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Wednesday 15 June 2022 – Afternoon

A Level Further Mathematics B (MEI)

Y431/01 Mechanics Minor

Time allowed: 1 hour 15 minutes

9452550438

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space, use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Answer **all** the questions.

1 Newton's gravitational constant, G, is approximately 6.67×10^{-11} N m^2 kg^{-2}.

(a) Find the dimensions of G. **[2]**

The escape velocity, v, of a body from a planet's surface, is given by the formula

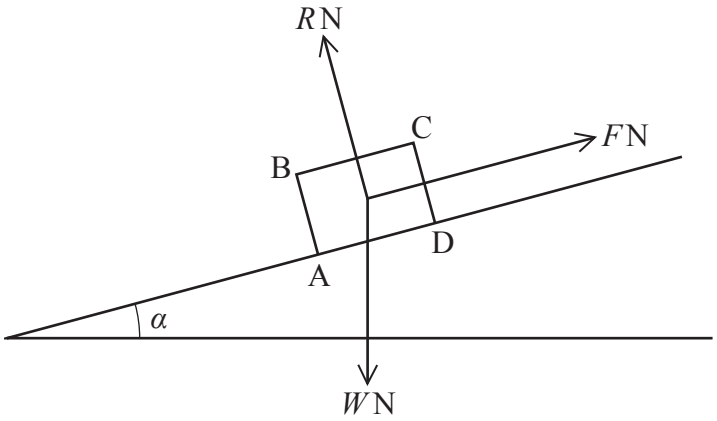
$v = kG^{\alpha}M^{\beta}r^{\gamma}$,

where M is the planet's mass, r is the planet's radius and k is a dimensionless constant.

(b) Use dimensional analysis to find α, β and γ. **[4]**

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2 The diagram below shows the cross-section through the centre of mass of a uniform block of weight WN, resting on a slope inclined at an angle α to the horizontal. The cross-section is a rectangle ABCD. The slope exerts a frictional force of magnitude FN and a normal contact force of magnitude RN.



(a) Explain why a triangle of forces may be used to model the scenario. **[2]**

(b) In the space provided in the Printed Answer Booklet, draw such a triangle, fully annotated, including the angle α in the correct position. **[2]**

The coefficient of friction between the block and the slope is μ.

(c) Given that the block is in limiting equilibrium, use your diagram in part **(b)** to show that $\mu = \tan\alpha$. **[2]**

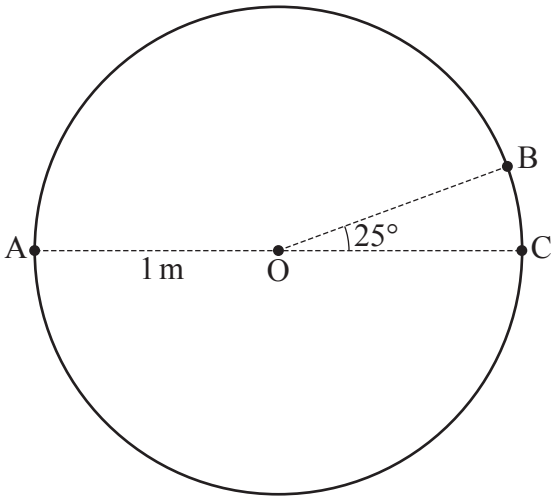
It is given that AB = 8.9 cm and AD = 11.6 cm. The coefficient of friction between the slope and the block is 1.35. The slope is slowly tilted so that α increases.

(d) Determine whether the block topples first without sliding or slides first without toppling. **[2]**

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3 A rough circular hoop, with centre O and radius 1 m, is fixed in a vertical plane. A, B and C are points on the hoop such that A and C are at the same horizontal level as O, and OB makes an angle of 25° above the horizontal, as shown in the diagram.



A bead P of mass 0.3 kg is threaded onto the hoop. P is projected vertically downwards from A on two separate occasions.

- The first time, when P is projected with a speed of $4\,\text{m}\,\text{s}^{-1}$, it first comes to rest at B.
- The second time, when P is projected with a speed of $v\,\text{m}\,\text{s}^{-1}$, it first comes to rest at C.

The situation is modelled by assuming that during the motion of P the magnitude of the frictional force exerted by the hoop on P is constant.

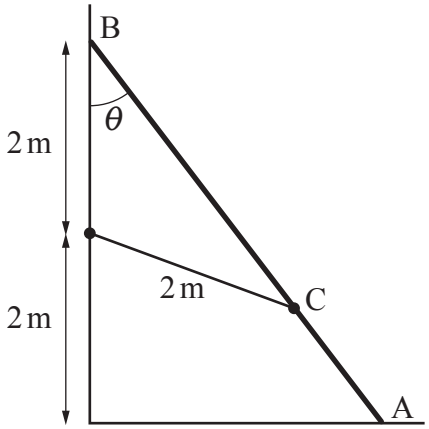
(a) Determine the value of v. **[5]**

(b) Comment on the validity of the modelling assumption used in this question. **[1]**

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4 A uniform beam AB of mass 6 kg and length 5 m rests with its end A on smooth horizontal ground and its end B against a smooth vertical wall. The vertical distance between the ground and B is 4 m, and the angle between the beam and the downward vertical is θ. To prevent the beam from sliding, one end of a light taut rope of length 2 m is attached to the beam at C and the other end of the rope is attached to a point on the wall 2 m above the ground, as shown in the diagram.



(a) By considering the value of $\cos\theta$, determine the distance BC. **[2]**

An object of mass 75 kg is placed on the beam at a point which is x m from A.

It is given that the tension in the rope is T N and the magnitude of the normal contact force between the ground and the beam is R N.

(b) By taking moments about B for the beam, show that $25R + 3675x - 16T = 19110$. **[4]**

(c) Given that the rope can withstand a maximum tension of 720 N, determine the largest possible value of x. **[4]**

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5 Point A lies 20 m vertically below a point B. A particle P of mass $4m$ kg is projected upwards from A, at a speed of $17.5\,\text{m s}^{-1}$. At the same time, a particle Q of mass m kg is released from rest at point B. The particles collide directly, and it is given that the coefficient of restitution in the collision between P and Q is 0.6.

(a) Show that, immediately after the collision, P continues to travel upwards at $0.7\,\text{m s}^{-1}$ and determine, at this time, the corresponding velocity of Q. **[8]**

In another situation, a particle of mass $3m$ kg is released from rest and falls vertically. After it has fallen 10 m, it explodes into two fragments. Immediately after the explosion, the lower fragment, of mass $2m$ kg, moves vertically downwards with speed $v_1\,\text{m s}^{-1}$, and the upper fragment, of mass m kg, moves vertically upwards with speed $v_2\,\text{m s}^{-1}$.

(b) Given that, in the explosion, the kinetic energy of the system increases by 72%, show that $2v_1^2 + v_2^2 = 1011.36$. **[3]**

(c) By finding another equation connecting v_1 and v_2, determine the speeds of the fragments immediately after the explosion. **[6]**

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6 **Fig. 6.1** shows a light rod ABC, of length 60 cm, where B is the midpoint of AC. Particles of masses 3.5 kg, 1.4 kg and 2.1 kg are attached to A, B and C respectively.

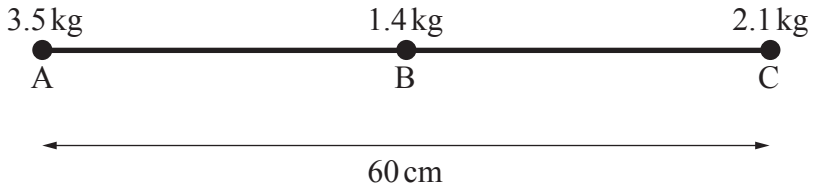


Fig. 6.1

The centre of mass is located at a point G along the rod.

(a) Determine the distance AG. **[2]**

Two light inextensible strings, each of length 40 cm, are attached to the rod, one at A, the other at C. The other ends of these strings are attached to a fixed point D. The rod is allowed to hang in equilibrium.

(b) Determine the angle AD makes with the vertical. **[3]**

The two strings are now replaced by a single light inextensible string of length 80 cm. One end of the string is attached to A and the other end of the string is attached to C. The string passes over a smooth peg fixed at D. The rod hangs in equilibrium, but is not vertical, as shown in **Fig. 6.2**.

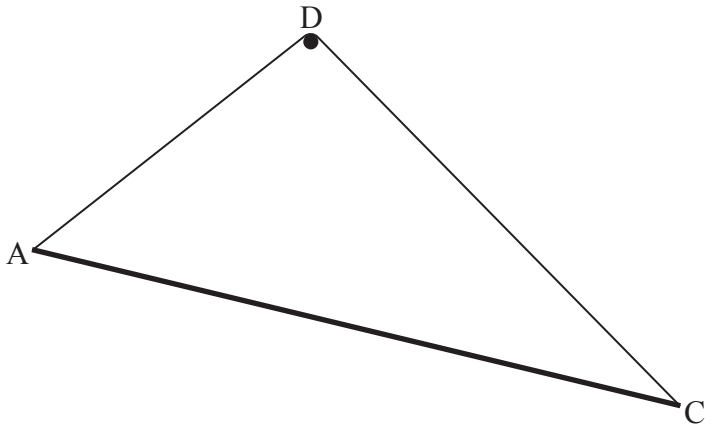


Fig. 6.2

(c) Explain why angle ADG and angle CDG must be equal. **[2]**

(d) Determine the tension in the string. **[6]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$F = \frac{GMm}{r^2} \Rightarrow G = \frac{Fr^2}{Mm}$

$[F] = [\text{mass} \times \text{acceleration}] = MLT^{-2}$

$[G] = \frac{MLT^{-2} \times L^2}{M^2}$

$[G] = M^{-1}L^3T^{-2}$

1 (b)

$[v] = LT^{-1}$, $[M] = M$, $[r] = L$, k dimensionless

$LT^{-1} = (M^{-1}L^3T^{-2})^{\alpha} M^{\beta} L^{\gamma}$

$M: \ 0 = -\alpha + \beta$

$L: \ 1 = 3\alpha + \gamma$

$T: \ -1 = -2\alpha$

$\Rightarrow \alpha = \frac{1}{2}$

$\beta = \alpha = \frac{1}{2}$

$\gamma = 1 - 3\alpha = 1 - \frac{3}{2} = -\frac{1}{2}$

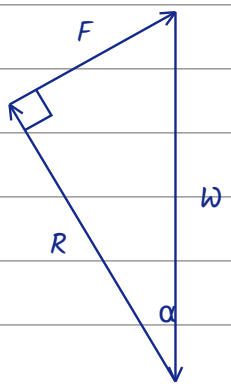
$\therefore \alpha = \frac{1}{2}, \ \beta = \frac{1}{2}, \ \gamma = -\frac{1}{2}$

2 (a)

The block is in equilibrium under exactly three forces (W, R and F), so the resultant force is zero. Hence the three force vectors, drawn end to end, join up to form a closed triangle.

Question number

2 (b)



Right angle between R and F; angle α between W and R.

2 (c) Limiting equilibrium, so $F = \mu R$.

From the triangle: $\tan\alpha = \frac{\text{opposite}}{\text{adjacent}} = \frac{F}{R}$

$\tan\alpha = \frac{\mu R}{R}$

$\therefore \tan\alpha = \mu$ as required

2 (d) Slides when $\tan\alpha = \mu = 1.35 \Rightarrow \alpha = 53.5°$

Topples when G is vertically above the lowest corner of the base,

(AD lies along the slope), i.e. when

$\tan\alpha = \frac{AD}{AB} = \frac{11.6}{8.9} = 1.303... \Rightarrow \alpha = 52.5°$

$52.5° < 53.5°$, so the block topples first (without sliding).

3 (a) Let the constant friction force be F N.

A to B: P travels through $180° + 25° = 205°$ of the hoop

Arc $AB = \frac{205}{360} \times 2\pi \times 1 = \frac{41\pi}{36} = 3.5779...$ m

Question number

Gain in GPE (A to B) = $0.3 \times 9.8 \times 1 \sin 25° = 1.2425...$ J

Work-energy: initial KE = gain in GPE + work done against friction

$\frac{1}{2}(0.3)(4^2) = 0.3 \times 9.8 \sin 25° + F \times \frac{41\pi}{36}$

$2.4 = 1.2425... + 3.5779...F$

$F = 0.32351...$ N

A to C: P travels 180°, arc length π m, and C is level with A,

so no change in GPE:

$\frac{1}{2}(0.3)v^2 = F \times \pi = 0.32351... \times \pi$

$v^2 = 6.7756...$

$v = 2.60$

3 (b) The normal reaction from the hoop changes as the speed and position of P change, so the friction (μR) is unlikely to be constant. The assumption is not very realistic.

4 (a) $\cos\theta = \frac{4}{5}$

Let W be the point on the wall 2 m above the ground, so BW = 2 m.

WC = WB = 2 m, so triangle BWC is isosceles with base angles θ.

$BC = 2 \times 2 \cos\theta = 4 \times 0.8$

BC = 3.2 m

4 (b) $\sin\theta = \frac{3}{5}$. Angle BCW = θ, so the perpendicular distance from B to the rope is $3.2 \sin\theta$.

Moments about B (wall reaction at B has no moment):

Question number

$SR \sin\theta = 3.2T \sin\theta + 6g(2.5 \sin\theta) + 75g(5 - x) \sin\theta$

Divide by $\sin\theta$:

$SR = 3.2T + 15g + 375g - 75gx$

$SR = 3.2T + 3822 - 735x$

$\times 5$: $25R = 16T + 19110 - 3675x$

$\therefore 25R + 3675x - 16T = 19110$ as required

4 (c) The rope makes angle 2θ with the vertical (angle at W is $180° - 2\theta$),

and $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{16}{25}\right) - 1 = \frac{7}{25} = 0.28$

Resolve vertically: $R + T\cos 2\theta = 6g + 75g$

$R + 0.28T = 793.8 \Rightarrow R = 793.8 - 0.28T$

Substitute into (b): $25(793.8 - 0.28T) + 3675x - 16T = 19110$

$19845 - 23T + 3675x = 19110 \Rightarrow 23T = 735 + 3675x$

T increases with x, so x is largest when $T = 720$:

$3675x = 23 \times 720 - 735 = 15825$

$x = 4.31$ m (4.3061...)

5 (a) Take upwards as positive. Let the particles collide t s after projection.

Height of P above A: $17.5t - 4.9t^2$

Height of Q above A: $20 - 4.9t^2$

$17.5t - 4.9t^2 = 20 - 4.9t^2 \Rightarrow 17.5t = 20$

$t = \frac{8}{7}$ s

Velocity of P: $17.5 - 9.8 \times \frac{8}{7} = 6.3 \text{ m s}^{-1}$ (upwards)

Velocity of Q: $0 - 9.8 \times \frac{8}{7} = -11.2 \text{ m s}^{-1}$ (11.2 downwards)

Question number

Let u_1, u_2 be the velocities (upwards) of P, Q after the collision.

Momentum: $4m(6.3) + m(-11.2) = 4mu_1 + mu_2$

$4u_1 + u_2 = 14$ (1)

Restitution: $u_2 - u_1 = 0.6(6.3 - (-11.2)) = 0.6 \times 17.5$

$u_2 - u_1 = 10.5$ (2)

(1) − (2): $5u_1 = 3.5$

$u_1 = 0.7 > 0$, so P continues upwards at 0.7 m s^{-1} as required

$u_2 = 10.5 + 0.7 = 11.2$

Q moves at 11.2 m s^{-1} upwards.

5 (b) Speed after falling 10 m: $v^2 = 0^2 + 2 \times 9.8 \times 10 = 196$

KE before explosion = $\frac{1}{2}(3m)(196) = 294m$ (= 30mg)

KE after = $1.72 \times 294m = 505.68m$

$\frac{1}{2}(2m)v_1^2 + \frac{1}{2}mv_2^2 = 505.68m$

$mv_1^2 + \frac{1}{2}mv_2^2 = 505.68m$

$\times \frac{2}{m}$: $2v_1^2 + v_2^2 = 1011.36$ as required

5 (c) Speed just before explosion = $\sqrt{196} = 14$ m s^{-1} (downwards)

Momentum (downwards positive): $3m(14) = 2mv_1 - mv_2$

$42 = 2v_1 - v_2 \Rightarrow v_2 = 2v_1 - 42$

Substitute: $2v_1^2 + (2v_1 - 42)^2 = 1011.36$

$2v_1^2 + 4v_1^2 - 168v_1 + 1764 = 1011.36$

$6v_1^2 - 168v_1 + 752.64 = 0$

$v_1^2 - 28v_1 + 125.44 = 0$

Question number

$(v_1 - 22.4)(v_1 - 5.6) = 0 \Rightarrow v_1 = 22.4$ or 5.6

$v_2 = 2(22.4) - 42 = 2.8$ or $v_2 = 2(5.6) - 42 = -30.8$

$v_2 = -30.8$ is impossible: the upper fragment would be moving down faster than the lower one (and v_2 is an upward speed, so $v_2 > 0$).

Lower fragment: 22.4 m s^{-1} downwards

Upper fragment: 2.8 m s^{-1} upwards

6 (a) Let AG = x cm. Total mass = 3.5 + 1.4 + 2.1 = 7 kg

$7x = 3.5(0) + 1.4(30) + 2.1(60) = 42 + 126 = 168$

AG = 24 cm

6 (b) AD = CD = 40 cm, so D is directly above B (the midpoint of AC).

$BD = \sqrt{(40^2 - 30^2)} = \sqrt{700} = 26.457...$ cm

In equilibrium G hangs vertically below D. GB = 30 − 24 = 6 cm.

$\angle ADB = \tan^{-1}\left(\frac{30}{\sqrt{700}}\right) = 48.590...°$

$\angle GDB = \tan^{-1}\left(\frac{6}{\sqrt{700}}\right) = 12.777...°$

Angle between AD and the vertical DG = 48.590...° − 12.777...°

= 35.8°

6 (c) The peg is smooth, so the tension T is the same in both parts of the string. The weight is vertical, so the only horizontal forces on the rod are the horizontal components of the tensions, which must balance:

T sin ADG = T sin CDG, and since DG is vertical, $\angle ADG = \angle CDG$.

6 (d) Let AD = x cm, so DC = 80 − x cm. AG = 24 cm, GC = 36 cm.

DG bisects angle ADC, so by the angle bisector theorem

Question number

$\frac{AD}{DC} = \frac{AG}{GC}$: $\frac{x}{80 - x} = \frac{24}{36}$

$36x = 1920 - 24x \Rightarrow 60x = 1920$

$AD = 32$ cm, $DC = 48$ cm

Cosine rule: $\cos ADC = \frac{32^2 + 48^2 - 60^2}{2 \times 32 \times 48} = -\frac{272}{3072}$

$\angle ADC = 95.079...° \Rightarrow \angle ADG = 47.539...°$

Let the tension be T N. Resolve vertically for the rod:

$2T \cos(47.539...°) = 7g = 68.6$

$T = 50.8$ N