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# Friday 17 June 2022 – Afternoon

## A Level Further Mathematics B (MEI)

**Y432/01** Statistics Minor

**Time allowed: 1 hour 15 minutes**

*9452616730*

**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

**ADVICE**
- Read each question carefully before you start your answer.

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Answer **all** the questions.

**1** In a quiz a contestant is asked up to four questions. The contestant's turn ends once the contestant gets a question wrong or has answered all four questions. The probability that a particular contestant gets any question correct is 0.6, independently of other questions. The discrete random variable $X$ models the number of questions which the contestant gets correct in a turn.

**(a)** Show that $P(X = 4) = 0.1296$. **[1]**

The probability distribution of $X$ is shown in **Fig. 1.1**.

| $r$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = r)$ | 0.4 | 0.24 | 0.144 | 0.0864 | 0.1296 |

**Fig. 1.1**

**(b)** Find each of the following.

- $E(X)$
- $Var(X)$ **[2]**

The number of points that a contestant scores is as shown in **Fig. 1.2**.

| Number of questions correct | Number of points scored |
|---|---|
| 0 or 1 | 0 |
| 2 | 2 |
| 3 | 3 |
| 4 | 5 |

**Fig. 1.2**

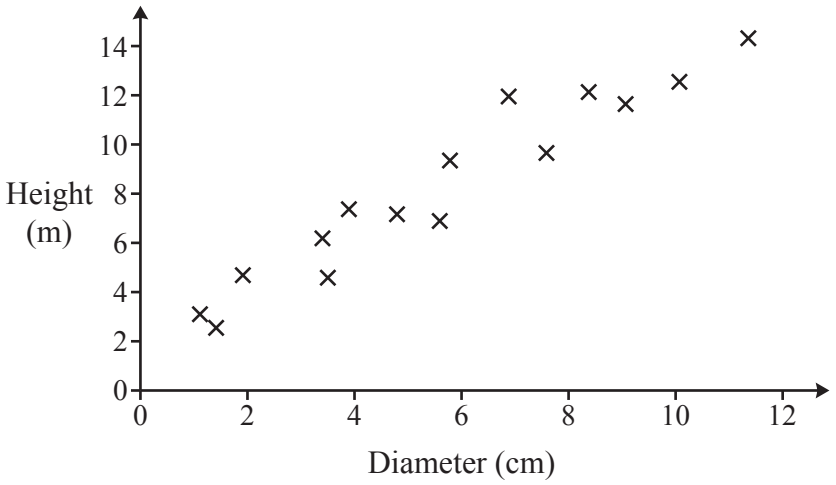
The discrete random variable $Y$ models the number of points which the contestant scores.

**(c)** Without doing any working, explain whether each of the following will be less than, equal to or greater than the corresponding value for $X$.

- $E(Y)$
- $Var(Y)$ **[3]**

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**2** A forester is investigating the relationship between the diameter and the height of young beech trees. She selects a random sample of 15 young beech trees in a forest and records their diameters, $d$ cm, and their heights, $h$ m. The data are illustrated in the scatter diagram.




**(a)** State whether either or both of the variables $d$ and $h$ are random variables. **[1]**

Summary data for the diameters and heights are as follows.

$n = 15 \quad \Sigma d = 84.9 \quad \Sigma h = 124.7 \quad \Sigma d^2 = 624.55 \quad \Sigma h^2 = 1230.57 \quad \Sigma dh = 866.63$

**(b)** Find the equation of the regression line of $h$ on $d$. Give your answer in the form $h = ad + b$, giving the values of $a$ and $b$ correct to **2** decimal places. **[4]**

**(c)** Use the regression line to predict the heights of beech trees with the following diameters.

- 7.5 cm
- 20.0 cm

**[2]**

**(d)** Comment on the reliability of your predictions. **[3]**

**(e)** There are many mature beech trees with diameter of 60 cm or greater. However, there are no beech trees with a height of more than 50 m.

Comment on this in relation to your regression line. **[2]**

**(f)** State the coordinates of the point at which the regression line of $d$ on $h$ meets the line which you calculated in part **(b)**. **[1]**

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**3** Jane wonders whether the number of wasps entering a wasp's nest per 5 second interval can be modelled by a Poisson distribution with mean $\mu$. She counts the number of wasps entering the nest over 60 randomly selected 5-second intervals. The results are shown in **Fig. 3.1**.

| Number of wasps | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ⩾10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 0 | 2 | 5 | 5 | 12 | 10 | 10 | 11 | 1 | 4 | 0 |

**Fig. 3.1**

**(a)** Show that a suitable estimate for the value of $\mu$ is 5.1. **[2]**

**Fig. 3.2** shows part of a screenshot for a $\chi^2$ test to assess the goodness of fit of a Poisson model. The sample mean has been used as an estimate for the population mean. Some of the values in the spreadsheet have been deliberately omitted.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Number of wasps | Observed frequency | Poisson probability | Expected frequency | Chi-squared contribution |
| 2 | ⩽2 | 7 | 0.1165 | 6.9887 | 0.0000 |
| 3 | 3 | 5 | | 8.0874 | 1.1786 |
| 4 | 4 | 12 | | | 0.2765 |
| 5 | 5 | 10 | | | 0.0255 |
| 6 | 6 | 10 | 0.1490 | 8.9400 | 0.1257 |
| 7 | 7 | 11 | 0.1086 | 6.5134 | 3.0904 |
| 8 | ⩾8 | 5 | 0.1440 | 8.6414 | |
| 9 | | | | | |

**Fig. 3.2**

**(b)** Determine the missing values in each of the following cells, giving your answers correct to **4** decimal places.

- C3
- D5
- E8

**[4]**

**(c)** Explain why some of the frequencies have been combined into the categories ⩽2 and ⩾8. **[1]**

**(d)** **In this question you must show detailed reasoning.**

Carry out the hypothesis test at the 5% significance level. **[6]**

**(e)** Jane also carries out a $\chi^2$ test for the number of wasps leaving another nest. As part of her calculations, she finds that the probability of no wasps leaving the nest in a 5-second period is 0.0053. She finds that a Poisson distribution is also an appropriate model in this case.

Find a suitable estimate for the value of the mean number of wasps leaving the nest per 5-second period. **[2]**

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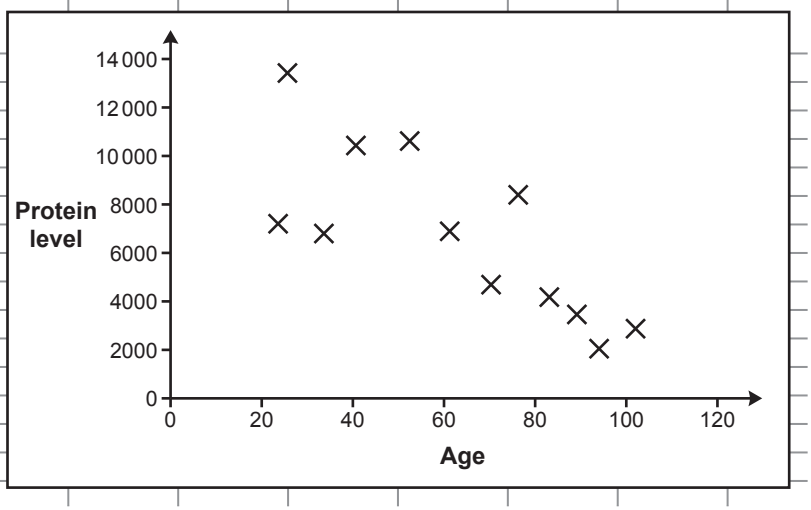
**4** Alex is practising bowling at a cricket wicket. Every time she bowls a ball, she has a 30% chance of hitting the wicket.

**(a)** Assuming that successive bowls are independent, determine the probability that Alex first hits the wicket on her third attempt. **[2]**

**(b)** Determine the probability that Alex hits the wicket for the fourth time on her tenth attempt. **[3]**

**5** A medical researcher is investigating whether there is any relationship between the age of a person and the level of a particular protein in the person's blood. She measures the levels of the protein (measured in suitable units) in a random sample of 12 hospital patients of various ages (in years). The spreadsheet shows the values obtained, together with a scatter diagram which illustrates the data.

| | A | B |
|---|---|---|
| 1 | **Age** | **Protein level** |
| 2 | 23 | 7200 |
| 3 | 25 | 13400 |
| 4 | 33 | 6800 |
| 5 | 40 | 10400 |
| 6 | 52 | 10600 |
| 7 | 61 | 6900 |
| 8 | 70 | 4700 |
| 9 | 76 | 8400 |
| 10 | 83 | 4200 |
| 11 | 89 | 3500 |
| 12 | 94 | 2100 |
| 13 | 102 | 2900 |




**(a)** The researcher decides that a test based on Pearson's product moment correlation coefficient may **not** be valid. Explain why she comes to this conclusion. **[2]**

**(b)** Calculate the value of Spearman's rank correlation coefficient. **[3]**

**(c)** Carry out a test based on this coefficient at the 5% significance level to investigate whether there is any association between age and protein level. **[5]**

**(d)** Explain why the researcher chose a sample that was random. **[2]**

**(e)** The researcher had originally intended to use a sample size of 6 rather than the 12 that she actually used.

Explain what advantage there is in using the larger sample size. **[2]**

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**6** The random variable $X$ has a uniform distribution over the values $\{1, 4, 7, \ldots, 3n - 2\}$, where $n$ is a positive integer.

**(a)** Determine $\text{Var}(X)$ in terms of $n$. **[3]**

**(b)** Given that $n = 100$, find the probability that $X$ is within one standard deviation of the mean. **[4]**

**END OF QUESTION PAPER**

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