OCR
Oxford Cambridge and RSA

# Thursday 25 May 2023 – Afternoon

## A Level Further Mathematics B (MEI)

**Y420/01** Core Pure

**Time allowed: 2 hours 40 minutes**

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**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

**QP**

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **144**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

**ADVICE**
- Read each question carefully before you start your answer.

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**Section A** (40 marks)

**1** **(a)** The complex number $a+\mathrm{i}b$ is denoted by $z$.

**(i)** Write down $z^*$. **[1]**

**(ii)** Find $\mathrm{Re}(\mathrm{i}z)$. **[2]**

**(b)** The complex number $w$ is given by $w = \dfrac{5+\mathrm{i}\sqrt{3}}{2-\mathrm{i}\sqrt{3}}$.

**(i)** **In this question you must show detailed reasoning.**

Express $w$ in the form $x+\mathrm{i}y$. **[2]**

**(ii)** Convert $w$ to modulus-argument form. **[2]**

**2** **In this question you must show detailed reasoning.**

Find the angle between the vector $3\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and the plane $-x+3y+2z=8$. **[5]**

**3** **(a)** Using partial fractions and the method of differences, show that

$$\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+\ldots+\frac{1}{n(n+2)} = \frac{3}{4}-\frac{an+b}{2(n+1)(n+2)},$$

where $a$ and $b$ are integers to be determined. **[5]**

**(b)** Deduce the sum to infinity of the series.

$$\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}+\ldots.$$ **[1]**

**4** **(a)** **(i)** Given that $\mathrm{f}(x) = \sqrt{1+2x}$, find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$. **[2]**

**(ii)** Hence, find the first three terms of the Maclaurin series for $\sqrt{1+2x}$. **[2]**

**(b)** Hence, using a suitable value for $x$, show that $\sqrt{5} \approx \dfrac{143}{64}$. **[2]**

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**5** **(a)** **In this question you must show detailed reasoning.**

Determine the sixth roots of $-64$, expressed in $re^{i\theta}$ form. **[4]**

**(b)** Represent the roots on an Argand diagram. **[3]**

**6** The matrices **M** and **N** are $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ respectively.

**(a)** **In this question you must show detailed reasoning.**

Determine whether **M** and **N** commute under matrix multiplication. **[3]**

**(b)** Specify the transformation of the plane associated with each of the following matrices.

**(i)** **M** **[1]**

**(ii)** **N** **[2]**

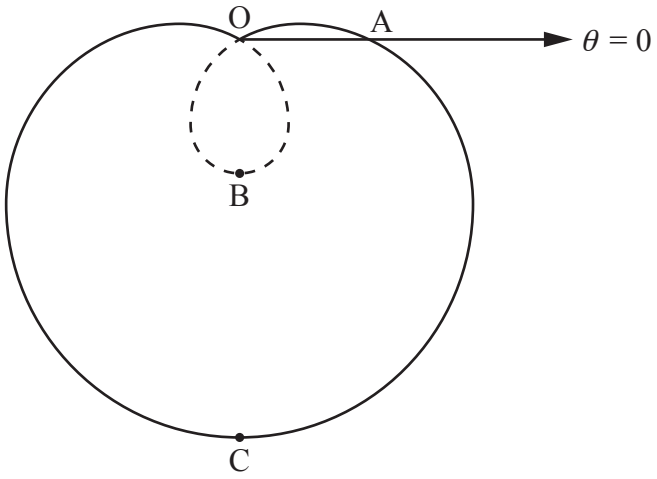
**(c)** State the significance of the result in part **(a)** for the transformations associated with **M** and **N**. **[1]**

**(d)** Use an algebraic method to show that all lines parallel to the $x$-axis are invariant lines of the transformation associated with **N**. **[2]**

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**Section B** (104 marks)

**7** The diagram below shows the curve with polar equation $r = a(1 - 2\sin\theta)$ for $0 \leqslant \theta \leqslant 2\pi$, where $a$ is a positive constant.




The curve crosses the initial line at A, and the points B and C are the lowest points on the two loops.

**(a)** Find the values of $r$ and $\theta$ at the points A, B and C. **[3]**

**(b)** Find the set of values of $\theta$ for the points on the inner loop (shown in the diagram with a broken line). **[3]**

**8** Prove by mathematical induction that $8^n - 3^n$ is divisible by 5 for all positive integers $n$. **[5]**

**9** In an electrical circuit, the alternating current $I$ amps is given by $I = a\sin nt$, where $t$ is the time in seconds and $a$ and $n$ are positive constants. The RMS value of the current, in amps, is defined to be the square root of the mean value of $I^2$ over one complete period of $\frac{2\pi}{n}$ seconds.

Show that the RMS value of the current is $\frac{a}{\sqrt{2}}$ amps. **[6]**

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**10** The equation $x^3 - 4x^2 + 7x + c = 0$, where $c$ is a constant, has roots $\alpha$, $\beta$ and $\alpha + \beta$.

**(a)** Determine the roots of the equation. **[6]**

**(b)** Find $c$. **[1]**

**11** Solve the differential equation $\cosh x \dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y \sinh x = \cosh x$, given that $y = 1$ when $x = 0$. **[7]**

**12** Show that $\sin^5\theta = a\sin 5\theta + b\sin 3\theta + c\sin\theta$, where $a$, $b$ and $c$ are constants to be determined. **[7]**

**13** **(a)** On separate Argand diagrams, show the set of points representing each of the following inequalities.

**(i)** $|z| \leqslant \sqrt{5}$ **[3]**

**(ii)** $|z + 2 - 4\mathrm{i}| \geqslant |z - 2 - 6\mathrm{i}|$ **[3]**

**(b)** Show that there is a unique value of $z$, which should be determined, for which both $|z| \leqslant \sqrt{5}$ and $|z + 2 - 4\mathrm{i}| \geqslant |z - 2 - 6\mathrm{i}|$. **[8]**

**14** Three planes have equations

$$\begin{aligned} kx \quad\quad - \ z &= 2, \\ -x + ky + 2z &= 1, \\ 2kx + 2y + 3z &= 0, \end{aligned}$$

where $k$ is a constant.

**(a)** By considering a suitable determinant, show that the three planes meet at a point for all values of $k$. **[5]**

**(b)** Using a matrix method, find, in terms of $k$, the coordinates of the point of intersection of the planes. **[8]**

**15** **In this question you must show detailed reasoning.**

Evaluate $\displaystyle\int_1^2 \frac{1}{\sqrt{1+2x-x^2}}\,\mathrm{d}x$, giving your answer in terms of $\pi$. **[5]**

**16** The point P (4, 1, 0) is equidistant from the plane $2x+y+2z=0$ and the line $\dfrac{x-3}{2}=\dfrac{y-1}{b}=\dfrac{z+5}{3}$, where $b>0$.

Determine the value of $b$. **[10]**

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**17** Two similar species, X and Y, of a small mammal compete for food and habitat. A model of this competition assumes, in a particular area, the following.

- In the absence of the other species, each species would increase at a rate proportional to the number present with the same constant of proportionality in each case.
- The competition reduces the rate of increase of each species by an amount proportional to the number of the other species present.

So if the numbers of species X and Y present at time $t$ years are $x$ and $y$ respectively, the model gives the differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = kx - ay \quad \text{and} \quad \frac{\mathrm{d}y}{\mathrm{d}t} = ky - bx,$$

where $k$, $a$ and $b$ are positive constants.

**(a) (i)** Show that the general solution for $x$ is $x = A\mathrm{e}^{(k+n)t} + B\mathrm{e}^{(k-n)t}$, where $n = \sqrt{ab}$ and $A$ and $B$ are arbitrary constants. **[6]**

**(ii)** Hence find the general solution for $y$ in terms of $A$, $B$, $k$, $n$, $a$ and $t$. **[2]**

Observations suggest that suitable values for the model are $k = 0.015$, $a = 0.04$ and $b = 0.01$. You should use these values in the rest of this question.

**(b)** When $t = 0$, the numbers present of species X and Y in this area are $x_0$ and $y_0$ respectively.

**(i)** Show that $x = \frac{1}{2}(x_0 - 2y_0)\mathrm{e}^{0.035t} + \frac{1}{2}(x_0 + 2y_0)\mathrm{e}^{-0.005t}$. **[3]**

**(ii)** Hence show that $y = \frac{1}{4}(x_0 + 2y_0)\mathrm{e}^{-0.005t} - \frac{1}{4}(x_0 - 2y_0)\mathrm{e}^{0.035t}$. **[1]**

**(c)** Use initial values $x_0 = 500$ and $y_0 = 300$ with the results in part **(b)** to determine what the model predicts for each of the following questions.

**(i)** What numbers of each species will be present after 25 years? **[2]**

**(ii)** **In this question you must show detailed reasoning.**

When will the numbers of the two species be equal? **[4]**

**(iii)** Does either species ever disappear from the area? Justify your answer. **[3]**

**(d)** Different initial values will apply in other areas where the two species compete, but previous studies indicate that one species or the other will eventually dominate in any given area.

**(i)** Identify a relationship between $x_0$ and $y_0$ where the model does **not** predict this outcome. **[1]**

**(ii)** Explain what the model predicts in the long term for this exceptional case. **[2]**

**END OF QUESTION PAPER**

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