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Thursday 13 June 2019 – Afternoon

A Level Further Mathematics B (MEI)

Y431/01 Mechanics Minor

Time allowed: 1 hour 15 minutes

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You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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Answer **all** the questions.

1 Dilip and Anna are doing an experiment to find the power at which they each work when running up a staircase at school. The top of the staircase is a vertical distance of 16 m above the bottom of the staircase.

Dilip, who has mass 75 kg, does the experiment first. Anna times him, and finds that he takes 5.6 seconds to run up the staircase.

(a) Find the average power generated by Dilip as he runs up the staircase. **[3]**

Anna, who has mass M kg, then does the same experiment and runs up the staircase in 5.0 seconds. She works out that the average power she has generated is **less than** the corresponding value for Dilip.

(b) Find an inequality satisfied by M. **[2]**

Gareth, who also has mass 75 kg, says that members of his sports club do an exercise similar to this, but they run up a 16 m high sand dune. Gareth can run up the sand dune in 8.4 seconds, but he claims that he generates more power than Dilip.

(c) Give a reason why Gareth's claim could be true. **[1]**

2 **(a)** Write down the dimensions of pressure. **[1]**

The SI unit of pressure is the pascal (Pa). 15 Pa is equivalent to Q newtons per square centimetre.

(b) Find the value of Q. **[1]**

Simon thinks the speed, v, of sound in a gas is given by the formula

$v = kP^x d^y V^z$,

where P is the pressure of the gas,

d is the density of the gas,

V is the volume of the gas,

k is a dimensionless constant.

(c) Use dimensional analysis to

- find the values of x and y and
- show that $z = 0$.

[4]

At normal atmospheric pressure the density of air at sea level is $1.29\,\text{kg m}^{-3}$. Under the same conditions the density of helium is $0.166\,\text{kg m}^{-3}$.

(d) Given that the speed of sound in air under these conditions is $340\,\text{m s}^{-1}$, use Simon's formula to find the speed of sound in helium under the same conditions. **[2]**

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3 Two identical uniform rectangular laminas, P and Q, each having length ka and width a are fixed together, in the same plane, to form a lamina R.
With reference to coordinate axes, the corners of P are at (0, 0), (ka, 0), (ka, a) and (0, a) and the corners of Q are at (ka, 0), ($ka + a$, 0), ($ka + a$, ka) and (ka, ka), as shown in Fig. 3.

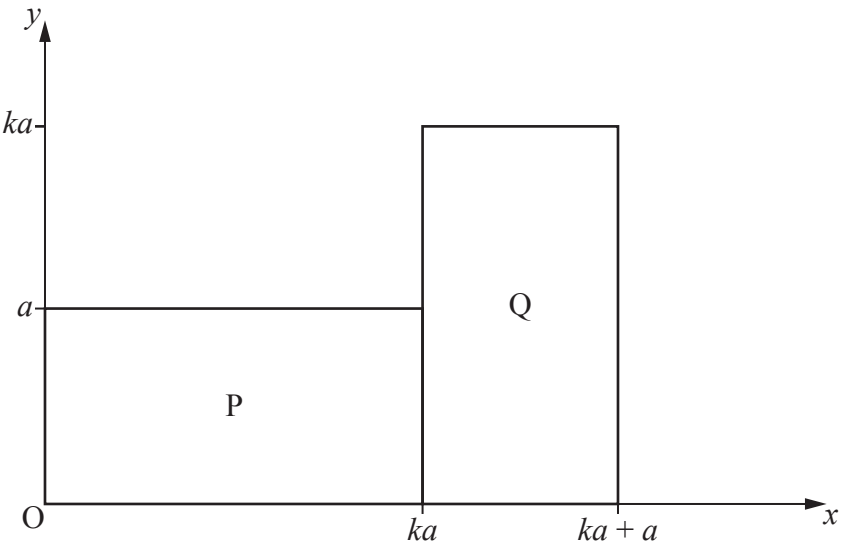


Fig. 3

Determine the range of values of k for which the centre of mass of R lies outside the boundary of R. **[7]**

4 Two model railway trucks, A of mass 0.1 kg and B of mass 0.2 kg, are constrained to move on a smooth straight level track.
Initially B is stationary and A is moving towards B with speed $u\,\text{m s}^{-1}$ before they collide. The coefficient of restitution between A and B is e.

(a) Find the speed of A and the speed of B after the collision, giving your answers in terms of e and u. **[5]**

(b) Show that the loss of kinetic energy in the collision is $\frac{1}{30}u^2(1-e^2)$. **[2]**

(c) For the case in which the **loss** of kinetic energy is **least**

- state the value of e
- state the loss in kinetic energy
- describe the subsequent motion of the trucks. **[3]**

(d) For the case in which the **loss** of kinetic energy is **greatest**

- state the value of e
- state the loss in kinetic energy
- describe the subsequent motion of the trucks. **[3]**

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5 Jack and Jemima are pulling a boat along a straight level canal.
The resistance to the motion of the boat is modelled as constant and equal to 1200 N.
Jack and Jemima walk in the same direction on paths on opposite sides of the canal. They each walk forwards at the same steady speed, keeping level with each other so that the distance between them is always 6 m. Jack and Jemima each pull a long light inextensible rope attached to the boat; initially they hold their ropes so the distance from each of them to the boat is 5 m, as shown in Fig. 5.1.

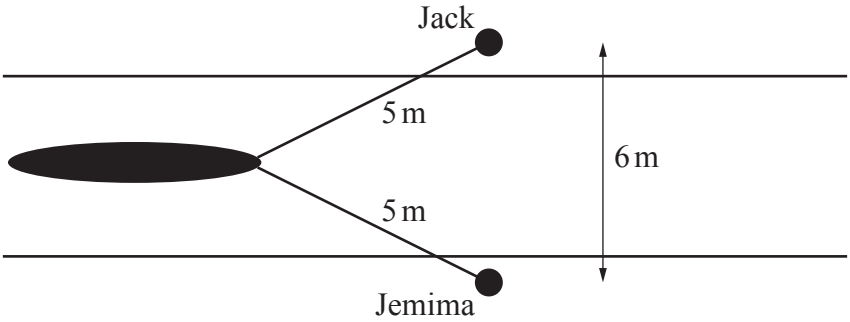


Fig. 5.1

(a) Explain why the tension will be the same in each rope. **[1]**

(b) Find the tension in each rope. **[3]**

Jemima then gradually releases more rope, so that the distance between her and the boat is 7 m. Jack and Jemima continue to walk at the same steady speed along the paths, but the position of the boat changes so that Jemima's rope makes an angle of θ with the path and Jack's rope makes an angle of ϕ with the path, as shown in Fig. 5.2.

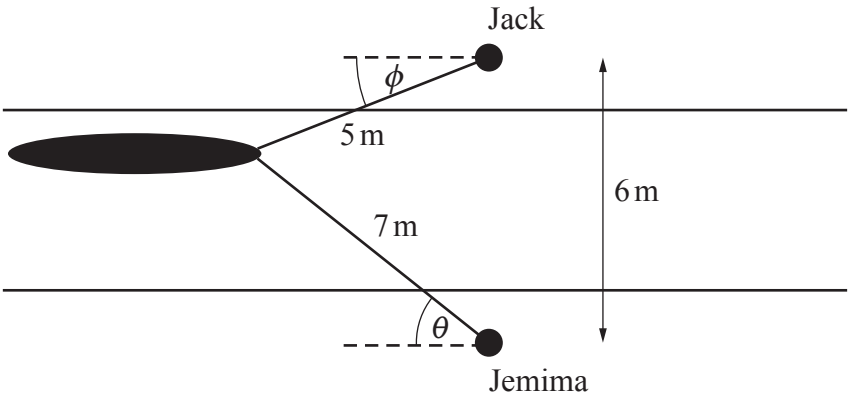


Fig. 5.2

(c) • Show that $\sin\phi = \frac{1}{5}$.
• Show that $\sin\theta = \frac{5}{7}$. **[2]**

(d) Find the tension in each rope in this new equilibrium position. **[5]**

(e) Without further calculation, state the effect on the tensions in the ropes if Jack now lengthens his rope to 7 m, the same length as Jemima's rope. **[2]**

(f) Suggest how the modelling assumption used in this question could be improved. **[1]**

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6 A uniform solid cylinder, L, has base radius 5 cm, height 24 cm and mass 5 kg.
L is placed on a rough plane inclined at an angle α to the horizontal, as shown in Fig. 6.

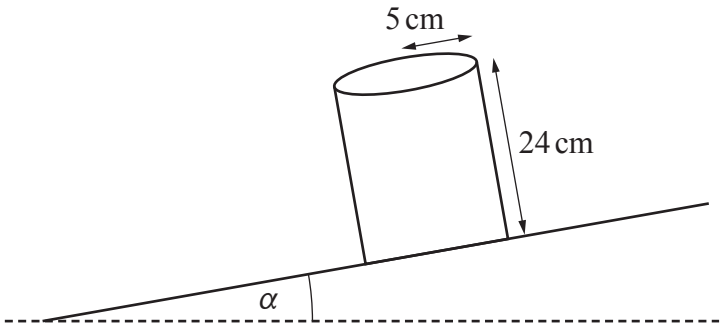


Fig. 6

(a) On the copy of Fig. 6 in the Printed Answer Booklet mark the forces acting on L. **[1]**

The coefficient of friction between L and the plane is 0.3. Initially α is 15°.

(b) Show that L rests in equilibrium on the plane. **[4]**

A couple is applied to L. It is given that L will topple if the couple is applied in an anticlockwise sense, but L will not topple if the couple is applied in a clockwise sense.

(c) Find the range of possible values of the magnitude of the couple. **[4]**

The couple is now removed and the plane is slowly tilted so that α increases.

(d) Determine whether L topples first without sliding or slides first without toppling. **[3]**

END OF QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) Work done against gravity = $mgh = 75 \times 9.8 \times 16 = 11760$ J

Average power = $\frac{11760}{5.6}$

Average power = 2100 W

1 (b) Anna's power = $\frac{M \times 9.8 \times 16}{5.0} = 31.36M$

$31.36M < 2100$

$M < \frac{2100}{31.36} = 66.96...$

$M < 67$ (kg)

1 (c) The sand gives way under his feet as he runs, so Gareth does more work than just mgh (he also does work moving the sand), so his power could be greater than Dilip's even though he takes longer.

2 (a) Pressure = force ÷ area

$[P] = MLT^{-2} \div L^2 = ML^{-1}T^{-2}$

2 (b) 15 Pa = 15 N m^{-2}, 1 m^2 = 10 000 cm^2

$Q = \frac{15}{10000} = 0.0015$

2 (c) $[v] = LT^{-1}$, $[d] = ML^{-3}$, $[V] = L^3$, k dimensionless

$LT^{-1} = (ML^{-1}T^{-2})^x(ML^{-3})^y(L^3)^z$

Question number

M: $0 = x + y$

L: $1 = -x - 3y + 3z$

T: $-1 = -2x$

$\Rightarrow x = \frac{1}{2}, \quad y = -\frac{1}{2}$

L: $1 = -\frac{1}{2} + \frac{3}{2} + 3z \Rightarrow 1 = 1 + 3z \Rightarrow 3z = 0$

$\therefore z = 0$ as required

2 (d) $v = k\sqrt{\frac{P}{d}}$ and P is the same for both gases

Air: $340 = k\sqrt{\frac{P}{1.29}} \Rightarrow k\sqrt{P} = 340\sqrt{1.29} = 386.17...$

Helium: $v = \frac{386.17...}{\sqrt{0.166}} \quad \left(= 340\sqrt{\frac{1.29}{0.166}}\right)$

$v = 948 \text{ m s}^{-1}$ (3 s.f.)

3 P and Q each have area ka^2, so each has the same mass m.

Centre of mass of P: $(\frac{1}{2}ka, \frac{1}{2}a)$ of Q: $(ka + \frac{1}{2}a, \frac{1}{2}ka)$

$2m\bar{x} = m(\frac{1}{2}ka) + m(ka + \frac{1}{2}a)$

$\bar{x} = \frac{3}{4}ka + \frac{1}{4}a$

$2m\bar{y} = m(\frac{1}{2}a) + m(\frac{1}{2}ka)$

$\bar{y} = \frac{1}{4}ka + \frac{1}{4}a$

Question number

ka is the length and a the width, so $k \geq 1$ (Q is at least as tall as P).

The only region of the bounding rectangle not in R is $0 < x < ka$, $a < y < ka$,

so G lies outside R when $\bar{x} < ka$ and $\bar{y} > a$.

$\bar{x} < ka$: $\frac{3}{4}ka + \frac{1}{4}a < ka \Rightarrow \frac{1}{4}a < \frac{1}{4}ka \Rightarrow k > 1$

$\bar{y} > a$: $\frac{1}{4}ka + \frac{1}{4}a > a \Rightarrow k + 1 > 4 \Rightarrow k > 3$

Both conditions are needed, so the centre of mass lies outside R

when $k > 3$ only.

4 (a) Let a and b be the velocities of A and B after the collision,

both measured in the direction A was moving initially.

Conservation of momentum: $0.1u = 0.1a + 0.2b \Rightarrow u = a + 2b$ (1)

Newton's law of restitution: $b - a = -e(0 - u) = eu$ (2)

(1) + (2): $3b = u + eu$

$b = \frac{1}{3}u(1 + e)$

$a = b - eu = \frac{1}{3}u(1 + e) - eu = \frac{1}{3}u(1 - 2e)$

Speed of A $= \frac{1}{3}u|1 - 2e|$ (reversed direction if $e > \frac{1}{2}$)

Speed of B $= \frac{1}{3}u(1 + e)$

4 (b) KE before $= \frac{1}{2}(0.1)u^2 = \frac{1}{20}u^2$

KE after $= \frac{1}{2}(0.1)\frac{u^2(1 - 2e)^2}{9} + \frac{1}{2}(0.2)\frac{u^2(1 + e)^2}{9}$

Question number

$= \frac{u^2}{180}[(1 - 4e + 4e^2) + 2(1 + 2e + e^2)]$

$= \frac{u^2}{180}(3 + 6e^2) = \frac{u^2(1 + 2e^2)}{60}$

$\text{Loss} = \frac{3u^2}{60} - \frac{u^2(1 + 2e^2)}{60} = \frac{u^2(2 - 2e^2)}{60}$

$= \frac{1}{30}u^2(1 - e^2)$ as required

4 (c) Least loss when e^2 is greatest, so $e = 1$

Loss in kinetic energy = 0

A moves back with speed $\frac{1}{3}u$ and B moves forward with speed $\frac{2}{3}u$,

so the trucks move away from each other.

4 (d) Greatest loss when $e = 0$

Loss in kinetic energy $= \frac{1}{30}u^2$

$a = b = \frac{1}{3}u$: the trucks coalesce (stay together) and move with

speed $\frac{1}{3}u$ in the direction A was originally moving.

5 (a) By symmetry both ropes make the same angle with the canal, and the components of tension perpendicular to the canal must balance, so the tensions are equal.

5 (b) The boat is midway, 3 m from the line of each path, rope length 5 m.

Question number

Angle α between each rope and the canal: $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

Steady speed, so forces balance along the canal:

$2T\cos\alpha = 1200$

$2T \times 0.8 = 1200$

$T = 750$ N in each rope

5 (c) Triangle Jack, Jemima, boat has sides 6, 5 and 7 m.

The line Jack to Jemima is perpendicular to the paths, so the angle in the triangle at Jack is $90° - \varphi$ and at Jemima is $90° - \theta$.

$\sin\varphi = \cos(90° - \varphi) = \frac{5^2 + 6^2 - 7^2}{2 \times 5 \times 6} = \frac{12}{60} = \frac{1}{5}$ as required

$\sin\theta = \cos(90° - \theta) = \frac{6^2 + 7^2 - 5^2}{2 \times 6 \times 7} = \frac{60}{84} = \frac{5}{7}$ as required

5 (d) Let T_1 = tension in Jack's rope, T_2 = tension in Jemima's rope.

$\cos\varphi = \frac{\sqrt{24}}{5}$, $\cos\theta = \frac{\sqrt{24}}{7}$

Perpendicular to canal: $T_1\sin\varphi = T_2\sin\theta$

$\frac{1}{5}T_1 = \frac{5}{7}T_2 \Rightarrow T_1 = \frac{25}{7}T_2$

Along canal: $T_1\cos\varphi + T_2\cos\theta = 1200$

$\frac{25}{7}T_2 \times \frac{\sqrt{24}}{5} + T_2 \times \frac{\sqrt{24}}{7} = 1200$

$\frac{6\sqrt{24}}{7}T_2 = 1200 \Rightarrow T_2 = \frac{1400}{\sqrt{24}} = 285.77...$

$T_1 = \frac{25}{7} \times 285.77... = 1020.6...$

Question number

Tension in Jack's rope = 1020 N (3 s.f.)

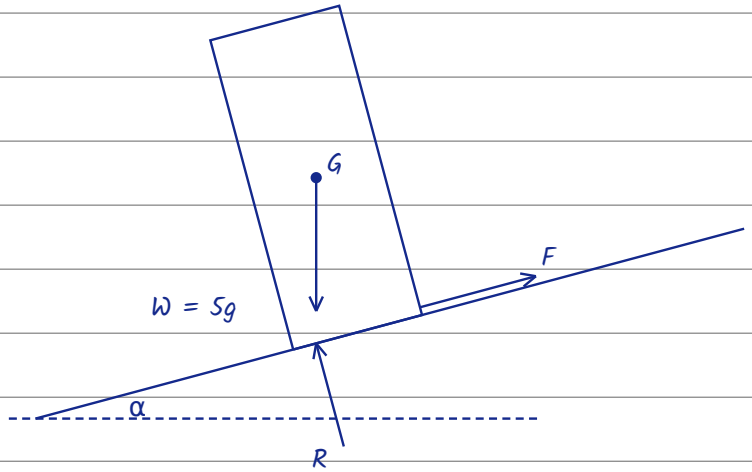
Tension in Jemima's rope = 286 N (3 s.f.)

5 (e) By symmetry the two tensions will be equal again.

The ropes now make a smaller angle with the canal than in (b), so cos is larger and each tension will be less than 750 N.

5 (f) The resistance is unlikely to be constant: it may depend on the speed of the boat and change when the boat is closer to the bank, as in (d). Model the resistance as varying with position/speed.

6 (a)



W = 5g vertically down through G; R normal to the plane, acting within the base; F (friction) up the slope along the plane.

6 (b) Sliding: resolving, $R = 5g\cos 15°$, $F = 5g\sin 15°$

$\frac{F}{R} = \tan 15° = 0.268 < 0.3 = \mu$

so $F < \mu R$ and the friction needed is available: L does not slide.

Toppling: G is 12 cm above the centre of the base, base radius 5 cm.

Question number

L topples when $\tan\alpha > \frac{5}{12}$, i.e. $\alpha > \tan^{-1}\frac{5}{12} = 22.6°$

$15° < 22.6°$, so the vertical through G passes within the base and L does not topple.

$\therefore$ L rests in equilibrium on the plane, as required.

6 (c) Let the couple have magnitude C N cm.

Anticlockwise (towards toppling down the slope), moments about the lower edge of the base: L topples if

$C + 5g(12\sin 15°) > 5g(5\cos 15°)$

$C > 49(5\cos 15° - 12\sin 15°) = 84.466...$

$C > 84.5$

Clockwise (towards toppling up the slope), moments about the upper edge of the base: L does not topple if

$C \leqslant 5g(12\sin 15°) + 5g(5\cos 15°) = 388.84...$

$C \leqslant 389$

$\therefore 84.5 < C \leqslant 389$ (N cm)

6 (d) L slides when $\tan\alpha = \mu = 0.3 \Rightarrow \alpha = \tan^{-1} 0.3 = 16.7°$

L topples when $\tan\alpha = \frac{5}{12} \Rightarrow \alpha = 22.6°$

$16.7° < 22.6°$, so L slides first without toppling.