OCR
Oxford Cambridge and RSA

Wednesday 22 June 2022 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

9453574818

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2022 [A/508/5598]
DC (LK/CT) 301596/6

OCR is an exempt Charity

stemup.co.uk

Answer **all** the questions.

1 $C = 3.7622$ and $S = 3.6269$ are used to approximate $\cosh 2$ and $\sinh 2$ respectively.

(a) Determine whether these approximations are the result of chopping or rounding the values of $\cosh 2$ and $\sinh 2$. **[1]**

(b) Calculate the relative error when $C^2 - S^2$ is used to approximate $\cosh^2 2 - \sinh^2 2$, giving your answer correct to **3** significant figures. **[2]**

(c) **Without** doing any further calculations, explain whether the same value for the relative error is obtained when $(C - S)^2$ is used to approximate $(\cosh 2 - \sinh 2)^2$. **[1]**

2 The table shows some values of x and the associated values of $y = \mathrm{f}(x)$.

x	2.75	3	3.25
$\mathrm{f}(x)$	0.920799	1	1.072858

(a) Calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$ using the forward difference method, giving your answer correct to **5** decimal places. **[2]**

(b) Calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$ using the central difference method, giving your answer correct to **5** decimal places. **[2]**

(c) Explain why your answer to part **(b)** is likely to be closer than your answer to part **(a)** to the true value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$. **[1]**

When $x = 5$ it is given that $y = 1.4645$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0.1820$, correct to **4** decimal places.

(d) Determine an estimate of the error when $\mathrm{f}(5)$ is used to estimate $\mathrm{f}(5.024)$. **[2]**

© OCR 2022

stemup.co.uk

3 The equation $f(x) = \sin^{-1}(x) - x + 0.1 = 0$ has a root α such that $-1 < \alpha < 0$.

Alex uses an iterative method to find a sequence of approximations to α. Some of the associated spreadsheet output is shown in the table.

	C	D	E
4	r	x_r	$f(x_r)$
5	0	−1	−0.4707963
6	1	−0.8	−0.0272952
7	2	−0.787691	−0.0193610
8	3	−0.7576546	−0.0020574
9	4	−0.7540834	−0.0001740
10	5		
11	6		

The formula in cell D7 is

=(D5*E6 − D6*E5)/(E6 − E5)

and equivalent formulae are in cells D8 and D9.

(a) State the method being used. **[1]**

(b) Use the values in the spreadsheet to calculate x_5 and x_6, giving your answers correct to **7** decimal places. **[3]**

(c) State the value of α as accurately as you can, justifying the precision quoted. **[1]**

Alex uses a calculator to check the value in cell D9, his result is -0.7540832686.

(d) Explain why this is different to the value displayed in cell D9. **[1]**

The value displayed in cell E11 in Alex's spreadsheet is $-1.4629\text{E-}09$.

(e) Write this value in standard mathematical notation. **[1]**

© OCR 2022

stemup.co.uk

4 **Fig. 4.1** shows part of the graph of $y = e^x - x^2 - x - 1.1$.

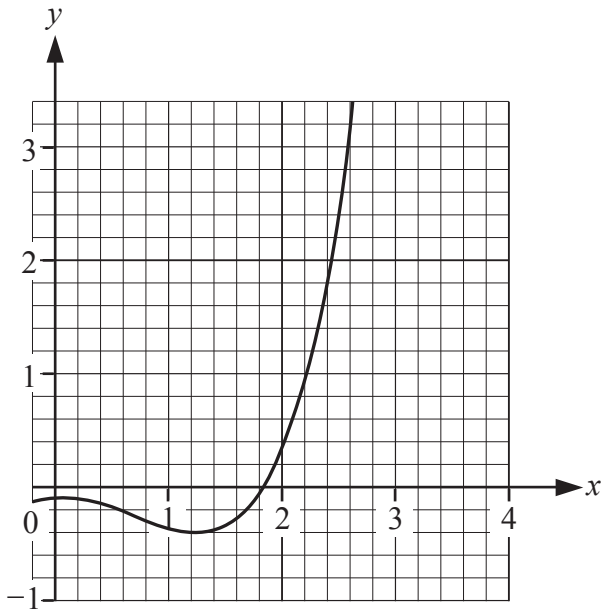


Fig. 4.1

The equation $e^x - x^2 - x - 1.1 = 0$ has a root α such that $1 < \alpha < 2$.

Ali is considering using the Newton-Raphson method to find α. Ali could use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$.

(a) **Without** doing any calculations, explain whether Ali should use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$, or whether using either starting value would work equally well. **[2]**

Ali is also considering using the method of fixed point iteration to find α. Ali could use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$.

Fig. 4.2 shows parts of the graphs of $y = x$ and $y = \ln(x^2 + x + 1.1)$.

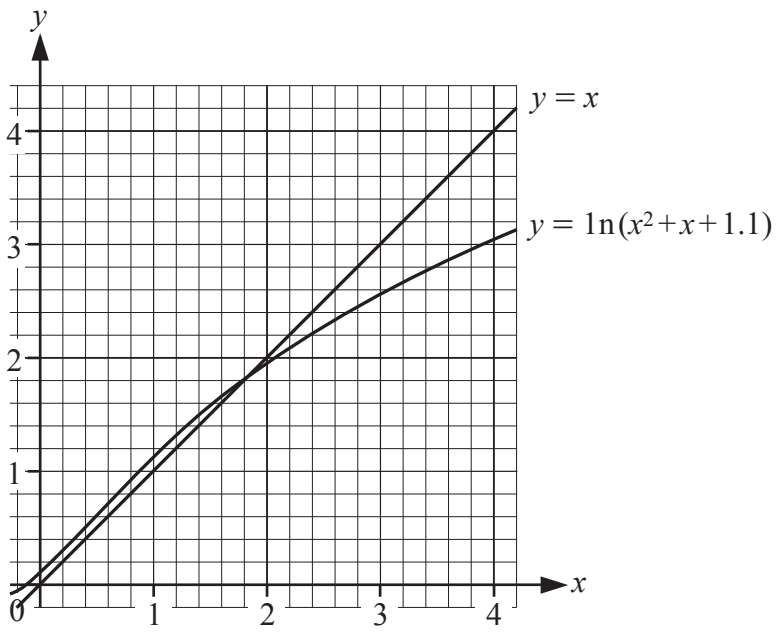


Fig. 4.2

© OCR 2022

stemup.co.uk

(b) **Without** doing any calculations, explain whether Ali should use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$ or whether either starting value would work equally well. **[2]**

Ali used one of the above methods to find a sequence of approximations to α. These are shown, together with some further analysis in the associated spreadsheet output in **Fig. 4.3**.

		M	N	O
	r	x_r		
4	0	2		
5	1	1.879008	−0.121	
6	2	1.858143	−0.021	0.172
7	3	1.857565	−6E-04	0.028
8	4	1.857564	−4E-07	8E-04
9	5	1.857564	−2E-13	6E-07

Fig. 4.3

The formula in cell N5 is `=M5−M4`

and the formula in cell O6 is `=N6/N5`

equivalent formulae are in cells N6 to N9 and O7 to O9 respectively.

(c) State what is being calculated in the following columns of the spreadsheet.

(i) Column N **[1]**

(ii) Column O **[1]**

(d) Explain whether the values in column O suggest that Ali used the Newton-Raphson method or the iterative formula $x_{n+1} = \ln\left(x_n^2 + x_n + 1.1\right)$ to find this sequence of approximations to α. **[2]**

stemup.co.uk

5 Kai uses the midpoint rule, trapezium rule and Simpson's rule to find approximations to $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$, where a and b are constants. The associated spreadsheet output is shown in the table. Some of the values are missing.

	F	G	H	I
3	n	M_n	T_n	S_{2n}
4	1	0.2436699	0.1479020	
5	2	0.2306967		

(a) Write down a suitable spreadsheet formula for cell H5. **[2]**

(b) Complete the copy of the table in the Printed Answer Booklet, giving the values correct to **7** decimal places. **[4]**

(c) Use your answers to part **(b)** to determine the value of $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$ as accurately as you can, justifying the precision quoted. **[3]**

6 Charlie uses fixed point iteration to find a sequence of approximations to the root of the equation $\sin^{-1}(x) - x^2 + 1 = 0$.

Charlie uses the iterative formula $x_{n+1} = \mathrm{g}(x_n)$, where $\mathrm{g}(x_n) = \sin\left(x_n^2 - 1\right)$.

Two sections of the associated spreadsheet output, showing x_0 to x_6 and x_{102} to x_{108}, are shown in **Fig. 6.1**.

r	x_r	difference	ratio
0	0		
1	−0.841471	−0.84147	
2	−0.287798	0.553673	−0.65798
3	−0.793885	−0.50609	−0.91405
4	−0.361379	0.432507	−0.85461
5	−0.763945	−0.40257	−0.93078
6	−0.404459	0.359486	−0.89299
102	−0.596302	0.004626	−0.95886
103	−0.600738	−0.00444	−0.95911
104	−0.596484	0.004254	−0.95887
105	−0.600564	−0.00408	−0.95910
106	−0.596652	0.003912	−0.95888
107	−0.600404	−0.00375	−0.95909
108	−0.596806	0.003598	−0.95889

Fig. 6.1

stemup.co.uk

(a) Use the information in **Fig. 6.1** to find the value of the root as accurately as you can, justifying the precision quoted. **[4]**

The relaxed iteration $x_{n+1} = (1-\lambda)x_n + \lambda g(x_n)$, with $\lambda = 0.51$ and $x_0 = 0$, is to be used to find the root of the equation $\sin^{-1}(x) - x^2 + 1 = 0$.

(b) Complete the copy of **Fig. 6.2** in the Printed Answer Booklet, giving the values of x_r correct to **7** decimal places and the values in the difference column and ratio column correct to **3** significant figures.

r	x_r	difference	ratio
0	0		
1			
2			
3			
4		-0.000192	
5		-1.99×10^{-7}	0.00103
6		-1.82×10^{-10}	0.000914

Fig. 6.2

[4]

(c) Write down the value of the root correct to **7** decimal places. **[1]**

(d) Explain why extrapolation could not be used in this case to find an improved approximation using this sequence of iterates. **[1]**

In this case the method of relaxation has been used to speed up the convergence of an iterative scheme.

(e) Name another application of the method of relaxation. **[1]**

stemup.co.uk

7 Sam decided to go on a high-protein diet. Sam's mass in kg, M, after t days of following the diet is recorded in **Fig. 7.1**.

t	0	10	20	30
M	88.3	80.05	78.7	78.85

Fig. 7.1

A difference table for the data is shown in **Fig. 7.2**.

t	M	ΔM	$\Delta^2 M$	$\Delta^3 M$
0	88.3			
10	80.05			
20	78.7			
30	78.85			

Fig. 7.2

(a) Complete the copy of the difference table in the Printed Answer Booklet. **[1]**

Sam's doctor uses these data to construct a cubic interpolating polynomial to model Sam's mass at time t days after starting the diet.

(b) Find the model in the form $M = at^3 + bt^2 + ct + d$, where a, b, c and d are constants to be determined. **[4]**

Subsequently it is found that when $t = 40$, $M = 78.7$ and when $t = 50$, $M = 80.05$.

(c) Determine whether the model is a good fit for these data. **[2]**

(d) By completing the extended copy of **Fig. 7.2** in the Printed Answer Booklet, explain why a quartic model may be more appropriate for the data. **[2]**

(e) Refine the doctor's model to include a quartic term. **[3]**

(f) Explain whether the new model for Sam's mass is likely to be appropriate over a longer period of time. **[2]**

END OF QUESTION PAPER

© OCR 2022

stemup.co.uk

BLANK PAGE

© OCR 2022

stemup.co.uk

BLANK PAGE

© OCR 2022

stemup.co.uk

BLANK PAGE

© OCR 2022

stemup.co.uk

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2022

stemup.co.uk

Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) $\cosh 2 = 3.762195\ldots$ and $\sinh 2 = 3.626860\ldots$

Chopping would give 3.7621 and 3.6268

$C = 3.7622$ and $S = 3.6269$, so both values come from rounding

1 (b) $C^2 - S^2 = 3.7622^2 - 3.6269^2$

$= 14.15414884 - 13.15440361 = 0.99974523$

True value: $\cosh^2 2 - \sinh^2 2 = 1$

$$\text{Relative error} = \frac{0.99974523 - 1}{1}$$

$= -0.000255$ (3 s.f.)

1 (c) No, not the same: the order of operations is different

$((C - S)^2$ subtracts first and then squares), so the errors propagate

differently and the relative error will be different

2 (a) $$\frac{dy}{dx} \approx \frac{f(3.25) - f(3)}{3.25 - 3} = \frac{1.072858 - 1}{0.25}$$

$$= \frac{0.072858}{0.25} = 0.29143 \text{ (5 d.p.)}$$

2 (b) $$\frac{dy}{dx} \approx \frac{f(3.25) - f(2.75)}{3.25 - 2.75} = \frac{1.072858 - 0.920799}{0.5}$$

$$= \frac{0.152059}{0.5} = 0.30412 \text{ (5 d.p.)}$$

Question number	
2 (c)	The central difference method is a second order method (it uses values either side of $x = 3$), whereas the forward difference method is only first order, so the central difference estimate is more accurate
2 (d)	$f(5.024) \approx f(5) + 0.024 \times f'(5)$ Error $\approx 0.024 \times 0.1820$ $= 0.004368$ So using $f(5)$ underestimates $f(5.024)$ by about 0.00437
3 (a)	Secant method
3 (b)	$x_5 = \dfrac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$ $= \dfrac{(-0.7576546)(-0.0001740) - (-0.7540834)(-0.0020574)}{-0.0001740 - (-0.0020574)}$ $x_5 = -0.7537535$ (7 d.p.) $f(x_5) = \sin^{-1}(x_5) - x_5 + 0.1 = -0.0000017290$ $x_6 = \dfrac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$ $x_6 = -0.7537502$ (7 d.p.)
3 (c)	$\alpha = -0.75375$, since the last two approximations (-0.7537535 and -0.7537502) agree to this precision
3 (d)	The spreadsheet works with the values stored to a higher precision than is displayed, whereas the calculator used the rounded (displayed) values from the table

Question number	
3 (e)	-1.4629×10^{-9}
4 (a)	Starting at $x_0 = 1$, the tangent there is shallow, so x_1 is further away from α (the iteration initially moves away from the root) Starting at $x_0 = 2$, the tangent meets the x-axis closer to α So Ali should use $x_0 = 2$
4 (b)	Either starting value would work ($x_0 = 2$ is nearer, so a little quicker), because the magnitude of the gradient of $y = \ln(x^2 + x + 1.1)$ is less than 1 at the point where it meets $y = x$, so the iteration converges to α from either side
4 (c)(i)	Column N: the differences between successive approximations, $x_n - x_{n-1}$
4 (c)(ii)	Column O: the ratio of successive differences
4 (d)	The ratios of differences are decreasing rapidly (not constant), so the convergence is faster than first order This suggests Ali used the Newton-Raphson method, not the fixed point iteration (which would give roughly constant ratios)
5 (a)	$T_2 = \frac{M_1 + T_1}{2}$ HS: =(G4+H4)/2
5 (b)	$T_2 = \frac{0.2436699 + 0.1479020}{2} = 0.1957860$

Question number

$$S_2 = \frac{2M_1 + T_1}{3} = \frac{2(0.2436699) + 0.1479020}{3} = 0.2117473$$

$$S_4 = \frac{2M_2 + T_2}{3} = \frac{2(0.2306967) + 0.1957860}{3} = 0.2190598$$

n	M_n	T_n	S_{2n}
1	0.2436699	0.1479020	0.2117473
2	0.2306967	0.1957860	0.2190598

5 (c) Simpson's rule is fourth order, so use Richardson's extrapolation:

$$\frac{16\,S_4 - S_2}{15} = \frac{16(0.2190598) - 0.2117473}{15}$$

$$= 0.2195473$$

$\int_a^b f(x)\,dx = 0.220$ (3 s.f.), this precision is justified

by the increased accuracy from Richardson's extrapolation

6 (a) The ratio of differences is roughly constant, $r \approx -0.95889$

so the convergence is first order: extrapolate using

$x \approx x_{108} + d \times \frac{r}{1 - r}$, with $d = 0.003598$

$$= -0.596806 + 0.003598 \times \frac{-0.95889}{1 + 0.95889}$$

$$= -0.596806 - 0.001761$$

$$= -0.598567$$

Root = -0.5986 (4 d.p.), this precision is justified by the

increased accuracy from extrapolation

Question number

6 (b)

$x_{n+1} = 0.49\,x_n + 0.51\sin(x_n^2 - 1)$

$x_1 = 0.51 \times \sin(-1) = -0.4291502$

r	x_r	difference	ratio
0	0		
1	−0.4291502	−0.429	
2	−0.5817139	−0.153	0.356
3	−0.5983772	−0.0167	0.109
4	−0.5985695	−0.000192	0.0115
5	−0.5985697	-1.99×10^{-7}	0.00103
6	−0.5985697	-1.82×10^{-10}	0.000914

6 (c)

Root = −0.5985697 (7 d.p.)

6 (d)

The ratio of differences is not constant (it is decreasing), so the differences do not form a geometric sequence and extrapolation cannot be used

6 (e)

To adapt an iterative scheme which does not converge into one which will converge

7 (a)

t	M	ΔM	$\Delta^2 M$	$\Delta^3 M$
0	88.3			
		−8.25		
10	80.05		6.9	
		−1.35		−5.4
20	78.7		1.5	
		0.15		

Question number

30 78.85

7 (b) Newton's forward difference formula with $h = 10$:

$$M = 88.3 - \frac{8.25t}{10} + \frac{6.9t(t-10)}{2! \times 10^2} - \frac{5.4t(t-10)(t-20)}{3! \times 10^3}$$

$= 88.3 - 0.825t + 0.0345(t^2 - 10t) - 0.0009(t^3 - 30t^2 + 200t)$

$= 88.3 - 0.825t + 0.0345t^2 - 0.345t - 0.0009t^3 + 0.027t^2 - 0.18t$

$M = -0.0009t^3 + 0.0615t^2 - 1.35t + 88.3$

7 (c) $t = 40$: $M = -0.0009(64000) + 0.0615(1600) - 1.35(40) + 88.3 = 75.1$

$t = 50$: $M = -0.0009(125000) + 0.0615(2500) - 1.35(50) + 88.3 = 62.05$

Actual values are 78.7 and 80.05, so the model is a poor fit for these data

7 (d)

t	M	ΔM	$\Delta^2 M$	$\Delta^3 M$	$\Delta^4 M$
0	88.3				
		−8.25			
10	80.05		6.9		
		−1.35		−5.4	
20	78.7		1.5		3.6
		0.15		−1.8	
30	78.85		−0.3		3.6
		−0.15		1.8	
40	78.7		1.5		
		1.35			
50	80.05				

The fourth differences are constant (both 3.6), so a quartic model fits all six data points and may be more appropriate

Question number

7 (e) Add the next term of the forward difference formula:

$$M = -0.0009t^3 + 0.0615t^2 - 1.35t + 88.3 + \frac{3.6t(t-10)(t-20)(t-30)}{4! \times 10^4}$$

$$\frac{3.6}{240000} = 0.000015, \quad t(t-10)(t-20)(t-30) = t^4 - 60t^3 + 1100t^2 - 6000t$$

extra term $= 0.000015t^4 - 0.0009t^3 + 0.0165t^2 - 0.09t$

$M = 0.000015t^4 - 0.0018t^3 + 0.078t^2 - 1.44t + 88.3$

7 (f) No: as $t \to \infty$, the t^4 term dominates so $M \to \infty$, i.e. the model predicts that Sam's mass will keep increasing without limit

So the model is not appropriate over a longer period of time