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Thursday 20 June 2019 – Morning

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

77702492246

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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Answer **all** the questions.

1 Fig. 1 shows some spreadsheet output concerning the values of a function, $f(x)$.

	A	B	C
1	x	$f(x)$	
2	1	0.367879441	0.367879441
3	2	0.018315639	0.38619508
4	3	0.00012341	0.38631849
5	4	1.12535E-07	0.386318602
6	5	1.38879E-11	0.386318602

Fig. 1

The formula in cell B2 is `=EXP(-(A2^2))`
and equivalent formulae are in cells B3 to B6.

The formula in cell C2 is `=B2`.

The formula in cell C3 is `=C2+B3`.

Equivalent formulae are in cells C4 to C6.

(a) Use sigma notation to express the formula in cell C5 in standard mathematical notation. **[2]**

(b) Explain why the same value is displayed in cells C5 and C6. **[2]**

Now suppose that the value in cell C2 is chopped to 3 decimal places and used to approximate the value in cell C2.

(c) Calculate the relative error when this approximation is used. **[1]**

Suppose that the values in cells B4, B5 and B6 are chopped to 3 decimal places and used as approximations to the original values in cells B4, B5 and B6 respectively.

(d) Explain why the relative errors in these approximations are all the same. **[1]**

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2 Fig. 2.1 shows the graph of $y = x^2e^{2x} - 5x^2 + 0.5$.

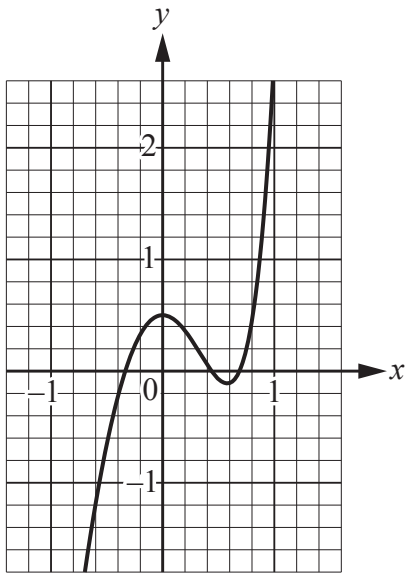


Fig. 2.1

There are three roots of the equation $x^2e^{2x} - 5x^2 + 0.5 = 0$. The roots are α, β and γ, where $\alpha < \beta < \gamma$.

(a) Explain why it is not possible to use the method of false position with $x_0 = 0$ and $x_1 = 1$ to find β and γ. **[1]**

The graph of the function indicates that the root γ lies in the interval [0.6, 0.8]. Fig. 2.2 shows some spreadsheet output using the method of false position using these values as starting points.

	A	B	C	D	E	F
1	*a*	f(*a*)	*b*	f(*b*)	approx	
2	0.6	-0.10476	0.8	0.469941	0.636457	-0.07876
3	0.636457	-0.07876	0.8	0.469941	0.659931	-0.04748
4	0.659931	-0.04748	0.8	0.469941	0.672783	-0.0249
5	0.672783	-0.0249	0.8	0.469941	0.679184	-0.01211
6	0.679184	-0.01211	0.8	0.469941	0.682218	-0.00567
7	0.682218	-0.00567	0.8	0.469941	0.683623	-0.00261
8	0.683623	-0.00261	0.8	0.469941	0.684266	-0.00119
9	0.684266	-0.00119	0.8	0.469941	0.684559	-0.00054
10	0.684559	-0.00054	0.8	0.469941	0.684692	-0.00025
11	0.684692	-0.00025	0.8	0.469941	0.684753	-0.00011
12	0.684753	-0.00011	0.8	0.469941	0.68478	-5.1E-05

Fig. 2.2

(b) **Without** doing any further calculation, write down the smallest possible interval which is certain to contain γ. **[1]**

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(c) State what is being calculated in column F. **[1]**

The formula in cell A3 is `=IF(F2<0,E2,A2)`.

(d) Explain the purpose of this formula in the application of the method of false position. **[2]**

The method of false position uses the same formula for obtaining new approximations as the secant method.

(e) Explain how the method of false position differs from the secant method. **[1]**

(f) Give one **advantage** and one **disadvantage** of using the method of false position instead of the secant method. **[2]**

3 In the first week of an outbreak of influenza, 9 patients were diagnosed with the virus at a medical practice in Pencaster. Records were kept of y, the total number of patients diagnosed with influenza in week n. The data are shown in Fig. 3.

n	1	2	3	4	5
y	9	32	63	96	125

Fig. 3

(a) Complete the difference table in the Printed Answer Booklet. **[3]**

(b) Explain why a cubic model is appropriate for the data. **[1]**

(c) Use Newton's method to find the interpolating polynomial of degree 3 for these data. **[4]**

In both week 6 and week 7 there were 145 patients in total diagnosed with influenza at the medical practice.

(d) Determine whether the model is a good fit for these data. **[2]**

(e) Determine the maximum number of weeks for which the model could possibly be valid. **[1]**

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4 Fig. 4 shows the graph of $y = x^5 - 6\sqrt{x} + 4$.

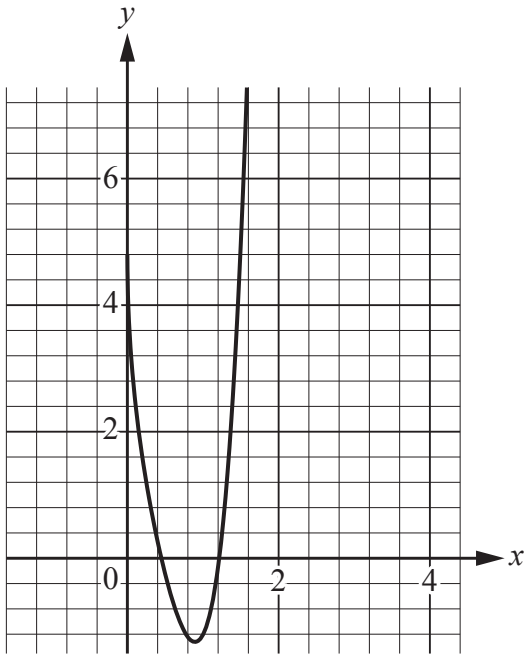


Fig. 4

There are two roots of the equation $x^5 - 6\sqrt{x} + 4 = 0$. The roots are α and β, such that $\alpha < \beta$.

(a) Show that $0 < \alpha < 1$ and $1 < \beta < 2$. **[2]**

(b) Obtain the Newton-Raphson iterative formula

$$x_{n+1} = x_n - \frac{x_n^{\frac{11}{2}} - 6x_n + 4\sqrt{x_n}}{5x_n^{\frac{9}{2}} - 3}.$$ **[3]**

(c) Use the iterative formula found in part **(b)** with a starting value of $x_0 = 1$ to obtain β correct to 6 decimal places. **[2]**

(d) Use the iterative formula found in part **(b)** with a starting value of $x_0 = 0$ to find x_1. **[1]**

(e) Give a geometrical explanation of why the Newton-Raphson iteration fails to find α in part **(d)**. **[1]**

(f) Obtain the iterative formula

$$x_{n+1} = \left(\frac{x_n^5 + 4}{6}\right)^2.$$ **[2]**

(g) Use the iterative formula found in part **(f)** with a starting value of $x_0 = 0$ to obtain α correct to 6 decimal places. **[2]**

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5 Fig. 5 shows spreadsheet output concerning the estimation of the derivative of a function f(x) at $x = 2$ using the forward difference method.

	A	B	C	D
1	h	estimate	difference	ratio
2	0.1	6.3050005		
3	0.01	6.0300512	-0.274949	
4	0.001	6.0030018	-0.027049	0.098379
5	0.0001	6.0003014	-0.0027	0.099835
6	0.00001	6.0000314	-0.00027	0.099983
7	0.000001	6.0000044	-2.7E-05	0.099994
8	1E-07	6.0000016	-2.71E-06	0.100352
9	1E-08	6.0000013	-3.02E-07	0.111457
10	1E-09	6.0000018	4.885E-07	-1.61765
11	1E-10	6.0000049	3.109E-06	6.363636
12	1E-11	6.0000005	-4.44E-06	-1.42857
13	1E-12	6.0005334	0.0005329	-120
14	1E-13	5.9952043	-0.005329	-10
15	1E-14	6.1284311	0.1332268	-25
16	1E-15	5.3290705	-0.799361	-6
17	1E-16	0	-5.329071	6.666667

Fig. 5

(a) Write down suitable cell formulae for

- cell C3,
- cell D4.

[2]

(b) Explain what the entries in cells D4 to D8 tell you about the order of the convergence of the forward difference method. **[2]**

(c) Write the entry in cell A10 in standard mathematical notation. **[1]**

(d) Explain what the values displayed in cells D10 to D17 suggest about the values in cells B10 to B16. **[2]**

(e) Write down the value of the derivative of f(x) at $x = 2$ to an accuracy that seems justified, explaining your answer. **[2]**

The formula in cell B2 is `=(LN(SQRT(SINH((2+A2)^3)))−LN(SQRT(SINH(2^3))))/A2`

and equivalent formulae are entered in cells B3 to B17.

(f) Write f(x) in standard mathematical notation. **[1]**

The value displayed in cell B17 is zero, even though the calculation results in a non-zero answer.

(g) Explain how this has arisen. **[2]**

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6 The spreadsheet output in Fig. 6 shows approximations to $\int_0^1 x^{-\sqrt{x}}\mathrm{d}x$ found using the midpoint rule, denoted by M_n, and the trapezium rule, denoted by T_n.

	A	B	C
1	n	M_n	T_n
2	1	1.632527	1
3	2	1.641461	1.316263
4	4	1.623053	1.478862
5	8	1.610295	1.550957
6	16	1.604132	1.580626
7	32	1.601505	1.592379

Fig. 6

(a) Write down an efficient spreadsheet formula for cell C3. **[2]**

(b) By first completing the table in the Printed Answer Booklet using the Simpson's rule, calculate the most accurate estimate of $\int_0^1 x^{-\sqrt{x}}\mathrm{d}x$ that you can, justifying the precision quoted. **[8]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) $C5 = B2 + B3 + B4 + B5$

$= \sum_{r=1}^{4} e^{-r^2}$

1 (b) $C6 = C5 + B6$, and $B6 = e^{-25} \approx 1.39 \times 10^{-11}$, which is very small

Adding it does not change the 9th decimal place of the sum,

so C5 and C6 are different numbers but both round to the same

value to 9 d.p., and the spreadsheet displays 9 d.p.

1 (c) Chopped value = 0.367

Relative error $= \dfrac{0.367 - 0.367879441}{0.367879441}$

$= -0.00239$ (3 s.f.)

1 (d) B4, B5 and B6 are all less than 0.001, so each chops to 0.000

Error = 0 − x = −x in each case, so relative error = −x / x = −1

for all three

2 (a) $f(0) = 0.5 > 0$ and $f(1) = e^2 - 4.5 > 0$, both positive

False position needs a change of sign between the starting values,

and there is none (the chord does not cross the x-axis between them)

2 (b) $0.68478 < \gamma < 0.8$

2 (c) Column F is the value of the function at the new approximation,

Question number

i.e. f(approx) = $f(x_{new})$

2 (d) It is a logical test for a change of sign, to set up the interval for the next iteration:
if F2 is negative, the value in E2 is copied into A3 (new a);
if F2 is not negative, the value in A2 is copied into A3 (a stays the same)

2 (e) The method of false position always keeps a change of sign (the root stays bracketed between a and b), whereas the secant method just uses the last two values and need not have a sign change

2 (f) Advantage: false position always brackets the root, so it always converges
Disadvantage: it can converge more slowly (here one end stays fixed at 0.8)

3 (a)

n	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1	9			
		23		
2	32		8	
		31		−6
3	63		2	
		33		−6
4	96		−4	
		29		
5	125			

Question number	
3 (b)	The third differences are constant (both −6), so the fourth difference is zero and a cubic fits the data
3 (c)	$y = 9 + 23(n-1) + \frac{8}{2!}(n-1)(n-2) + \frac{-6}{3!}(n-1)(n-2)(n-3)$
	$= 9 + 23n - 23 + 4(n^2 - 3n + 2) - (n^3 - 6n^2 + 11n - 6)$
	$= 9 + 23n - 23 + 4n^2 - 12n + 8 - n^3 + 6n^2 - 11n + 6$
	$y = 10n^2 - n^3$
3 (d)	$n = 6$: $y = 360 - 216 = 144$, compared with 145: good fit
	$n = 7$: $y = 490 - 343 = 147$, compared with 145: good fit
	So the model is a good fit for these data
3 (e)	$10n^2 - n^3 = n^2(10 - n) = 0$ gives $n = 0$ or $n = 10$
	For $n > 10$ the model predicts a negative total number of patients,
	so the model could be valid for at most 10 weeks
4 (a)	$f(x) = x^5 - 6\sqrt{x} + 4$
	$f(0) = 4 > 0$, $f(1) = 1 - 6 + 4 = -1 < 0$
	Change of sign (f continuous) so a root lies in (0, 1): $0 < \alpha < 1$
	$f(2) = 32 - 6\sqrt{2} + 4 = 27.51\ldots > 0$
	$f(1) < 0 < f(2)$, change of sign so $1 < \beta < 2$
4 (b)	$f'(x) = 5x^4 - 3x^{-\frac{1}{2}}$
	$x_{n+1} = x_n - \frac{x_n^5 - 6\sqrt{x_n} + 4}{5x_n^4 - 3x_n^{-\frac{1}{2}}}$

Question number

Multiply numerator and denominator by $x_n^{\frac{1}{2}} = \sqrt{x_n}$:

$$x_{n+1} = x_n - \frac{x_n^{\frac{11}{2}} - 6x_n + 4\sqrt{x_n}}{5x_n^{\frac{9}{2}} - 3}$$ as required

4 (c)

$x_0 = 1$

$x_1 = 1.5$

$x_2 = 1.3143166$

$x_3 = 1.2295168$

$x_4 = 1.2115408$

$x_5 = 1.2107830$

$x_6 = 1.2107817$

$x_7 = 1.2107817$

$\beta = 1.210782$ (6 d.p.)

4 (d)

$$x_1 = 0 - \frac{0 - 0 + 0}{0 - 3} = 0$$

4 (e)

At $x = 0$ the tangent to the curve is vertical (along the y-axis), so it meets the x-axis at $x = 0$ again and the iteration just repeats the value 0

4 (f)

$x^5 - 6\sqrt{x} + 4 = 0$

$$6\sqrt{x} = x^5 + 4 \Rightarrow \sqrt{x} = \frac{x^5 + 4}{6}$$

Square both sides: $x = \left(\frac{x^5 + 4}{6}\right)^2$

Question number	
	$x_{n+1} = \left(\frac{x_n^5 + 4}{6}\right)^2$ as required
4 (g)	$x_0 = 0$
	$x_1 = 0.4444444$
	$x_2 = 0.4483065$
	$x_3 = 0.4484776$
	$x_4 = 0.4484853$
	$x_5 = 0.4484857$
	$x_6 = 0.4484857$
	$\alpha = 0.448486$ (6 d.p.)
5 (a)	C3: =B3−B2
	D4: =C4/C3
5 (b)	The ratios of differences in D4 to D8 are approximately equal,
	tending to 0.1 (the factor by which h is reduced each time)
	This suggests the forward difference method is first order:
	the error is roughly proportional to h
5 (c)	$h = 10^{-9} = 0.000000001$
5 (d)	The ratios in D10 to D17 are no longer close to 0.1; they are erratic
	and diverging
	This suggests the estimates in B10 to B16 are becoming less accurate
	(rounding errors now dominate as h gets very small)
5 (e)	$f'(2) = 6.00000$ (5 d.p.) seems certain, and 6.000001 seems likely

Question number

The estimates agree to this precision (down to about 6.0000013) before they start to diverge, after which the values get larger and less reliable

5 (f) $f(x) = \ln(\sqrt{\sinh(x^3)})$

5 (g) The numerator is the subtraction of two nearly equal numbers, $f(2 + 10^{-16}) - f(2)$

To the precision the spreadsheet works to, $2 + 10^{-16}$ is stored as 2, so both values are stored as the same (non-zero) number, the difference is 0 and the estimate displays 0

6 (a) $T_{2n} = \frac{M_n + T_n}{2}$

C3: =(B2+C2)/2

6 (b) $S_{2n} = \frac{2M_n + T_n}{3}$

M_n	T_n	S_{2n}	difference	ratio
1.632527	1	1.421685		
1.641461	1.316263	1.533062	0.1113771	
1.623053	1.478862	1.574989	0.0419277	0.376448
1.610295	1.550957	1.590516	0.0155263	0.370312
1.604132	1.580626	1.596297	0.0057810	0.372335
1.601505	1.592379	1.598463	0.0021663	0.374733

The ratios of differences are roughly constant, $r \approx 0.375$

Question number

Use this to extrapolate (geometric series of remaining differences):

$$S_\infty \approx S_{64} + \frac{0.0021663 \times 0.375}{1 - 0.375}$$

$= 1.598463 + 0.0012998$

$= 1.59976$

The extrapolated value is about 1.5998, and $S_{64} = 1.59846$,

so the estimates all agree with 1.60 to 3 s.f.

Estimate: $\int_0^1 x^{-\sqrt{x}}\, dx = 1.60$ (certain to 3 s.f.),

and 1.600 is probable to 4 s.f.