GCE

Further Mathematics B (MEI)

Y434/01: Numerical methods

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to **2 s.f.**

Follow through should be used so that only one mark is lost for each distinct accuracy error.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question	Answer	Marks	AOs		Guidance

Question			Answer	Marks	AOs		Guidance
1	(a)		$\sum e^{-i^2}$ seen **oe** $\sum_{i=1}^{4} e^{-i^2}$ **oe**	**M1** **A1** **[2]**	**1.1** **1.1**	must see i = oe	
1	(b)		e^{-5^2} is (very) small and doesn't affect the 9th decimal place in the sum	**B1** **B1** **[2]**	**2.2a** **2.4**	eg [the numbers are different oe] but both round to the same value at 9dp and the spreadsheet displays to 9 dp	condone eg B6 is (very) small
1	(c)		– 0.00239 to – 0.0024 **isw**	**B1** **[1]**	**1.1**	NB – 0.002390568491	
1	(d)		[the approximations are all zero], so the relative error is the number divided by itself in each case oe	**B1** **[1]**	**2.4**	allow the relative errors are all ±1	
2	(a)		f(0) and f(1) are both positive – a sign change is needed for this method **oe**	**B1** **[1]**	**3.1a**	allow the chord does not cut the x-axis **oe**	there are 2 roots **oe** is insufficient
2	(b)		$0.68478 < \gamma < 0.8$	**B1** **[1]**	**2.2a**		
2	(c)		the value of f(x_{new}) **oe**	**B1** **[1]**	**1.1**	must allude to the value of the function	allow eg f(approx.)

Question			Answer	Marks	AOs		Guidance
2	(d)		the formula represents a logical test for a change of sign **oe** if the value in F2 is negative, the value in E2 will be copied into A3; if it is not negative, the value in A2 will be copied into A3	**B1** **B1** **[2]**	**3.1a** **2.4**	allow eg establishing the interval for the next iteration must consider both possibilities and must not refer to value in column C	
2	(e)		the secant method does not (necessarily) incorporate a sign change whereas the method of false position does require a change of sign **oe**	**B1** **[1]**	**1.2**	must mention both methods	
2	(f)		eg false position always straddles the root so always converges eg converges more slowly eg harder to implement because it incorporates the programming of a sign check	**B1** **B1** **[2]**	**2.4** **2.4**	advantage disadvantage	

Question			Answer	Marks	AOs		Guidance
3	(a)		<table><tr><th>n</th><th>f(n)</th><th>Δ</th><th>Δ^2</th><th>Δ^3</th></tr><tr><td>1</td><td>9</td><td></td><td></td><td></td></tr><tr><td></td><td></td><td>23</td><td></td><td></td></tr><tr><td>2</td><td>32</td><td></td><td>8</td><td></td></tr><tr><td></td><td></td><td>31</td><td></td><td>-6</td></tr><tr><td>3</td><td>63</td><td></td><td>2</td><td></td></tr><tr><td></td><td></td><td>33</td><td></td><td>-6</td></tr><tr><td>4</td><td>96</td><td></td><td>-4</td><td></td></tr><tr><td></td><td></td><td>29</td><td></td><td></td></tr><tr><td>5</td><td>125</td><td></td><td></td><td></td></tr></table>	**M1** **A1** **A1** **[3]**	**1.1** **1.1** **1.1**	3 columns of differences seen for Δ column correct Δ^2 and Δ^3 correct **FT**	
3	(b)		Δ^3 column has same values or Δ^4 is zero	**B1** **[1]**	**2.3**		
3	(c)		$9+(n-1)\times 23+(n-1)(n-2)\times\frac{8}{2!}$ $+(n-1)(n-2)(n-3)\times\frac{-6}{3!}$ $10n^2-n^3$ may be eg in terms of x **FT** their cubic [y] in terms of n	**M1** **A1** **A1** **A1** **[4]**	**3.3** **1.1** **1.1** **1.1**	all four terms all correctly placed, may be implied by correct answer both terms correct with no additional terms	allow up to **M1A1A1** for polynomial in x
3	(d)		f(6) = 144 good fit f(7) = 147 good fit	**B1** **B1** **[2]**	**3.4** **3.4**	allow **SC1** for 144 and 147 seen and no comment or incorrect comment	**FT** their $10n^2-n^3$ for each mark

Question			Answer	Marks	AOs		Guidance
3	(e)		$10n^2 - n^3 = 0$ gives $n = 10$ (or 0) so maximum possible number of weeks it could be viable is 10, since a negative number of cases is predicted after week 10	**B1** **[1]**	**3.5b**	$n = 10$ unsupported is insufficient	**FT** their cubic
4	(a)		f(0) = 4 and f(1) = –1 so root in this interval f(1) = – 1 and f(2) = 27.5(1…) or 28 so root in this interval	**B1** **B1** **[2]**	**1.1** **1.1**	simply stating f(0) > 0 and f(1) < 0 is insufficient	condone omission of supporting comments but the values must be correct
4	(b)		$\frac{dy}{dx} = 5x^4 - 3x^{-\frac{1}{2}}$ **oe** substitution of their derivative in Newton-Raphson formula multiplication of numerator and denominator by $x^{½}$ to obtain $x_{n+1} = x_n - \frac{x_n^{\frac{11}{2}} - 6x_n + 4\sqrt{x_n}}{5x_n^{\frac{9}{2}} - 3}$ AG	**B1** **M1** **A1** **[3]**	**2.1** **2.4** **1.1**	this step must be explicitly shown must include subscripts as soon as they are introduced with no wrong working	

Question			Answer	Marks	AOs		Guidance
4	(c)		eg 1 1.5 1.3143166 1.2295168 1.2115408 1.210783 1.2107817 1.2107817 1.210782	**M1** **A1** **[2]**	**1.1** **1.1**	must see at least three iterates eg [1], 1.5, 1.314(3…)	
4	(d)		0	**B1** **[1]**	**1.1**		
4	(e)		eg the **tangent** is along the *y*-axis oe so the iteration will simply repeat the value 0	**B1** **[1]**	**2.4**		must be a geometrical explanation, not an algebraic one
4	(f)		constructive step in rearrangement seen eg $\sqrt{x} = \frac{x^5 + 4}{6}$ square both sides to obtain $x_{n+1} = \left(\frac{x_n^5 + 4}{6}\right)^2$. **AG**	**M1** **A1** **[2]**	**1.1** **1.1**	must include subscripts	

Question			Answer	Marks	AOs		Guidance
4	(g)		eg 0 0.4444444 0.4483065 0.4484776 0.4484853 0.4484857 0.4484857 0.448486	**M1** **A1** **[2]**	**1.1** **1.1**	must see at least three iterates eg [0], 0.444(…), 0.448(…)	
5	(a)		= B3 – B2 = C4/C3	**B1** **B1** **[2]**	**1.1** **1.1**	in cell C3 in cell D4	if **B0B0** allow **SC1** for B3 – B2 and C4/C3 seen
5	(b)		these values are approximately equal / appear to be tending to 0.1 which suggests 1st order	**B1** **B1** **[2]**	**2.4** **2.2b**		 **B0** for eg just "it's a first order method"
5	(c)		[1×] 10^{-9} or 0.000 000 001	**B1** **[1]**	**1.1**	ignore cell references	

Question			Answer	Marks	AOs		Guidance
5	(d)		the ratios are diverging **oe** (which suggests that) the approximations to the derivative are (becoming) less accurate towards the bottom of the column **oe**	**B1** **B1** **[2]**	**2.4** **2.2b**		
5	(e)		6.000001 seems likely or 6.00000 seems certain approximation after 6.0000013 is larger oe	**B1** **B1** **[2]**	**2.2b** **2.4**	allow eg values agree to this precision before they start to diverge	
5	(f)		[f(x) =] ln($\sqrt{}$sinh(x^3))	**B1** **[1]**	**2.5**	or ½ln(sinhx^3)	allow omission of inner brackets; **B0** for modulus instead of brackets
5	(g)		the calculation involves the subtraction of nearly equal numbers (in the numerator) **oe** they are stored as the same (non-zero) number to the precision the spreadsheet operates **oe**	**B1** **B1** **[2]**	**3.1b** **3.2b**		

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th></th><th>Guidance</th></tr>
<tr><td>6</td><td>(a)</td><td></td><td>evidence of $(M_1 + T_1)/2$
=(B2+C2)/2 or =0.5*(B2+C2)</td><td>M1
A1
[2]</td><td>1.1
1.1</td><td></td><td>need to see = for A1</td></tr>
<tr><td>6</td><td>(b)</td><td></td><td>
<table>
<tr><th>M_n</th><th>T_n</th><th>S_{2n}</th><th>difference</th><th>ratio</th></tr>
<tr><td>1.632527</td><td>1</td><td>1.421685</td><td></td><td></td></tr>
<tr><td>1.641461</td><td>1.316263</td><td>1.533062</td><td>0.1113771</td><td></td></tr>
<tr><td>1.623053</td><td>1.478862</td><td>1.574989</td><td>0.0419276</td><td>0.376447</td></tr>
<tr><td>1.610295</td><td>1.550957</td><td>1.590516</td><td>0.0155267</td><td>0.370321</td></tr>
<tr><td>1.604132</td><td>1.580626</td><td>1.596297</td><td>0.0057806</td><td>0.372299</td></tr>
<tr><td>1.601505</td><td>1.592379</td><td>1.598463</td><td>0.0021665</td><td>0.374787</td></tr>
</table>
extrapolation attempted with their S_{64}, their last difference and their r
r = 0.37 to 0.4, difference = 0.0021665 to 0.00217 and S_{64} = 1.59846(3)
1.599735…to 1.59991
1.60 is certain or 1.600 is probable
</td><td>M1
A1
M1
M1
M1
A1
M1
A1
[8]</td><td>3.1a
1.1
2.1
1.1
3.2a
1.1
1.1
2.2b</td><td>Simpson’s estimates
All correct to 5 dp or better
Differences
ratios</td><td>For method marks, at least two correct in each case: estimates to 5 or 6 dp, differences and ratios awrt to 3 dp or better
if M0, allow SC2 for 1.598607 to 1.58961 obtained from $\frac{16\times1.598463-1.596297}{15}$ oe</td></tr>
</table>

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