

GCSE MATHS • GRADE 9

MATHS



MODEL ANSWERS

PROOF

SCORE

/92

PERCENTAGE

%

STEM UP

1 n is an integer.

Show that the product of two consecutive integers is even.

*Let the integers be n and $n + 1$
 one of them must be even
 even $\times$ any integer is even
 so $n(n + 1)$ is even*

Proved

1 / 1

2 n is an integer.

Prove that the sum of three consecutive odd numbers is odd.

*Let the numbers be $2n + 1$, $2n + 3$, $2n + 5$
 $\text{sum} = (2n + 1) + (2n + 3) + (2n + 5)$
 $= 6n + 9$
 $6n$ is even
 9 is odd
 even + odd = odd*

Proved

1 / 1

3 Prove that the difference between the squares of any two consecutive integers is equal to the sum of the two integers.

*Let the integers be n and $n + 1$
 difference of squares $= (n + 1)^2 - n^2$
 $= n^2 + 2n + 1 - n^2$
 $= 2n + 1$
 sum of the integers $= n + (n + 1)$
 $= 2n + 1$
 the two are equal, so the statement is proved*

Proved

1 / 1

- 4 Show that the product of two consecutive integers is even.

*Let the integers be n and $n + 1$
 one of them must be even
 even $\times$ any integer is even
 so $n(n + 1)$ is even*

Proved

1 / 1

- 5 n is an integer.

Prove that the sum of six consecutive integers is 3 more than a multiple of 6.

*Let the integers be $n, n + 1, n + 2, n + 3, n + 4, n + 5$
 $sum = n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5)$
 $= 6n + 15$
 $= 6(n + 2) + 3$
 $6(n + 2)$ is a multiple of 6
 so the sum is 3 more than a multiple of 6*

Proved

1 / 1

- 6 Prove that the difference between the squares of any two consecutive integers is equal to the sum of the two integers.

*Let the integers be n and $n + 1$
 difference of squares $= (n + 1)^2 - n^2$
 $= n^2 + 2n + 1 - n^2$
 $= 2n + 1$
 sum of the integers $= n + (n + 1)$
 $= 2n + 1$
 the two are equal, so the statement is proved*

Proved

1 / 1

7 Prove that $n^2 + n$ is always even, where n is an integer.

$$n^2 + n = n(n + 1)$$

n and $n + 1$ are consecutive so one is even

even $\times$ any integer is even

so $n^2 + n$ is even

Proved

1 / 1

8 Prove algebraically that the sum of the squares of any two even positive integers is a multiple of 4.

Let the integers be $2m$ and $2n$

$$(2m)^2 + (2n)^2$$

$$= 4m^2 + 4n^2$$

$$= 4(m^2 + n^2)$$

it is 4 times an integer so it is a multiple of 4

Proved

1 / 1

9 Prove that the sum of any 2 consecutive integers is equal to the difference between their squares.

Let the integers be n and $n + 1$

$$\text{difference of squares} = (n + 1)^2 - n^2$$

$$= n^2 + 2n + 1 - n^2$$

$$= 2n + 1$$

$$\text{sum of the integers} = n + (n + 1)$$

$$= 2n + 1$$

the two are equal, so the statement is proved

Proved

1 / 1

10 n is an integer.

Prove that the sum of six consecutive integers is 3 more than a multiple of 6.

Let the integers be $n, n + 1, n + 2, n + 3, n + 4, n + 5$

$$\text{sum} = n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5)$$

$$= 6n + 15$$

$$= 6(n + 2) + 3$$

$6(n + 2)$ is a multiple of 6

so the sum is 3 more than a multiple of 6

Proved

1 / 1

11 n is an integer.

Prove that $(n + 1)^2 - (n - 1)^2$ is a multiple of 4.

$$(n + 1)^2 = n^2 + 2n + 1$$

$$(n - 1)^2 = n^2 - 2n + 1$$

$$(n + 1)^2 - (n - 1)^2 = (n^2 + 2n + 1) - (n^2 - 2n + 1)$$

$$= 4n$$

which is 4 lots of n

so it is a multiple of 4

Proved

1 / 1

12 Prove algebraically that the difference between the squares of any 2 consecutive integers is equal to the sum of these integers.

Let the integers be n and $n + 1$

$$\text{difference of squares} = (n + 1)^2 - n^2$$

$$= n^2 + 2n + 1 - n^2$$

$$= 2n + 1$$

$$\text{sum of the integers} = n + (n + 1)$$

$$= 2n + 1$$

the two are equal, so the statement is proved

Proved

1 / 1

13 n is an integer.

Prove algebraically that the sum of seven consecutive integers is a multiple of 7.

Let the integers be $n, n + 1, n + 2, n + 3, n + 4, n + 5, n + 6$

$$\text{sum} = n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) + (n + 6)$$

$$= 7n + 21$$

$$= 7(n + 3)$$

7 is a factor so the sum is a multiple of 7

Proved

/ / 1

14 n is an integer.

Prove that the sum of three consecutive even numbers is a multiple of 6.

Let the numbers be $2n, 2n + 2, 2n + 4$

$$\text{sum} = (2n) + (2n + 2) + (2n + 4)$$

$$= 6n + 6$$

$$= 6(n + 1)$$

so the sum is a multiple of 6

Proved

/ / 1

15 Show that the sum of the squares of any two multiples of 7 is a multiple of 49.

Let the numbers be $7m$ and $7n$

$$(7m)^2 + (7n)^2$$

$$= 49m^2 + 49n^2$$

$$= 49(m^2 + n^2)$$

it is 49 times an integer so it is a multiple of 49

Proved

/ / 1

16 Prove algebraically that $(a + b)^2 - (a - b)^2 = 4ab$

$$\begin{aligned} \text{LHS} &= a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) \\ &= a^2 + 2ab + b^2 - a^2 + 2ab - b^2 \\ &= 4ab \\ &= \text{RHS} \end{aligned}$$

Proved

1 / 1

17 n is an integer.

Show that the sum of three consecutive odd numbers is a multiple of 3.

$$\begin{aligned} \text{Let the numbers be } &2n + 1, 2n + 3, 2n + 5 \\ \text{sum} &= (2n + 1) + (2n + 3) + (2n + 5) \\ &= 6n + 9 \\ &= 3(2n + 3) \\ \text{so the sum is a multiple of } &3 \end{aligned}$$

Proved

1 / 1

18 Prove algebraically that the sum of the squares of any two multiples of 5 is a multiple of 25.

$$\begin{aligned} \text{Let the numbers be } &5m \text{ and } 5n \\ (5m)^2 + (5n)^2 &= 25m^2 + 25n^2 \\ &= 25(m^2 + n^2) \\ \text{it is 25 times an integer so it is a multiple of } &25 \end{aligned}$$

Proved

1 / 1

- 19 Prove that the sum of the squares of any two consecutive integers is always an odd number.

Let the integers be n and $n + 1$

$$n^2 + (n + 1)^2$$

$$= n^2 + n^2 + 2n + 1$$

$$= 2n^2 + 2n + 1$$

$$= 2(n^2 + n) + 1$$

$2(n^2 + n)$ is even so the sum is odd

Proved

1 / 1

- 20 n is an integer.

Prove that the sum of three consecutive even numbers is a multiple of 6.

Let the numbers be $2n$, $2n + 2$, $2n + 4$

$$\text{sum} = (2n) + (2n + 2) + (2n + 4)$$

$$= 6n + 6$$

$$= 6(n + 1)$$

so the sum is a multiple of 6

Proved

1 / 1

- 21 Prove algebraically that the sum of the squares of any two multiples of 3 is a multiple of 9.

You must show your working.

Let the numbers be $3m$ and $3n$

$$(3m)^2 + (3n)^2$$

$$= 9m^2 + 9n^2$$

$$= 9(m^2 + n^2)$$

it is 9 times an integer so it is a multiple of 9

Proved

2 / 2

22 n is an integer.

Show that $(n + 3)(n + 7) + 4$ is a perfect square.

You must show your working.

$$(n + 3)(n + 7) = n^2 + 10n + 21$$

$$\text{adding } 4 \text{ gives } n^2 + 10n + 25$$

$$n^2 + 10n + 25 = (n + 5)^2$$

so it is a square number

Proved

2 / 2

23 n is an integer.

Prove algebraically that the sum of four consecutive even numbers is a multiple of 4.

You must show your working.

Let the numbers be $2n, 2n + 2, 2n + 4, 2n + 6$

$$\text{sum} = (2n) + (2n + 2) + (2n + 4) + (2n + 6)$$

$$= 8n + 12$$

$$= 4(2n + 3)$$

so the sum is a multiple of 4

Proved

2 / 2

24 n is an integer.

Prove that the sum of $n(n + 1)$ and $n + 1$ is always a square number.

$$n(n + 1) = n^2 + n$$

$$\text{adding } n + 1 \text{ gives } n^2 + 2n + 1$$

$$n^2 + 2n + 1 = (n + 1)^2$$

so the sum is a square number

Proved

2 / 2

- 25 Show that the difference between the squares of two consecutive even numbers is a multiple of 4.

Let the numbers be $2n$ and $2n + 2$

$$\begin{aligned}(2n + 2)^2 - (2n)^2 \\&= 4n^2 + 8n + 4 - 4n^2 \\&= 8n + 4 \\&= 4(2n + 1) \\&\text{so it is a multiple of 4}\end{aligned}$$

Proved

2 / 2

- 26 Show that the sum of the squares of any two odd positive integers is even.

You must show your working.

Let the integers be $2m + 1$ and $2n + 1$

$$\begin{aligned}(2m + 1)^2 + (2n + 1)^2 \\&= 4m^2 + 4m + 1 + 4n^2 + 4n + 1 \\&= 4m^2 + 4m + 4n^2 + 4n + 2 \\&= 2(2m^2 + 2m + 2n^2 + 2n + 1) \\&\text{it is 2 times an integer so it is even}\end{aligned}$$

Proved

2 / 2

- 27 n is an integer.

Prove that the sum of $(n + 1)(n + 2)$ and $n + 2$ is always a square number.

$$\begin{aligned}(n + 1)(n + 2) &= n^2 + 3n + 2 \\&\text{adding } n + 2 \text{ gives } n^2 + 4n + 4 \\n^2 + 4n + 4 &= (n + 2)^2 \\&\text{so the sum is a square number}\end{aligned}$$

Proved

2 / 2

- 28 Prove that the difference between the squares of two consecutive even numbers is a multiple of 4.

You must show your working.

Let the numbers be $2n$ and $2n + 2$

$$\begin{aligned}(2n + 2)^2 - (2n)^2 \\&= 4n^2 + 8n + 4 - 4n^2 \\&= 8n + 4 \\&= 4(2n + 1) \\&\text{so it is a multiple of 4}\end{aligned}$$

Proved

2 / 2

- 29 Show that the sum of the squares of any two odd positive integers is even.

Let the integers be $2m + 1$ and $2n + 1$

$$\begin{aligned}(2m + 1)^2 + (2n + 1)^2 \\&= 4m^2 + 4m + 1 + 4n^2 + 4n + 1 \\&= 4m^2 + 4m + 4n^2 + 4n + 2 \\&= 2(2m^2 + 2m + 2n^2 + 2n + 1) \\&\text{it is 2 times an integer so it is even}\end{aligned}$$

Proved

2 / 2

- 30 n is an integer.

Prove algebraically that the sum of $(n + 3)(n + 2)$ and $n + 3$ is always a square number.

$$\begin{aligned}(n + 3)(n + 2) &= n^2 + 5n + 6 \\&\text{adding } n + 3 \text{ gives } n^2 + 6n + 9 \\n^2 + 6n + 9 &= (n + 3)^2 \\&\text{so the sum is a square number}\end{aligned}$$

Proved

2 / 2

- 31 Show that the sum of the squares of any two consecutive integers is 1 more than twice the product of the two integers.

You must show your working.

Let the integers be n and $n + 1$

sum of squares = $n^2 + (n + 1)^2 = 2n^2 + 2n + 1$

twice the product = $2n(n + 1) = 2n^2 + 2n$

the sum of squares is 1 more than this

so the statement is true

Proved

2 / 2

- 32 A two-digit number has tens digit a and units digit b .

The digits are reversed to make a new number.

Prove algebraically that the difference between the two numbers is a multiple of 9.

number = $10a + b$

reversed = $10b + a$

difference = $(10a + b) - (10b + a)$

= $9a - 9b$

= $9(a - b)$

so it is a multiple of 9

Proved

2 / 2

- 33 n is an integer.

Prove that the sum of $(n + 4)(n + 3)$ and $n + 4$ is always a square number.

You must show your working.

$(n + 4)(n + 3) = n^2 + 7n + 12$

adding $n + 4$ gives $n^2 + 8n + 16$

$n^2 + 8n + 16 = (n + 4)^2$

so the sum is a square number

Proved

2 / 2

- 34 Prove that the difference between the squares of two consecutive odd numbers is a multiple of 8.

Let the numbers be $2n + 1$ and $2n + 3$

$$\begin{aligned}(2n + 3)^2 - (2n + 1)^2 \\&= 4n^2 + 12n + 9 - (4n^2 + 4n + 1) \\&= 8n + 8 \\&= 8(n + 1) \\&\text{so it is a multiple of 8}\end{aligned}$$

Proved

2 / 2

- 35 Prove that the difference between the squares of any two odd positive integers is a multiple of 4.

Let the integers be $2m + 1$ and $2n + 1$

$$\begin{aligned}(2m + 1)^2 - (2n + 1)^2 \\&= 4m^2 + 4m + 1 - (4n^2 + 4n + 1) \\&= 4m^2 + 4m - 4n^2 - 4n \\&= 4(m^2 + m - n^2 - n) \\&\text{so it is a multiple of 4}\end{aligned}$$

Proved

2 / 2

- 36 Prove algebraically that the sum of the squares of any two consecutive multiples of 3 is a multiple of 9.

Let the numbers be $3n$ and $3n + 3$

$$\begin{aligned}(3n)^2 + (3n + 3)^2 \\&= 9n^2 + 9n^2 + 18n + 9 \\&= 18n^2 + 18n + 9 \\&= 9(2n^2 + 2n + 1) \\&\text{so it is a multiple of 9}\end{aligned}$$

Proved

2 / 2

37 Prove that $x^2 - 10x + 32$ is always greater than 0 for all values of x .

complete the square

$$x^2 - 10x + 32 = (x - 5)^2 - 25 + 32$$

$$= (x - 5)^2 + 7$$

a square is never negative

$$(x - 5)^2 \geq 0$$

so the expression is ≥ 7 which is positive

Proved

2 / 2

38 Prove that the sum of the squares of any two odd positive integers is 2 more than a multiple of 4.

You must show your working.

Let the integers be $2m + 1$ and $2n + 1$

$$(2m + 1)^2 + (2n + 1)^2$$

$$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$$

$$= 4m^2 + 4m + 4n^2 + 4n + 2$$

$$= 4(m^2 + m + n^2 + n) + 2$$

so it is 2 more than a multiple of 4

Proved

2 / 2

39 Show that the sum of the squares of any two consecutive multiples of 5 is a multiple of 25.

Let the numbers be $5n$ and $5n + 5$

$$(5n)^2 + (5n + 5)^2$$

$$= 25n^2 + 25n^2 + 50n + 25$$

$$= 50n^2 + 50n + 25$$

$$= 25(2n^2 + 2n + 1)$$

so it is a multiple of 25

Proved

3 / 3

40 Prove algebraically that $x^2 - 4x + 12$ is always greater than 0 for all values of x .

complete the square

$$x^2 - 4x + 12 = (x - 2)^2 - 4 + 12$$

$$= (x - 2)^2 + 8$$

a square is never negative

$$(x - 2)^2 \geq 0$$

so the expression is ≥ 8 which is positive

Proved

3 / 3

41 n is an integer.

Prove that $(5n)^2 - (3n - 2)^2$ is a multiple of 4.

difference of two squares

$$(5n)^2 - (3n - 2)^2 = (5n - (3n - 2))(5n + (3n - 2))$$

$$= (2n + 2)(8n - 2)$$

$$= 2(n + 1) \times 2(4n - 1)$$

$$= 4(n + 1)(4n - 1)$$

so it is a multiple of 4

Proved

3 / 3

42 Show that the sum of the squares of any two consecutive even numbers is 4 more than a multiple of 8.

Let the numbers be $2n$ and $2n + 2$

$$(2n)^2 + (2n + 2)^2$$

$$= 4n^2 + 4n^2 + 8n + 4$$

$$= 8n^2 + 8n + 4$$

$$= 8(n^2 + n) + 4$$

so it is 4 more than a multiple of 8

Proved

3 / 3

- 43 Prove that the product of two consecutive even numbers is a multiple of 8.

Let the numbers be $2n$ and $2n + 2$

$$\text{product} = 2n(2n + 2)$$

$$= 4n(n + 1)$$

$n(n + 1)$ is even so $n(n + 1) = 2k$

$$4 \times 2k = 8k$$

so it is a multiple of 8

Proved

3 / 3

- 44 n is an integer.

Prove that $(5n + 3)^2 - (3n - 1)^2$ is a multiple of 4.

difference of two squares

$$(5n + 3)^2 - (3n - 1)^2 = (5n + 3 - (3n - 1))(5n + 3 + (3n - 1))$$

$$= (2n + 4)(8n + 2)$$

$$= 2(n + 2) \times 2(4n + 1)$$

$$= 4(n + 2)(4n + 1)$$

so it is a multiple of 4

Proved

3 / 3

- 45 Prove algebraically that the sum of the squares of any two consecutive even numbers is 4 more than a multiple of 8.

You must show your working.

Let the numbers be $2n$ and $2n + 2$

$$(2n)^2 + (2n + 2)^2$$

$$= 4n^2 + 4n^2 + 8n + 4$$

$$= 8n^2 + 8n + 4$$

$$= 8(n^2 + n) + 4$$

so it is 4 more than a multiple of 8

Proved

3 / 3

- 46 A three-digit number has digits a , b and c , where a is the hundreds digit.
Prove that the number is a multiple of 3 when $a + b + c$ is a multiple of 3.

$\text{number} = 100a + 10b + c$
 $= 99a + 9b + (a + b + c)$
 $= 3(33a + 3b) + (a + b + c)$
 $3(33a + 3b)$ is a multiple of 3
 if $a + b + c$ is a multiple of 3 then so is the number

Proved

3 / 3

- 47 n is an integer.
Prove that $(4n + 3)^2 - (n + 2)^2$ is a multiple of 5.

difference of two squares
 $(4n + 3)^2 - (n + 2)^2 = (4n + 3 - (n + 2))(4n + 3 + (n + 2))$
 $= (3n + 1)(5n + 5)$
 $= 1(3n + 1) \times 5(n + 1)$
 $= 5(3n + 1)(n + 1)$
 so it is a multiple of 5

Proved

3 / 3

- 48 Prove that $(m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2$

$(m^2 - n^2)^2 = m^4 - 2m^2n^2 + n^4$
 $(2mn)^2 = 4m^2n^2$
 $LHS = m^4 + 2m^2n^2 + n^4$
 $= (m^2 + n^2)^2$
 $= RHS$

Proved

3 / 3

49 n is an integer.

Prove algebraically that $(n + 5)^2 - (3n - 1)^2$ is a multiple of 8.

difference of two squares

$$\begin{aligned}(n + 5)^2 - (3n - 1)^2 &= (n + 5 - (3n - 1))(n + 5 + (3n - 1)) \\&= (-2n + 6)(4n + 4) \\&= 2(-n + 3) \times 4(n + 1) \\&= 8(-n + 3)(n + 1) \\&\text{so it is a multiple of 8}\end{aligned}$$

Proved

3 / 3

50 n is an integer.

Prove algebraically that $(4n - 2)^2 - (2n - 4)^2$ is a multiple of 12.

difference of two squares

$$\begin{aligned}(4n - 2)^2 - (2n - 4)^2 &= (4n - 2 - (2n - 4))(4n - 2 + (2n - 4)) \\&= (2n + 2)(6n - 6) \\&= 2(n + 1) \times 6(n - 1) \\&= 12(n + 1)(n - 1) \\&\text{so it is a multiple of 12}\end{aligned}$$

Proved

3 / 3