			$R - mg\cos\theta = \frac{m}{r}\left\{u^2 + gr + 2gr\cos\theta\right\}$	M1dep*	3.4	Substitute expression for v^2 (with correct number of terms)
			$R = 3mg\cos\theta + mg + \frac{mu^2}{r}$	A1	2.2a	Must be simplified to three terms
				[7]		
	(b)		At A, $R + mg\cos 60 = \frac{mu^2}{r}$	B1	3.3	
			$\left(3mg\cos\theta + mg + \frac{mu^2}{r}\right) - \left(\frac{mu^2}{r} - mg\cos 60\right) = 4mg$	M1*	1.1	The difference of two expressions for R (their expression from **(a)** with three terms, and their two term expression for R at A with the equivalent of a single term for the resolved weight) equated to $\pm 4mg$
			$3\cos\theta + \frac{3}{2} = 4 \Rightarrow \cos\theta = \frac{5}{6}$	M1dep*	1.1	Obtain $\cos\theta = k$ where $0 < k < 1$ (or corresponding value of θ)
			Vertical distance = $r - \frac{5}{6}r = \frac{1}{6}r$	A1	2.2a	
				[4]		

Question			Answer	Marks	AO	Guidance
	(c)		Diameter of the rim of the bowl is $2r\cos 30$ $(=r\sqrt{3})$	B1	1.1	
			$r\sqrt{3}=(u\cos 60)t \quad \left(\Rightarrow t=\frac{2r\sqrt{3}}{u}\right)$	M1*	3.1b	Realise that P leaves the inner surface with speed u and applies $s=ut$ horizontally with s = their diameter $(\neq r, 2r)$ and component of u
			$y=(u\sin 60)\left(\frac{2r\sqrt{3}}{u}\right)-\frac{1}{2}g\left(\frac{2r\sqrt{3}}{u}\right)^2 \quad \left(=3r-\frac{6gr^2}{u^2}\right)$	M1dep*	3.4	Applies $s=ut+0.5at^2$ vertically with their value of t
			$y>0\Rightarrow 3r-\frac{6gr^2}{u^2}>0$	M1	3.1b	Sets their expression for y (in terms of r, g and u) >0 – dependent on both previous M marks (condone 'equals' or 'greater than or equal to' zero for this mark)
			$3r\left(u^2-2gr\right)>0\Rightarrow u^2>2gr$	A1	2.2a	**AG** (if using non-strict inequality or equals then argument for strict inequality must be convincing)
				[5]		
			Diameter of the rim of the bowl is $2r\cos 30$ $(=r\sqrt{3})$	B1		
			$u\cos 30t-\frac{1}{2}gt^2=0\Rightarrow t=...$	M1*		Finding the time (from $s=ut+0.5at^2$ with $s=0$, a component of u and $a=\pm g$) when P is at the same horizontal level as the rim of the bowl – if correct then $t=\frac{u\sqrt{3}}{g}$
			$x=(u\sin 30)t=\frac{1}{2}u\left(\frac{u\sqrt{3}}{g}\right)$	M1dep*		Using their t and $s=ut$ horizontally (with a component of u) to find the horizontal distance travelled by P when it is at the same horizontal level as the rim
			$\frac{1}{2}u\left(\frac{u\sqrt{3}}{g}\right)>r\sqrt{3}$	M1		Sets their x > their $r\sqrt{3}$ (not just r or $2r$) – condone 'equals' or 'greater than or equal to' – dependent on both previous M marks
			$u^2>2gr$	A1		**AG**

Question			Answer	Marks	AO	Guidance
11	(a)		For reference:  	**M1***	**3.3**	Attempt to resolve horizontally **or** vertically (with correct number of terms) allow if using 15*g* for the **M** mark (condone in words '*P* = Fr at A + Reaction at C' for **M1** only – for the **A** mark must be using symbols)
			$R_A = F_C + 15,\ P = F_A + R_C$	**A1**	**1.1**	Both correct
			(Least value of *P* implies) $F_A = 0.5R_A,\quad F_C = 0.5R_C$	**B1***	**3.1b**	For both (no justification required)
			Taking moments to form an equation containing all relevant forces	**M1***	**3.3**	Taking moments about e.g. A, B, C etc.– correct number of terms (at least 4 terms if A or C and 5 terms if B) – one for each force), dimensionally consistent – allow sin/cos confusion. Must have numerical values for all distances and each term must be resolved with a term in $\sin\theta / \cos\theta$
			$P(3\cos\theta) + 4(15\cos\theta) = 1(15\sin\theta) + 12(R_C \sin\theta) - 12(F_C \cos\theta)$ $8(15\cos\theta) + 1(15\sin\theta) + P(12\sin\theta - 3\cos\theta)$ $= R_A(12\cos\theta) + F_A(12\sin\theta)$	**A2**	**1.1** **1.1**	Award **A1** for any two terms correct **A2** for a fully correct equation $P(3\cos\theta) = 15\sqrt{17}\sin(\theta - \arctan 4) + 12(R_C \sin\theta) - 12(F_C \cos\theta)$
			$3P\cos\theta + 60\cos\theta - 15\sin\theta$ $= 12\sin\theta\left(\frac{4P-30}{5}\right) - 6\cos\theta\left(\frac{4P-30}{5}\right)$	**M1dep***	**1.1**	Deriving an equation containing P and θ only (Reference: $R_A = 0.5R_C + 15 \therefore 2(P - R_C) = 0.5R_C + 15$ $\Rightarrow R_C = \frac{4P-30}{5}$)
			$\Rightarrow P = \frac{120\cos\theta + 285\sin\theta}{48\sin\theta - 39\cos\theta} = \frac{40\cos\theta + 95\sin\theta}{16\sin\theta - 13\cos\theta}$	**A1**	**2.2a**	**AG** Possible intermediate step is $15P\cos\theta + 300\cos\theta - 75\sin\theta$ $= 12(4P-30)\sin\theta - 6(4P-30)\cos\theta$
				[8]		

Question			Answer	Marks	AO	Guidance
11	(b)		$P > 0 \Rightarrow 16\sin\theta - 13\cos\theta > 0$	M1	3.4	Considers denominator of given answer in **(a)** either > 0 or = 0 or $\geqslant 0$
			$\tan\theta > \frac{13}{16} \Rightarrow \theta > 39.09...$	A1	1.1	awrt 39
			For the prism to be in equilibrium $\tan\theta \leqslant 4$	M1	3.1b	Considers $\tan\theta = \frac{\bar{y}}{\bar{x}}$ (or reciprocal) for triangular lamina
			$\theta \leqslant 75.96...$	A1	1.1	awrt 76 (For full marks must imply an interval between these two values)
				[4]		

Question			Answer	Marks	AO	Guidance
12	(a)	(i)	Spheres are smooth and so the impulse acts in a direction parallel to the line of centres (which is in the **i** direction)	**B1**	**2.4**	
				[1]		
	(a)	**(ii)**		**M1***	**3.3**	Use of conservation of linear momentum (parallel to the line of centres) – correct number of terms (condone no masses present for this mark) but condone sign errors
				M1*	**3.3**	Use of Newton's experimental law (parallel to the line of centres) – correct number of terms
			$mu_1 = mv_A + \lambda mv_B$ $v_A - v_B = -eu_1$	**A1**	**1.1**	Use of NEL must be consistent with CLM e.g. $mu_1 = -mv_A + \lambda mv_B$ and $v_A + v_B = eu_1$ (v_A and v_B are the horizontal components of the velocities of A and B after impact parallel to the line of centres)
			Solving simultaneously to find either v_A or v_B in terms u_1, e and λ	**M1dep***	**1.1**	For reference: $v_A = \frac{u_1(1+e)}{1+\lambda} - eu_1 = \frac{u_1(1-e\lambda)}{1+\lambda}$ $v_B = \frac{u_1(1+e)}{1+\lambda}$
				M1	**3.3**	Using either $\mathbf{I} = \pm\lambda mw\mathbf{i}$ or $\mathbf{I} = \pm m(v_A - u_1)\mathbf{i}$ (condone lack of **i**) to get **I** in terms of λ, m, e and u_1 only. Dependent on all previous M marks. Must be using the correct mass
			Impulse = $\lambda m\left(\frac{1+e}{1+\lambda}\right)u_1\mathbf{i}$	**A1**	**2.5**	Allow unsimplified e.g. $-m\left\{\frac{u_1(1-e\lambda)}{1+\lambda} - u_1\right\}\mathbf{i}$ - condone lack of **i**
				[6]		

Question	Answer	Marks	AO	Guidance
(b)	$\frac{1}{2}mu_1^2 \quad (+\frac{1}{2}mu_2^2)$	B1	1.1	KE before collision - or just considering energy in the **i** direction (so **B1** for $\frac{1}{2}mu_1^2$)
	$\frac{1}{2}m\left(\frac{u_1^2(1-e\lambda)^2}{(1+\lambda)^2}\right)+\frac{1}{2}(\lambda m)u_1^2\left(\frac{1+e}{1+\lambda}\right)^2 \quad (+\frac{1}{2}mu_2^2)$	B1 B1	1.1	KE after collision – **B1** for each correct term (need only consider the energy in the **i** direction) For reference: $v_A=\frac{u_1(1+e)}{1+\lambda}-eu_1=\frac{u_1(1-e\lambda)}{1+\lambda}$ $v_B=\frac{u_1(1+e)}{1+\lambda}$
	$\frac{1}{2}-\frac{1}{2}\left(\frac{1-e\lambda}{1+\lambda}\right)^2-\frac{1}{2}\lambda\left(\frac{1+e}{1+\lambda}\right)^2=\frac{1}{8}$	M1*	1.1	Using KE before – KE after = $\pm\frac{1}{8}mu_1^2$ to obtain an equation in λ and e only For reference only : $e^2=\frac{3\lambda^2+2\lambda-1}{4\lambda(\lambda+1)}$ and $\lambda^2(4e^2-3)+\lambda(4e^2-2)+1=0$
	$\lambda\left(\frac{1-e^2}{1+\lambda}\right)=\frac{1}{4}\Rightarrow e^2=\frac{3\lambda-1}{4\lambda}\Rightarrow 0\leqslant\frac{3\lambda-1}{4\lambda}\leqslant 1$	M1dep*	3.1b	Re-arranging to make e^2 (or e) the subject from an equation containing terms in λ (possibly λ^2) and e^2 (not e) only **and** considering either $e^2\geqslant 0$ or $e^2\leqslant 1$ (condone 'equals' or 'strict' comparison). Or obtaining an equation in λ (possibly λ^2) and e^2 (not e) only and setting $e = 1$ or $e = 0$
	$\lambda\geqslant\frac{1}{3}$	A1	2.2a	**A0** if strict inequality or an upper limit for the value of λ stated
		[6]		

Question			Answer	Marks	AO	Guidance
13	(a)		$T_0 = mg$	M1	1.1	Resolving vertically for P
			$T_0 = \frac{man^2 e_0}{a}$	M1	3.3	Apply Hooke's law – where e_0 is the extension of the spring from its natural length
			$\frac{man^2 e_0}{a} = mg \Rightarrow e_0 = gn^{-2}$	A1	1.1	
				[3]		
	(b)		$y + a + e_0 = a + x + z$	M1	3.1b	Attempt an equation containing z where z is the upward displacement of P from its initial position (at time t) – must include terms in y, x and e_0 (allow sign errors **only**)
			$(z =) y + gn^{-2} - x$	A1	1.1	
				[2]		
	(c)		$(\ddot{z} =) kt - \ddot{x}$	B1	1.1	
				[1]		
	(d)		$T - mg = m\ddot{z}$	M1*	2.1	Apply N2L vertically – correct number of terms but allow sign errors (allow for $T - mg = ma$) – implied by a correct equation
			$\frac{man^2 x}{a} - mg = m(kt - \ddot{x})$ or $m\ddot{x} = mkt + mg - mn^2 x$	M1dep*	3.4	Use Hooke's law with extension = x and acceleration equal to their expression from part **(c)** – if **(c)** blank then must be correct
			$n^2 x - g = kt - \ddot{x} \Rightarrow \ddot{x} + n^2 x = kt + g$	A1	2.2a	**AG** – so sufficient working must be shown – **any** errors then **A0**
				[3]		

Question			Answer	Marks	AO	Guidance
	(e)		$x = \frac{1}{n^3}(knt + gn - k\sin(nt))$			For reference: $\ddot{x} + n^2 x = kt + g$ and $x = \frac{1}{n^3}(knt + gn - k\sin(nt))$
			$\dot{x} = \frac{1}{n^3}(kn - kn\cos(nt))$	**B1**	**1.1**	Correct derivative
			$\ddot{x} = \frac{1}{n^3}(kn^2\sin(nt))$	**B1ft**	**1.1**	Follow through their first derivative
			$\frac{1}{n^3}(kn^2\sin(nt)) + n^2\left(\frac{1}{n^3}(knt + gn - k\sin(nt)\right)$ $= \frac{k\sin(nt)}{n} + \frac{ktn + gn - k\sin(nt)}{n} = kt + g$	**B1**	**1.1**	Correctly confirm given differential equation - at least one stage of working before obtaining $kt + g$
			When $t = 0$, $x = \frac{1}{n^3}(gn) = gn^{-2}$ and $\dot{x} = \frac{1}{n^3}(kn - kn\cos(0)) = 0$	**B1**	**3.4**	Confirm initial conditions
				[4]		
			CF: $A\cos nt + B\sin nt$	**B1***		
			GS: $x = A\cos nt + B\sin nt + \frac{kt}{n^2} + \frac{g}{n^2}$	**B1***		From a trial solution of the form $x = \lambda t + \mu \Rightarrow \dot{x} = \lambda, \ddot{x} = 0 \therefore \mu = \frac{g}{n^2}, \lambda = \frac{k}{n^2}$
			Initial conditions: $t = 0, x = \frac{g}{n^2}$ and $\dot{x} = 0$	**B1dep***		
			$A = 0, B = -\frac{k}{n^3} \Rightarrow x = -\frac{k}{n^3}\sin nt + \frac{kt}{n^2} + \frac{g}{n^2}$	**B1**		

Question			Answer	Marks	AO	Guidance
	(f)		$\dot{x} = \frac{k}{n^2}(1-\cos(nt))$ but $\lvert\cos(nt)\rvert \leqslant 1$ for all values of t so $\dot{x} \geqslant 0$	**B1***	**3.1b**	Using correct $\dot{x}$ (possibly seen in part **(c)** and allow if correctly stated there) to conclude that $\dot{x}$ is always greater than or equal to 0 **or** between 0 and $\frac{2k}{n^2}$ (must imply that it could be zero)
			We can therefore infer that P does not move closer to the ceiling of the lift in the subsequent motion	**B1dep***	**2.2b**	or equivalent comment e.g. in the subsequent motion P is never nearer the ceiling of the lift as it was when $t = 0$, or 'is always moving away from the ceiling'
				[2]		

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