OCR
Oxford Cambridge and RSA

GCE

Further Mathematics B MEI

Y434/01: Numerical methods

A Level

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range. NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		sinh2 = 3.62686(041)… and cosh2 = 3.76219(5691)..			**Both** values shown, need to see3.62686…**and** 3.76219**…**
			so both values are from rounding	**B1**	**1.1**	
				[1]		
1	(b)		$\frac{0.99974523-1}{1}$	**M1**	**1.1**	allow $\frac{1-0.99974523}{1}$ **NB** ± 0.00025477 to 2 or more sf unsupported implies **M1**
			–0.000255 or 0.000255	**A1**	**1.1**	**B2** for ± 0.000255 unsupported
				[2]		
1	(c)		not the same, since the order of operations is different **oe**	**B1**	**2.4**	must refer to **order** of operations being different
				[1]		
2	(a)		$\frac{1.072858-1}{3.25-3}$ **oe**	**M1**	**1.1a**	**NB** $\frac{0.072858}{0.25}$ implies **M1**
			0.29143 **cao**	**A1**	**1.1**	
				[2]		
2	(b)		$\frac{1.072858-0.920799}{3.25-2.75}$ **oe**	**M1**	**1.1a**	**NB** $\frac{0.152059}{0.5}$ implies **M1**
			0.30412 **cao**	**A1**	**1.1**	
				[2]		

Question			Answer	Marks	AO	Guidance
2	(c)		central difference method is a 2nd order method whereas forward difference method is 1st order **oe** *or* central difference method uses values either side of 3 **oe** whereas forward difference method steps in the positive *x*-direction.	**E1** **[1]**	**2.4**	must refer to both methods must refer to both methods
2	(d)		0.024×0.182 **soi** ± 0.004368 **or** ± 0.00437 **or** $\pm$ 0.0044	**M1** **A1** **[2]**	**3.1a** **1.1**	may be embedded mark the final answer
3	(a)		Secant (method)	**B1** **[1]**	**2.2a**	
3	(b)		$\frac{-0.7576546\times-0.000174-(-0.7540834)\times(-0.0020574)}{(-0.000174-(-0.0020574))}$ **oe** –0.7537535 –0.7537502	**M1** **A1** **A1** **[3]**	**1.1** **1.1** **1.1**	allow sign errors; may be implied by equivalent cell formulae or by correct answers to 3 or more dp allow –0.7537501 from working with values rounded to 7 dp
3	(c)		–0.75375 **cao** since the last two approximations agree to this precision **or** –0.753750 is possible since convergence is faster than 1st order **oe**	**B1** **[1]**	**2.2b**	must see value **and** justification

Question			Answer	Marks	AO	Guidance
3	(d)		the spreadsheet uses the numbers to a higher precision than is displayed; the calculator uses the displayed values **oe**	**B1** **[1]**	**2.4**	**B0** if answer spoiled by eg spreadsheet stores exact values
3	(e)		-1.4629×10^{-9} or – 0.000 000 001 462 9	**B1** **[1]**	**1.1**	
4	(a)		with $x_0 = 1$ the value of x_1 is further away from α **oe** with $x_0 = 2$ the value of x_1 is closer to α so better to use $x_0 = 2$	**B1** **B1** **[2]**	**2.2a** **2.2a**	(initially) diverges starting at 1 allow converges more slowly with $x_0 = 1$ **B1** for one correct statement and **B1** for 2nd correct statement **and** better to start at 2
4	(b)		can use either (but better to start at $x = 2$ because it's nearer) because the magnitude of the gradient of $(y =) \ln(x^2 + x + 1.1)$ is less than 1 at the point of intersection with $y = x$ **oe**	**B1** **B1** **[2]**	**2.4** **2.2a**	 allow gradient of the curve for gradient of $(y =) \ln(x^2 + x + 1.1)$

Question			Answer	Marks	AO	Guidance
4	(c)	(i)	difference between x_{n+1} and x_n **oe**	**B1** **[1]**	**2.2a**	
4	(c)	(ii)	ratio of differences	**B1** **[1]**	**2.2a**	
4	(d)		ratio of differences decreasing so convergence faster than first order which suggests she used Newton-Raphson method	**B1** **B1** **[2]**	**2.2a** **2.2b**	**or** ratio of differences not constant so convergence not first order which means she **didn't** use fixed point iteration – she probably used Newton-Raphson **B0B0** for reasoning based on values in columns M or N
5	(a)		$\frac{M_1+T_1}{2}$ used =(G4+H4)/2 **oe**	**M1** **A1** **[2]**	**1.1a** **1.1**	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>5</td><td>(b)</td><td></td><td>$S_2 = \frac{2M_1+T_1}{3}$ or $S_4 = \frac{2M_2+T_2}{3}$ or $\frac{4T_2-T_1}{3}$ or $T_2 = \frac{M_1+T_1}{2}$ used soi
<table><tr><th>M_n</th><th>T_n</th><th>S_{2n}</th></tr><tr><td>0.2436699</td><td>0.1479020</td><td>0.2117473</td></tr><tr><td>0.2306967</td><td>0.1957860</td><td>0.2190598</td></tr></table></td><td>M1

A1
A1
A1

[4]</td><td>3.1a

1.1
1.1
1.1</td><td>use of appropriate formula, may be implied by one correct answer

if all correct values given to greater precision allow M1A1A1A0

if table incorrect or incomplete, allow up to M1A1A1 for work seen in the space below</td></tr>
<tr><td>5</td><td>(c)</td><td></td><td>$\frac{16\times their\ 0.2190598 - their\ 0.2117472}{15}$ oe

0.2195473 – 0.21954731

0.220 is possible due to increased accuracy from Richardson’s extrapolation</td><td>M1

A1

A1

[3]</td><td>3.1a

1.1

1.1</td><td>or $0.2190598 + \frac{0.2190598-0.2117473}{16}$; may be implied by 0.2195168…

allow 0.22 is secure by comparison with S_4</td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>6</td><td>(a)</td><td></td><td>$x + d \times \frac{r}{1-r}$ or $x + d \times r$

$x = -0.596806, d = 0.003598,$
$r = -0.95889\ or\ -0.9589\ or\ -0.959\ or\ -0.96$

awrt -0.598567 to awrt -0.598568

-0.5986 or -0.599 is possible due to increased accuracy from extrapolation oe</td><td>M1

A1

A1

A1

[4]</td><td>3.1a

2.1

1.1

2.2b</td><td>at least two values from information in table; allow sign errors

allow -0.60 is certain by comparison with x_{108} oe
the final mark is only available if all other marks earned

if M0 allow SC1 for -0.60 is certain by comparison of x_{108} and x_{107} and confirmation by correct change of sign test</td></tr>
<tr><td>6</td><td>(b)</td><td></td><td>
<table>
<tr><th>r</th><th>x_r</th><th>difference</th><th>ratio</th></tr>
<tr><td>0</td><td>0</td><td></td><td></td></tr>
<tr><td>1</td><td>–0.4291502</td><td>–0.429</td><td></td></tr>
<tr><td>2</td><td>–0.5817139</td><td>–0.153</td><td>0.356</td></tr>
<tr><td>3</td><td>–0.5983772</td><td>–0.0167</td><td>0.109</td></tr>
<tr><td>4</td><td>–0.5985695</td><td>–0.000192</td><td>0.0115</td></tr>
<tr><td>5</td><td>–0.5985697</td><td>-1.99×10^{-7}</td><td>0.00103</td></tr>
<tr><td>6</td><td>–0.5985697</td><td>-1.82×10^{-10}</td><td>0.000914</td></tr>
</table>
</td><td>M1
A1
A1
A1
[4]</td><td>1.1
1.1
1.1
1.1</td><td>two iterates seen to 3 or more dp
iterates
differences
ratios

allow M1A1A1A0 for correct values to greater precision</td></tr>
<tr><td>6</td><td>(c)</td><td></td><td>–0.5985697 cao</td><td>B1
[1]</td><td>2.2a</td><td></td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th colspan="2">Guidance</th></tr>
<tr><td>6</td><td>(d)</td><td></td><td>ratio of differences is not constant</td><td>B1
[1]</td><td>2.4</td><td colspan="2"></td></tr>
<tr><td>6</td><td>(e)</td><td></td><td>to adapt an iterative scheme which does not converge into one which will converge oe</td><td>B1
[1]</td><td>1.2</td><td colspan="2"></td></tr>
<tr><td>7</td><td>(a)</td><td></td><td>
<table>
<tr><td>t</td><td>M</td><td>ΔM</td><td>$\Delta^2 M$</td><td>$\Delta^3 M$</td></tr>
<tr><td>0</td><td>88.3</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>–8.25</td><td></td><td></td></tr>
<tr><td>10</td><td>80.05</td><td></td><td>6.9</td><td></td></tr>
<tr><td></td><td></td><td>–1.35</td><td></td><td>–5.4</td></tr>
<tr><td>20</td><td>78.7</td><td></td><td>1.5</td><td></td></tr>
<tr><td></td><td></td><td>0.15</td><td></td><td></td></tr>
<tr><td>30</td><td>78.85</td><td></td><td></td><td></td></tr>
</table>
</td><td>B1
[1]</td><td>1.1</td><td colspan="2"></td></tr>
<tr><td>7</td><td>(b)</td><td></td><td>$M = 88.3 - \frac{8.25t}{10} + \frac{6.9t(t-10)}{2!\times 10^2} - \frac{5.4t(t-10)(t-20)}{3!\times 10^3}$

$M = -0.0009t^3 + 0.0615t^2 - 1.35t + 88.3$</td><td>M1

A1
A1

A1

[4]</td><td>3.3

1.1
1.1

1.1</td><td>soi; allow bracket and/or sign errors; allow use of different variable

two correct coefficients in cubic
three correct coefficients

all correct

NB $b = \frac{123}{2000}$</td><td>FT their –5.4 and their 6.9

FT first A1 only

A0 if different variable; must see “$M =$” at some stage</td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th><th></th></tr>
<tr><td>7</td><td>(c)</td><td></td><td>evaluation of M at $t = 40$ and $t = 50$

75.1 and 62.05 obtained respectively,
so a poor fit for these data oe</td><td>M1

A1FT
[2]</td><td>3.4

3.5a</td><td>FT their cubic, dependent on award of M1 in (b)

may be stated separately</td><td></td></tr>
<tr><td>7</td><td>(d)</td><td></td><td>
<table>
<tr><th>t</th><th>W</th><th>ΔW</th><th>$\Delta^2 W$</th><th>$\Delta^3 W$</th><th>$\Delta^4 W$</th></tr>
<tr><td>0</td><td>88.3</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>–8.25</td><td></td><td></td><td></td></tr>
<tr><td>10</td><td>80.05</td><td></td><td>6.9</td><td></td><td></td></tr>
<tr><td></td><td></td><td>–1.35</td><td></td><td>–5.4</td><td></td></tr>
<tr><td>20</td><td>78.7</td><td></td><td>1.5</td><td></td><td>3.6</td></tr>
<tr><td></td><td></td><td>0.15</td><td></td><td>–1.8</td><td></td></tr>
<tr><td>30</td><td>78.85</td><td></td><td>–0.3</td><td></td><td>3.6</td></tr>
<tr><td></td><td></td><td>–0.15</td><td></td><td>1.8</td><td></td></tr>
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<tr><td></td><td></td><td>1.35</td><td></td><td></td><td></td></tr>
<tr><td>50</td><td>80.05</td><td></td><td></td><td></td><td></td></tr>
</table>
</td><td>B1

B1

[2]</td><td>1.1

2.4</td><td>difference table correct

4th differences same (so quartic may be good model)</td><td></td></tr>
<tr><td>7</td><td>(e)</td><td></td><td>$-0.0009t^3 + 0.0615t^2 - 1.35t + 88.3$

$+ \frac{3.6t(t-10)(t-20)(t-30)}{4! \times 10^4}$

$W = 0.000015t^4 - 0.0018t^3 + 0.078t^2 - 1.44t + 88.3$</td><td>M1

M1

A1
[3]</td><td>3.5c

3.3

1.1</td><td>for addition of their cubic to extra term

quartic term; FT their 3.6

condone use of other variable</td><td>NB
may see
$\frac{3}{200000}, -\frac{9}{5000}, \frac{39}{500}, -\frac{36}{25}$</td></tr>
</table>

Question			Answer	Marks	AO	Guidance	
7	(f)		the model predicts that Sam's weight will continue to increase **oe** allow eg $t \to \infty, M \to \infty$ so not appropriate (in long run)	**M1** **A1** **[2]**	**3.5a** **3.5b**	must be in context; dependent on award of 2nd **M1** in **(e)**	do not allow eg increases exponentially

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