OCR
Oxford Cambridge and RSA

Thursday 23 June 2022 – Afternoon

AS/A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Duration: 1 hour 15 minutes

MAXIMUM MARK 60

Post-Standardisation

Last updated: 07.07.22

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

2. **Subject-specific Marking Instructions for AS/A Level Further Mathematics B (MEI)**

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
if there is nothing written at all in the answer space and no attempt elsewhere in the script
OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.
Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows thatthe method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads “3 s.f”
Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:
If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” and “Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			a_n: periodic...	B1	2.2b		
			with period 2 oe	B1	2.2b	eg "the smallest infinitely repeating sub-sequence containing two numbers".	If **B0B0** for description of sequence a_n then **SC1** for stating either that sequence a_n does not converge or that it oscillates.
			b_n: oscillatory...	B1	2.2b		
			convergent...	B1	2.2b		
			...limit is 2 (or 2.0 or 2.00)	B1	2.2b	**Not** $1.\dot{9}$ or $2.\dot{0}$ or 1.9... or 2.0....	
			c_n: increasing...	B1	2.2b	Must be a correct statement.	
			...divergent	B1	2.2b	or "not convergent" or "tends to infinity" or "(increases) without limit"	
				[7]			

2	(a)		$\det(\mathbf{A}-\lambda\mathbf{I}) = \begin{vmatrix} 10-\lambda & 12 & -8 \\ -1 & 2-\lambda & 4 \\ 3 & 6 & 2-\lambda \end{vmatrix}$	M1	1.1a	**DR** Formation of appropriate determinant	May be implied
			$= (10-\lambda)\left((2-\lambda)^2 - 4\times 6\right)$ $-12\left(-(2-\lambda)-4\times 3\right)-8\left(-1\times 6-3(2-\lambda)\right)$	M1	1.1	Attempt to expand determinant. Condone one minor slip	
			$= (10-\lambda)(\lambda^2-4\lambda+4-24)-12(-2+\lambda-12)$ $-8(-6-6+3\lambda)$				
			$= (10-\lambda)(\lambda^2-4\lambda-20)-12(\lambda-14)$ $-8(-12+3\lambda)$				
			$= 10\lambda^2-40\lambda-200-\lambda^3+4\lambda^2+20\lambda-12\lambda+168$ $+96-24\lambda$			**AG** Some intermediate working must be shown.	
			$= -\lambda^3+14\lambda^2-56\lambda+64$				
			So char eqn is $-\lambda^3 + 14\lambda^2 - 56\lambda + 64 = 0$	A1	1.1	Must be exactly the *equation* given.	

Question			Answer	Marks	AO	Guidance	
2	(a)		**Alternative method:** $\text{tr}(\mathbf{A}) = 10 + 2 + 2 = 14$ $\det(\mathbf{A}) == 10(4-24) - 12(-2-12) - 8(-6-6)$ $= -200 + 168 + 96 = 64$	**B1**		AG. Calculation of determinant and trace. Clear calculation must be seen; just tr(**A**) = 14 is **B0**.	
			$\text{tr}(\mathbf{A}^2) = \text{tr}\begin{pmatrix} 64 & 96 & -48 \\ 0 & 16 & 24 \\ 30 & 60 & 4 \end{pmatrix} = 64 + 16 + 4 = 84$ or $\begin{vmatrix} 2 & 4 \\ 6 & 2 \end{vmatrix} + \begin{vmatrix} 10 & -8 \\ 3 & 2 \end{vmatrix} + \begin{vmatrix} 10 & 12 \\ -1 & 2 \end{vmatrix}$ $= (4-24) + (20+24) + (20+12) = 56$	**M1**		Attempt to find either tr($\mathbf{A}^2$) or the trace of the minor matrix. Condone one minor slip.	
			So characteristic equation is $-\lambda^3 + 14\lambda^2 - \frac{1}{2}(14^2 - 84)\lambda + 64 = 0$ $\therefore -\lambda^3 + 14\lambda^2 - 56\lambda + 64 = 0$	**A1**		AG. From correct working only. Must be exactly the *equation* given.	
				[3]			

Question			Answer	Marks	AO	Guidance	
2	(b)		C-H $\Rightarrow -\mathbf{A}^3 + 14\mathbf{A}^2 - 56\mathbf{A} + 64\mathbf{I} = \mathbf{0}$	B1	1.1	Correct statement of C-H theorem using char eqn. Condone 0 or **O** for **0** but must be an equation	
			$\Rightarrow -\mathbf{A}^2 + 14\mathbf{A} - 56\mathbf{I} + 64\mathbf{A}^{-1} = \mathbf{0}$	M1	1.1	Multiplying throughout by $\mathbf{A}^{-1}$ to leave $\mathbf{A}^2$, $\mathbf{A}$ and $\mathbf{A}^{-1}$ terms. $\mathbf{OA}^{-1}$ must be **0** (or 0 or **O**).	
			$\Rightarrow \mathbf{A}^{-1} = \frac{1}{64}(\mathbf{A}^2 - 14\mathbf{A} + 56\mathbf{I})$	A1FT	1.1	Correct rearrangement of quoted characteristic eqn to find a matrix expression for $\mathbf{A}^{-1}$ (or $k\mathbf{A}^{-1}$)	
			$\mathbf{A}^{-1} = \frac{1}{64}\left(\begin{pmatrix}10&12&-8\\-1&2&4\\3&6&2\end{pmatrix}^2 - 14\begin{pmatrix}10&12&-8\\-1&2&4\\3&6&2\end{pmatrix} + 56\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right)$	M1	1.1	Clear evidence of substitution of **A** into their matrix equation for $k\mathbf{A}^{-1}$. NB $14\mathbf{A} = \begin{pmatrix}140&168&-112\\-14&28&56\\42&84&28\end{pmatrix}$	$\mathbf{A}^2 = \begin{pmatrix}64&96&-48\\0&16&24\\30&60&4\end{pmatrix}$ $-14\mathbf{A} + 56\mathbf{I} = \begin{pmatrix}-84&-168&112\\14&28&-56\\-42&-84&28\end{pmatrix}$
			$\mathbf{A}^{-1} = \frac{1}{64}\begin{pmatrix}-20&-72&64\\14&44&-32\\-12&-24&32\end{pmatrix}$	A1	1.1	Must be derived from correct working. or $\frac{1}{32}\begin{pmatrix}-10&-36&32\\7&22&-16\\-6&-12&16\end{pmatrix}$ or $\begin{pmatrix}-0.3125&-1.125&1\\0.21875&0.6875&-0.5\\-0.1875&-0.375&0.5\end{pmatrix}$	Correct answer from no working or other method: 0/5 or $\begin{pmatrix}-\frac{5}{16}&-\frac{9}{8}&1\\\frac{7}{32}&\frac{11}{16}&-\frac{1}{2}\\-\frac{3}{16}&-\frac{3}{8}&\frac{1}{2}\end{pmatrix}$
				[5]			

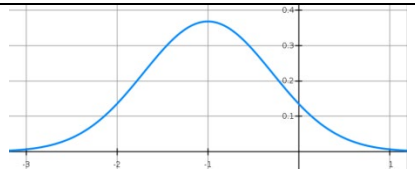
Question			Answer	Marks	AO	Guidance	
2	(c)		$-2^3 + 14\times2^2 - 56\times2 + 64$ $= -8 + 56 - 112 + 64 = 0 \Rightarrow (\lambda - 2)$ is a factor	B1	3.1a	For one linear factor soi	$(\lambda - 2)$, $(\lambda - 4)$ or $(\lambda - 8)$
			$-\lambda^3 + 14\lambda^2 - 56\lambda + 64$ $= \lambda^2(2 - \lambda) - 12\lambda(2 - \lambda) + 32(2 - \lambda)$ $= (2 - \lambda)(\lambda^2 - 12\lambda + 32) = (2 - \lambda)(4 - \lambda)(8 - \lambda)$	M1	1.1	Attempt to factorise (could also be by eg symbolic division or comparing coefficients)	
			So eigenvalues are 2, 4 and 8	A1	3.1a	soi	
			$\mathbf{D} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$	A1FT	1.1	FT their solutions in correct order	If **M0** then **SC2** for correct answer. If **M0** then **SC1** for incorrect order of correct diagonal elements.
				[4]			

Question			Answer	Marks	AO	Guidance	
3	(a)		CF: $5t_{n+1} - 4t_n = 0 \Rightarrow t_n = \alpha(4/5)^n$ PS: Try $t_n = an^2 + bn + c$ $5(a(n+1)^2 + b(n+1) + c) - 4(an^2 + bn + c) = 3n^2 + 28n + 6$ or $5(an^2 + bn + c) - 4(a(n-1)^2 + b(n-1) + c) = 3(n-1)^2 + 28(n-1) + 6$ oe $an^2 + (10a + b)n + 5a + 5b + c \equiv 3n^2 + 28n + 6$ $a = 3$, $10a + b = 28$, $5a + 5b + c = 6$ $a = 3$, $b = -2$, $c = 1$ so Gen Sol: $t_n = \alpha(4/5)^n + 3n^2 - 2n + 1$ $t_0 = 7 \Rightarrow \alpha = 6 \Rightarrow t_n = 6(4/5)^n + 3n^2 - 2n + 1$	B1 M1 M1 M1 A1FT B1FT [6]	1.1 1.1 1.1 1.1 1.1 1.1	Correct general form for particular solution Substituting their form correctly into recurrence relation. Expanding, collecting terms and comparing coefficients Their CF + correct PS Substituting $t_0 = 7$ into their CF + PS solution with a single arbitrary constant to derive a solution with a correct first term	
3	(b)	(i)	If $m = 3$, $v_n = \frac{6\times 0.8^n}{n^3} + \frac{3}{n} - \frac{2}{n^2} + \frac{1}{n^3}$ so $\lim_{n\to\infty} v_n$ exists and is equal to 0	B1FT [1]	2.2a	Some justification (eg expressing v_n in the form shown) must be given.	For parts **(i)**, **(ii)** and **(iii)** marks can be gained for answers correctly derived from solutions of the form $t_n = \alpha r^n + an^2 + bn + c$ provided that $\lvert r\rvert < 1$.
3	(b)	(ii)	If $m = 2$, $v_n = 3 + \frac{6\times 0.8^n}{n^2} - \frac{2}{n} + \frac{1}{n^2}$ so $\lim_{n\to\infty} v_n$ exists and is equal to "3"	B1FT [1]	2.2a	Their a (see guidance above). Some justification (eg expressing v_n in the form shown) must be given.	

Question			Answer	Marks	AO	Guidance	
3	(b)	(iii)	If $m = 1$, $v_n = 3n - 2 + \dfrac{6 \times 0.8^n}{n} + \dfrac{1}{n}$ so $\lim\limits_{n \to \infty} v_n$ does not exist	**B1FT** **[1]**	**2.2a**	Some justification (eg expressing v_n in the form shown) must be given.	Allow "infinity" or "infinite" or ∞ or "diverges" or "no limit" or "increases without bound" etc

Question			Answer	Marks	AO	Guidance	
4	(a)		$0 \circ 0 = 2 \Rightarrow k_3 = 2$	B1	3.1a		
			$a \circ e = k_1 a - k_2 e + k_3 = a$ for any a	M1	3.1a	Using identity correctly, one order of a/e, general or specific a.	eg $0 \circ e = -k_2 e + k_3 = 0$ but not $e \circ e$
			$k_1 = 1$	A1	1.1	Comparing coefficients (oe) to derive the value of k_1 or k_2.	(may be seen later)
			$e \circ a = k_1 e - k_2 a + k_3 = a$ for any a	M1	3.1a	Using identity correctly, other order of a/e, general or specific a.	eg $e \circ 0 = k_1 e + k_3 = 0$ but not $e \circ e$
			$k_1 = 1, k_2 = -1, k_3 = 2$ so $a \circ b = a + b + 2$ **AG**	A1	1.1	AG. Solving system of equations and concluding correctly	Do not award credit for work done based on assumptions (eg $e = -2$ or $\circ$ is associative).
				[5]			
4	(b)		So the identity element, e, is -2	B1	2.2a		
				[1]			
4	(c)		Because $b \circ a = b + a + 2 = a + b + 2 = a \circ b$, the operation $\circ$ **is** commutative over A	B1	2.1	Or stating that $a + b + 2$ is symmetrical in a and b.	Simply giving one or more examples is insufficient for **B1**.
				[1]			
4	(d)		$a, b \in \mathbb{R}$ (or A) $\Rightarrow a + b + 2 \in \mathbb{R}$ (or A) so closed	B1	2.1		
			$(a \circ b) \circ c = \ldots$ and $a \circ (b \circ c) = \ldots$	M1	3.1a	Consideration of both $(a \circ b) \circ c$ and $a \circ (b \circ c)$. Must be bracketed correctly	
			$a \circ (b \circ c) = a + b + c + 4 = (a \circ b) \circ c$ so associative	A1	2.5	Must be bracketed correctly (ie same order of a, b and c but different pair bracketed). Must have conclusion.	Or argument from symmetry of a, b and c.
			Need $a \circ b = -2$ or $a \circ a^{-1} = -2$ (or e)	M1	1.2	Knowledge that element and its inverse combine to give identity.	
			$a^{-1} (= b) = -4 - a$ which (is unique and) in A(or $\mathbb{R}$) so inverse property satisfied	A1FT	1.1	FT their e provided $e \neq 0$. $a^{-1} = e - 2 - a$	
			Identity property given so all 4 axioms satisfied so $(A, \circ)$ **is** a group	B1FT	2.4	Must have consistent conclusion. This mark can be awarded even if not all other marks awarded. If commutativity implied as a necessary axiom then **B0**	Ignore attempt to prove identity (unless the conclusion is that there is no identity).

Question			Answer	Marks	AO	Guidance	
				[6]			
4	(e)	(i)	If $A = \mathbb{Z}$ then the arguments in **(d)** still apply so there is no change	**B1** [1]	**2.2a**	From correct conclusion in **4(d)** only.	
4	(e)	(ii)	If $A = \{2m : m \in \mathbb{Z}\}$ then the arguments in **(d)** still apply so there is no change	**B1** [1]	**2.2a**	From correct conclusion in **4(d)** only.	
4	(e)	(iii)	If $A = \{n : n \in \mathbb{Z}, n \geq -2\}$ then the inverse property is not satisfied (eg $2^{-1} = -6 \notin A$) so $(A, \circ)$ is **not** a group	**B1** [1]	**2.2a**	From correct conclusion in **4(d)** only. An example does not need to be given but if given must be correct. Ignore other irrelevant comments but if incorrect statement about any of the other 3 axioms then **B0**.	Condone slightly incorrect statements (eg "No element in A has an inverse") provided that it is established that only the inverse property is not satisfied.

Question			Answer	Marks	AO	Guidance	
5	(a)	(i)	$\frac{\partial f}{\partial x} = y\,e^{-(x^2+2x+2)y} \times \left(-y(2x+2)\right)$ oe	B1 [1]	1.1	$-2y^2(x+1)e^{-(x^2+2x+2)y}$	
5	(a)	(ii)	$\frac{\partial f}{\partial y} = e^{-(x^2+2x+2)y} - y(x^2+2x+2)e^{-(x^2+2x+2)y}$ $= (1-y(x^2+2x+2))e^{-(x^2+2x+2)y}$ $= -(x^2y+2xy+2y-1)e^{-(x^2+2x+2)y}$	B1 [1]	2.1	Use of product and chain rules must be clear. **AG** Intermediate working must be shown.	
5	(a)	(iii)	$\frac{\partial f}{\partial x} = 0 \Rightarrow y = 0$ or $x = -1$ $y = 0 \Rightarrow \frac{\partial f}{\partial y} = 1$ so no SP $x = -1$ and $\frac{\partial f}{\partial y} = 0 \Rightarrow (1-y)e^{-y} = 0 \Rightarrow y = 1$ $(-1\ ,\ 1,\ e^{-1})$	M1 A1 M1 A1 [4]	1.1 1.1 1.1 1.1	Solving $f_x = 0$ to produce at least one numerical solution Eliminating $y = 0$ Using x-value and $f_y = 0$ condition to find a y-value. This mark can be awarded even with 'rogue' SPs or $y = 0$ not properly considered. Must be the only SP	
5	(b)		Translation, 1 unit in the negative x-direction and stretch, parallel to the z-axis, scale factor e^{-1}	B1 B1 [2]	3.1a 1.1	or –1 unit in the x-direction or defined by vector (may be 2D). Condone stretch "in y-direction" or "vertical" if intention clear. Must be exact value.	$f(x, 1) = e^{-(x^2+2x+2)} = e^{-1}e^{-(x+1)^2}$ The transformations can be given in either order.
5	(c)			B1FT [1]	1.1	Sketch of given graph with peak marked at $(-1,\ e^{-1})$. Condone awrt 0.37. FT their transformation if properly defined and stretch and translation only.	

Question			Answer	Marks	AO	Guidance	
5	(d)		The SP is a maximum point... ...since $e^{-1} > 0.25$ and if moving from the SP to the contour line in any direction we must go down since the contour line is closed.	**B1** **B1** **[2]**	**3.2a** **2.4**	Or sketch of section shows a maximum and a closed contour line around a single SP can only contain a maximum or a minimum	
5	(e)		$z = 0 \Rightarrow y = 0$	**B1**	**3.1a**	Could be seen at any stage	
			$\nabla \mathrm{f} = \begin{pmatrix} y\,\mathrm{e}^{-(x^2+2x+2)y} \times (-y(2x+2)) \\ \mathrm{e}^{-(x^2+2x+2)y} - y(x^2+2x+2)\,\mathrm{e}^{-(x^2+2x+2)y} \\ -1 \end{pmatrix}$	**M1**	**1.1**	Use of $\nabla \mathrm{f} = \frac{\partial \mathrm{f}}{\partial x}\mathbf{i} + \frac{\partial \mathrm{f}}{\partial y}\mathbf{j} - \mathbf{k}$ Could be derivative from **(a)**. Could be used once values found.	$\begin{pmatrix} -2y^2(x+1)\,\mathrm{e}^{-(x^2+2x+2)y} \\ \left(1 - y(x^2+2x+2)\right)\mathrm{e}^{-(x^2+2x+2)y} \\ -1 \end{pmatrix}$
			$y = 0 \Rightarrow \mathbf{n} = \nabla \mathrm{f} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ cwo	**A1**	**1.1**	Must be from the correct $\nabla \mathrm{f}$.	
			$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} . \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} = 0$ for any x so equation of tangent plane is $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$ oe	**A1FT**	**3.2a**	FT their normal with numerical components. Normal could be changed to eg $\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$	
			Alternative method for last 3 marks: $y = 0 \Rightarrow \frac{\partial \mathrm{f}}{\partial x} = 0,\ \frac{\partial \mathrm{f}}{\partial y} = 1$ $\therefore z - 0 = 0(x - a) + 1(y - 0)$ $\therefore$ Cartesian equation of tangent plane is $z = y$	**M1** **A1**		Calculating the values of the partial derivatives (possibly from**(a)**) and using $z - c = \frac{\partial \mathrm{f}}{\partial x}(x - a) + \frac{\partial \mathrm{f}}{\partial y}(y - b)$	

Question			Answer	Marks	AO	Guidance	
			So vector equation of plane is $\begin{pmatrix}0\\1\\-1\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}=0$	**A1**		or eg $\begin{pmatrix}0\\-1\\1\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}=0$	
				[4]			
5	**(f)**		Eqn of line is $\mathbf{r}=\lambda\begin{pmatrix}1\\0\\0\end{pmatrix}$	**B1**	**3.2a**	Or $\mathbf{r}=\begin{pmatrix}a\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\0\end{pmatrix}$ for any numerical a.	No justification for correct equation is necessary but must be in correct vector form with "**r** =" oe.
				[1]			

APPENDIX

Exemplar responses for Qxx

Response	Mark