

GCSE MATHS • GRADE 9

MATHS



MODEL ANSWERS

PROBABILITY EQUATION

SCORE

/92

PERCENTAGE

%

STEM UP

- 1 A spinner has three colours: red, blue and green.

$$P(\text{red}) = 3x, P(\text{blue}) = x \text{ and } P(\text{green}) = \frac{1}{10}.$$

Work out the value of x .

$$\text{total probability} = 1$$

$$3x + x + \frac{1}{10} = 1$$

$$4x = 1 - \frac{1}{10}$$

$$4x = \frac{9}{10}$$

$$x = \frac{9}{40}$$

$$\boxed{1} / 1$$

- 2 There are 8 black buttons and y grey buttons in a tin.

Oscar takes a button at random, replaces it, then takes another button at random.

$$\text{The probability that both buttons are black is } \frac{64}{361}.$$

Work out the value of y .

$$P(\text{black}) = \frac{8}{8+y}$$

$$\left(\frac{8}{8+y}\right)^2 = \frac{64}{361}$$

take square roots

$$\frac{8}{8+y} = \frac{8}{19}$$

$$8+y = 8 \times \frac{19}{8}$$

$$8+y = 19$$

$$y = 11$$

$$\boxed{1} / 1$$

- 3 A spinner has three colours: red, blue and green.

$$P(\text{red}) = 4x, P(\text{blue}) = x \text{ and } P(\text{green}) = \frac{1}{5}.$$

Work out the value of x .

$$\text{total probability} = 1$$

$$4x + x + \frac{1}{5} = 1$$

$$5x = 1 - \frac{1}{5}$$

$$5x = \frac{4}{5}$$

$$x = \frac{4}{25}$$

$$\boxed{1} / 1$$

- 4 There are some orange cubes and some purple cubes in a box.
The ratio of orange cubes to purple cubes is 4:3.
Two cubes are removed at random.

The probability that both the cubes taken are orange is $\frac{4}{13}$.

Work out how many orange cubes were in the box before any were removed.

$$\begin{aligned} \text{let orange cubes} &= 4x \text{ and all cubes} = 7x \\ P(\text{both orange}) &= \frac{4x}{7} \times \frac{4x-1}{7x-1} \\ &= \frac{4(4x-1)}{7} (7x-1) \\ \frac{4(4x-1)}{7} (7x-1) &= \frac{4}{13} \\ 52(4x-1) &= 28(7x-1) \\ 208x - 52 &= 196x - 28 \\ 12x &= 24 \\ x &= 2 \\ \text{orange cubes} &= 4 \times 2 = 8 \end{aligned}$$

8

1 / 1

- 5 There are 9 black buttons and y grey buttons in a tin.
Zara takes a button at random, replaces it, then takes another button at random.

The probability that both buttons are black is $\frac{81}{121}$.

Work out the value of y .

$$\begin{aligned} P(\text{black}) &= \frac{9}{9+y} \\ \left(\frac{9}{9+y}\right)^2 &= \frac{81}{121} \\ \text{take square roots} \\ \frac{9}{9+y} &= \frac{9}{11} \\ 9+y &= 9 \times \frac{11}{9} \\ 9+y &= 11 \end{aligned}$$

 $y = 2$

1 / 1

- 6 A bag contains x red counters and 8 blue counters.
A counter is taken at random.

The probability that it is red is $\frac{11}{19}$.

Work out the value of x .

$$\begin{aligned} \frac{x}{x+8} &= \frac{11}{19} \\ 19x &= 11(x+8) \\ 19x &= 11x + 88 \\ 8x &= 88 \end{aligned}$$

 $x = 11$

1 / 1

- 7 There are 7 black buttons and x grey buttons in a tin.
Two buttons are taken from the tin at random without replacement.

The probability that both the buttons taken are grey is $\frac{1}{36}$.

Work out the value of x .

$$\begin{aligned}
 P(\text{both grey}) &= \frac{x}{x+7} \times \frac{x-1}{x+6} \\
 \frac{x(x-1)}{(x+7)(x+6)} &= \frac{1}{36} \\
 36x(x-1) &= 1(x+7)(x+6) \\
 36x^2 - 36x &= x^2 + 13x + 42 \\
 35x^2 - 49x - 42 &= 0 \\
 5x^2 - 7x - 6 &= 0 \\
 (5x+3)(x-2) &= 0 \\
 x = -\frac{3}{5} \text{ or } x = 2 \\
 -\frac{3}{5} \text{ is not possible so } x &= 2
 \end{aligned}$$

$$x = 2$$

/ 1

- 8 There are 4 green counters and y yellow counters in a bag.
Maya takes a counter at random, replaces it, then takes another counter at random.

The probability that both counters are green is $\frac{16}{289}$.

Work out the value of y .

$$\begin{aligned}
 P(\text{green}) &= \frac{4}{4+y} \\
 \left(\frac{4}{4+y}\right)^2 &= \frac{16}{289} \\
 \text{take square roots} \\
 \frac{4}{4+y} &= \frac{4}{17} \\
 4+y &= 4 \times \frac{17}{4} \\
 4+y &= 17
 \end{aligned}$$

$$y = 13$$

/ 1

- 9 A box contains x orange cubes and 13 purple cubes.
A cube is taken at random.

The probability that it is orange is $\frac{15}{28}$.

Work out the value of x .

$$\begin{aligned}
 \frac{x}{x+13} &= \frac{15}{28} \\
 28x &= 15(x+13) \\
 28x &= 15x + 195 \\
 13x &= 195
 \end{aligned}$$

$$x = 15$$

/ 1

- 10** There are 6 red beads and x green beads in a jar.
Two beads are taken from the jar at random without replacement.

The probability that both the beads taken are green is $\frac{1}{28}$.

Work out the value of x .

$$\begin{aligned}
 P(\text{both green}) &= \frac{x}{x+6} \times \frac{x-1}{x+5} \\
 \frac{x(x-1)}{(x+6)(x+5)} &= \frac{1}{28} \\
 28x(x-1) &= 1(x+6)(x+5) \\
 28x^2 - 28x &= x^2 + 11x + 30 \\
 27x^2 - 39x - 30 &= 0 \\
 9x^2 - 13x - 10 &= 0 \\
 (9x+5)(x-2) &= 0 \\
 x = -\frac{5}{9} \text{ or } x = 2 \\
 -\frac{5}{9} \text{ is not possible so } x &= 2
 \end{aligned}$$

$$x = 2$$

1 / 1

- 11** There are 9 mint sweets and y toffee sweets in a bag.
Chloe takes a sweet at random, replaces it, then takes another sweet at random.

The probability that both sweets are mint is $\frac{9}{25}$.

Work out the value of y .

$$\begin{aligned}
 P(\text{mint}) &= \frac{9}{9+y} \\
 \left(\frac{9}{9+y}\right)^2 &= \frac{9}{25} \\
 \text{take square roots} \\
 \frac{9}{9+y} &= \frac{3}{5} \\
 9+y &= 9 \times \frac{5}{3} \\
 9+y &= 15
 \end{aligned}$$

$$y = 6$$

1 / 1

- 12** A bag contains x red counters and 6 blue counters.
A counter is taken at random.

The probability that it is red is $\frac{3}{5}$.

Work out the value of x .

$$\begin{aligned}
 \frac{x}{x+6} &= \frac{3}{5} \\
 5x &= 3(x+6) \\
 5x &= 3x + 18 \\
 2x &= 18
 \end{aligned}$$

$$x = 9$$

1 / 1

- 13 There are 5 orange cubes and x purple cubes in a box.
Two cubes are taken from the box at random without replacement.

The probability that both the cubes taken are purple is $\frac{3}{28}$.

Work out the value of x .

$$P(\text{both purple}) = \frac{x}{x+5} \times \frac{x-1}{x+4}$$

$$\frac{x(x-1)}{(x+5)(x+4)} = \frac{3}{28}$$

$$28x(x-1) = 3(x+5)(x+4)$$

$$28x^2 - 28x = 3x^2 + 27x + 60$$

$$25x^2 - 55x - 60 = 0$$

$$5x^2 - 11x - 12 = 0$$

$$(5x+4)(x-3) = 0$$

$$x = -\frac{4}{5} \text{ or } x = 3$$

$$-\frac{4}{5} \text{ is not possible so } x = 3$$

$$x = 3$$

1 / 1

- 14 There are 9 red beads and y green beads in a jar.
Ethan takes a bead at random, replaces it, then takes another bead at random.

The probability that both beads are red is $\frac{9}{49}$.

Work out the value of y .

$$P(\text{red}) = \frac{9}{9+y}$$

$$\left(\frac{9}{9+y}\right)^2 = \frac{9}{49}$$

take square roots

$$\frac{9}{9+y} = \frac{3}{7}$$

$$9+y = 9 \times \frac{7}{3}$$

$$9+y = 21$$

$$y = 12$$

1 / 1

- 15 A box has 5 orange cubes and 11 purple cubes.
 m orange cubes are removed from the box.
A cube is then taken at random from the box.

The probability that it is orange is $\frac{2}{13}$.

Work out the value of m .

$$\text{orange left} = 5 - m$$

$$\text{all left} = 16 - m$$

$$\frac{5-m}{16-m} = \frac{2}{13}$$

$$13(5-m) = 2(16-m)$$

$$65 - 13m = 32 - 2m$$

$$65 - 32 = 13m - 2m$$

$$33 = 11m$$

$$m = 3$$

1 / 1

- 16** There are 3 white balls and x black balls in a box.
Two balls are removed at random from the box.

The probability that both the balls taken are black is $\frac{1}{5}$.

Work out the value of x .

$$\begin{aligned}
 P(\text{both black}) &= \frac{x}{x+3} \times \frac{x-1}{x+2} \\
 \frac{x(x-1)}{(x+3)(x+2)} &= \frac{1}{5} \\
 5x(x-1) &= 1(x+3)(x+2) \\
 5x^2 - 5x &= x^2 + 5x + 6 \\
 4x^2 - 10x - 6 &= 0 \\
 2x^2 - 5x - 3 &= 0 \\
 (2x+1)(x-3) &= 0 \\
 x = -\frac{1}{2} \text{ or } x = 3 \\
 -\frac{1}{2} \text{ is not possible so } x &= 3
 \end{aligned}$$

$$x = 3$$

1 / 1

- 17** There are 9 red counters and y blue counters in a bag.
Matt takes a counter at random, replaces it, then takes another counter at random.

The probability that both counters are red is $\frac{1}{4}$.

Work out the value of y .

$$\begin{aligned}
 P(\text{red}) &= \frac{9}{9+y} \\
 \left(\frac{9}{9+y}\right)^2 &= \frac{1}{4} \\
 \text{take square roots} \\
 \frac{9}{9+y} &= \frac{1}{2} \\
 9+y &= 9 \times 2 \\
 9+y &= 18
 \end{aligned}$$

$$y = 9$$

1 / 1

- 18** There are some mint sweets and some toffee sweets in a bag.
The ratio of mint sweets to toffee sweets is 2:1.
Two sweets are removed at random.

The probability that both the sweets taken are toffee is $\frac{1}{12}$.

Work out how many sweets were in the bag before any were removed.

$$\begin{aligned}
 \text{let toffee sweets} &= x \text{ and all sweets} = 3x \\
 P(\text{both toffee}) &= \frac{x}{3x} \times \frac{x-1}{3x-1} \\
 &= \frac{x-1}{3(3x-1)} \\
 \frac{x-1}{3(3x-1)} &= \frac{1}{12} \\
 12(x-1) &= 3(3x-1) \\
 12x - 12 &= 9x - 3 \\
 3x &= 9 \\
 x &= 3 \\
 \text{all sweets} &= 3 \times 3 = 9
 \end{aligned}$$

$$9$$

1 / 1

- 19 There are 5 orange cubes and x purple cubes in a box.
Two cubes are removed at random from the box.

The probability that both the cubes taken are purple is $\frac{2}{9}$.

Work out the value of x .

$$P(\text{both purple}) = \frac{x}{x+5} \times \frac{x-1}{x+4}$$

$$\frac{x(x-1)}{(x+5)(x+4)} = \frac{2}{9}$$

$$9x(x-1) = 2(x+5)(x+4)$$

$$9x^2 - 9x = 2x^2 + 18x + 40$$

$$7x^2 - 27x - 40 = 0$$

$$(7x+8)(x-5) = 0$$

$$x = -\frac{8}{7} \text{ or } x = 5$$

$$-\frac{8}{7} \text{ is not possible so } x = 5$$

$$x = 5$$

1 / 1

- 20 There are 6 mint sweets and y toffee sweets in a bag.
Alan takes a sweet at random, replaces it, then takes another sweet at random.

The probability that both sweets are mint is $\frac{4}{49}$.

Work out the value of y .

$$P(\text{mint}) = \frac{6}{6+y}$$

$$\left(\frac{6}{6+y}\right)^2 = \frac{4}{49}$$

take square roots

$$\frac{6}{6+y} = \frac{2}{7}$$

$$6+y = 6 \times \frac{7}{2}$$

$$6+y = 21$$

$$y = 15$$

1 / 1

- 21 There are some black buttons and some grey buttons in a tin.
The ratio of black buttons to grey buttons is 4:1.
Two buttons are removed at random.

The probability that both the buttons taken are grey is $\frac{1}{35}$.

Work out how many grey buttons were in the tin before any were removed.

let grey buttons = x and all buttons = $5x$

$$P(\text{both grey}) = \frac{x}{5x} \times \frac{x-1}{5x-1}$$

$$= \frac{x-1}{5(Sx-1)}$$

$$\frac{x-1}{5(Sx-1)} = \frac{1}{35}$$

$$35(x-1) = 5(Sx-1)$$

$$35x - 35 = 25x - 5$$

$$10x = 30$$

$$x = 3$$

$$\text{grey buttons} = 1 \times 3 = 3$$

$$3$$

2 / 2

- 22 There are 2 black buttons and x grey buttons in a tin.
Two buttons are taken from the tin at random without replacement.

The probability that both the buttons taken are black is $\frac{1}{66}$.

Work out the value of x .

$$P(\text{both black}) = \frac{2}{x+2} \times \frac{1}{x+1}$$

$$\frac{2}{(x+2)(x+1)} = \frac{1}{66}$$

$$66 \times 2 = 1(x+2)(x+1)$$

$$132 = x^2 + 3x + 2$$

$$x^2 + 3x - 130 = 0$$

$$(x+13)(x-10) = 0$$

$$x = -13 \text{ or } x = 10$$

$$-13 \text{ is not possible so } x = 10$$

$$x = 10$$

2 / 2

- 23 There are x white balls and 3 black balls in a box.
Charles takes a ball at random, replaces it, then takes another ball at random.

The probability that both balls are black is $\frac{9}{256}$.

Work out the value of x .

$$P(\text{black}) = \frac{3}{3+x}$$

$$\left(\frac{3}{3+x}\right)^2 = \frac{9}{256}$$

take square roots

$$\frac{3}{3+x} = \frac{3}{16}$$

$$3+x = 3 \times \frac{16}{3}$$

$$3+x = 16$$

$$x = 13$$

2 / 2

- 24 There are some blue marbles and some white marbles in a bag.
The ratio of blue marbles to white marbles is 3:1.
Two marbles are removed at random.

The probability that both the marbles taken are white is $\frac{1}{20}$.

Work out how many marbles were in the bag before any were removed.

let white marbles = x and all marbles = $4x$

$$P(\text{both white}) = \frac{x}{4x} \times \frac{x-1}{4x-1}$$

$$= \frac{x-1}{4} (4x-1)$$

$$\frac{x-1}{4} (4x-1) = \frac{1}{20}$$

$$20(x-1) = 4(4x-1)$$

$$20x - 20 = 16x - 4$$

$$4x = 16$$

$$x = 4$$

$$\text{all marbles} = 4 \times 4 = 16$$

$$16$$

2 / 2

- 25 There are 5 blue marbles and x white marbles in a bag.
Two marbles are removed at random from the bag.

The probability that both the marbles taken are white is $\frac{33}{68}$.

Work out the value of x .

$$\begin{aligned} P(\text{both white}) &= \frac{x}{x+5} \times \frac{x-1}{x+4} \\ \frac{x(x-1)}{(x+5)(x+4)} &= \frac{33}{68} \\ 68x(x-1) &= 33(x+5)(x+4) \\ 68x^2 - 68x &= 33x^2 + 297x + 660 \\ 35x^2 - 365x - 660 &= 0 \\ 7x^2 - 73x - 132 &= 0 \\ (7x+11)(x-12) &= 0 \\ x = -\frac{11}{7} \text{ or } x = 12 \\ -\frac{11}{7} \text{ is not possible so } x &= 12 \end{aligned}$$

$$x = 12$$

2 / 2

- 26 There are n buttons in a tin. 8 of the buttons are black and the rest are grey. Jack takes a button from the tin at random and does not replace it. He then takes another button at random from the tin.

The probability that Jack takes two grey buttons is $\frac{1}{5}$.

(a) Show that $n^2 - 21n + 90 = 0$

(b) Find the value of n .

$$\begin{aligned} \frac{(n-8)(n-9)}{n(n-1)} &= \frac{1}{5} \\ 5 \times (n-8)(n-9) &= 1 \times n(n-1) \\ 5n^2 - 85n + 360 &= n^2 - n \\ 4n^2 - 84n + 360 &= 0 \\ n^2 - 21n + 90 &= 0 \\ (n-6)(n-15) &= 0 \\ n = 6 \text{ or } n = 15 \\ n = 15 \text{ (} n = 6 \text{ is too small)} \end{aligned}$$

$$(b) n = 15$$

2 / 2

- 27 There are some orange cubes and some purple cubes in a box.
The ratio of orange cubes to purple cubes is 3:2.
Two cubes are removed at random.

The probability that both the cubes taken are orange is $\frac{33}{95}$.

Work out how many orange cubes were in the box before any were removed.

$$\begin{aligned} \text{let orange cubes} &= 3x \text{ and all cubes} = 5x \\ P(\text{both orange}) &= \frac{3x}{5} \times \frac{3x-1}{5x-1} \\ &= \frac{3(3x-1)}{5} \times \frac{1}{5x-1} \\ \frac{3(3x-1)}{5} &= \frac{33}{95} \\ 285(3x-1) &= 165(5x-1) \\ 855x - 285 &= 825x - 165 \\ 30x &= 120 \\ x &= 4 \\ \text{orange cubes} &= 3 \times 4 = 12 \end{aligned}$$

$$12$$

2 / 2

- 28 There are 7 mint sweets and x toffee sweets in a bag.
Two sweets are taken from the bag at random without replacement.

The probability that both the sweets taken are mint is $\frac{21}{190}$.

Work out the value of x .

$$\begin{aligned} P(\text{both mint}) &= \frac{7}{x+7} \times \frac{6}{x+6} \\ \frac{42}{(x+7)(x+6)} &= \frac{21}{190} \\ 190 \times 42 &= 21(x+7)(x+6) \\ 7980 &= 21x^2 + 273x + 882 \\ 21x^2 + 273x - 7098 &= 0 \\ x^2 + 13x - 338 &= 0 \\ (x+26)(x-13) &= 0 \\ x &= -26 \text{ or } x = 13 \\ -26 &\text{ is not possible so } x = 13 \end{aligned}$$

$$x = 13$$

2 / 2

- 29 There are n sweets in a bag. 4 of the sweets are mint and the rest are toffee. Ava takes a sweet from the bag at random and does not replace it. She then takes another sweet at random from the bag.

The probability that Ava takes two mint sweets is $\frac{2}{7}$.

(a) Show that $n^2 - n - 42 = 0$

(b) Find the value of n .

$$\begin{aligned} \frac{4}{n(n-1)} &= \frac{2}{7} \\ 7 \times 4 &= 2 \times n(n-1) \\ 28 &= 2n^2 - 2n \\ 2n^2 - 2n - 28 &= 0 \\ n^2 - n - 14 &= 0 \\ (n+6)(n-7) &= 0 \\ n &= -6 \text{ or } n = 7 \\ n &= 7 \text{ (n cannot be negative)} \end{aligned}$$

$$(b) n = 7$$

2 / 2

- 30 There are some orange cubes and some purple cubes in a box.
The ratio of orange cubes to purple cubes is 5:2.
Two cubes are removed at random.

The probability that both the cubes taken are purple is $\frac{9}{119}$.

Work out how many purple cubes were in the box before any were removed.

$$\begin{aligned} \text{let purple cubes} &= 2x \text{ and all cubes} = 7x \\ P(\text{both purple}) &= \frac{2x}{7} \times \frac{2x-1}{7x-1} \\ &= \frac{2(2x-1)}{7(7x-1)} \\ \frac{2(2x-1)}{7(7x-1)} &= \frac{9}{119} \\ 238(2x-1) &= 63(7x-1) \\ 476x - 238 &= 441x - 63 \\ 35x &= 175 \\ x &= 5 \\ \text{purple cubes} &= 2 \times 5 = 10 \end{aligned}$$

$$10$$

2 / 2

- 31 There are 8 mint sweets and x toffee sweets in a bag.
Two sweets are removed at random from the bag.

The probability that the two sweets taken are the same colour is $\frac{43}{91}$.

Work out the value of x .

$$\begin{aligned}
 P(\text{same}) &= \frac{x}{x+8} \times \frac{x-1}{x+7} + \frac{8}{x+8} \times \frac{7}{x+7} \\
 \frac{x(x-1)+56}{(x+8)(x+7)} &= \frac{43}{91} \\
 91(x(x-1)+56) &= 43(x+8)(x+7) \\
 91x^2 - 91x + 5096 &= 43x^2 + 645x + 2408 \\
 48x^2 - 736x + 2688 &= 0 \\
 3x^2 - 46x + 168 &= 0 \\
 (x-6)(3x-28) &= 0 \\
 x = 6 \text{ or } x = \frac{28}{3} \\
 \frac{28}{3} \text{ is not possible so } x &= 6
 \end{aligned}$$

$$x = 6$$

2 / 2

- 32 There are n counters in a bag. 2 of the counters are red and the rest are blue. Ethan takes a counter from the bag at random and does not replace it. He then takes another counter at random from the bag.

The probability that Ethan takes two red counters is $\frac{1}{28}$.

(a) Show that $n^2 - n - 56 = 0$

(b) Find the value of n .

$$\begin{aligned}
 P(\text{two red}) &= \frac{2}{n} \times \frac{1}{n-1} \\
 \frac{2}{n(n-1)} &= \frac{1}{28} \\
 28 \times 2 &= 1 \times n(n-1) \\
 56 &= n^2 - n \\
 n^2 - n - 56 &= 0 \\
 (b) (n+7)(n-8) &= 0 \\
 n = -7 \text{ or } n &= 8 \\
 n = 8 \text{ (} n \text{ cannot be negative)}
 \end{aligned}$$

$$(b) n = 8$$

2 / 2

- 33 There are some black buttons and some grey buttons in a tin.
The ratio of black buttons to grey buttons is 1:1.
Two buttons are removed at random.

The probability that both the buttons taken are grey is $\frac{2}{9}$.

Work out how many buttons were in the tin before any were removed.

$$\begin{aligned}
 \text{let grey buttons} &= x \text{ and all buttons} = 2x \\
 P(\text{both grey}) &= \frac{x}{2x} \times \frac{x-1}{2x-1} \\
 &= \frac{x-1}{2(2x-1)} \\
 \frac{x-1}{2(2x-1)} &= \frac{2}{9} \\
 9(x-1) &= 4(2x-1) \\
 9x - 9 &= 8x - 4 \\
 1x &= 5 \\
 x &= 5 \\
 \text{all buttons} &= 2 \times 5 = 10
 \end{aligned}$$

$$10$$

2 / 2

- 34 There are 7 black buttons and x grey buttons in a tin.
Two buttons are taken from the tin at random without replacement.

The probability that the two buttons taken are the same colour is $\frac{19}{40}$.

Work out the value of x .

$$P(\text{same}) = \frac{x}{x+7} \times \frac{x-1}{x+6} + \frac{7}{x+7} \times \frac{6}{x+6}$$

$$\frac{x(x-1)+42}{(x+7)(x+6)} = \frac{19}{40}$$

$$40(x(x-1)+42) = 19(x+7)(x+6)$$

$$40x^2 - 40x + 1680 = 19x^2 + 247x + 798$$

$$21x^2 - 287x + 882 = 0$$

$$3x^2 - 41x + 126 = 0$$

$$(3x-14)(x-9) = 0$$

$$x = \frac{14}{3} \text{ or } x = 9$$

$$\frac{14}{3} \text{ is not possible so } x = 9$$

$$x = 9$$

2 / 2

- 35 There are n marbles in a bag. 4 of the marbles are blue and the rest are white. Priya takes a marble from the bag at random and does not replace it. She then takes another marble at random from the bag.

The probability that Priya takes two blue marbles is $\frac{6}{55}$.

(a) Show that $n^2 - n - 110 = 0$

(b) Find the value of n .

$$\frac{12}{n(n-1)} = \frac{6}{55}$$

$$55 \times 12 = 6 \times n(n-1)$$

$$660 = 6n^2 - 6n$$

$$6n^2 - 6n - 660 = 0$$

$$n^2 - n - 110 = 0$$

$$(b) (n+10)(n-11) = 0$$

$$n = -10 \text{ or } n = 11$$

$$n = 11 \text{ (n cannot be negative)}$$

$$(b) n = 11$$

2 / 2

- 36 There are some red counters and some blue counters in a bag.
The ratio of red counters to blue counters is 1:1.
Two counters are removed at random.

The probability that both the counters taken are blue is $\frac{5}{22}$.

Work out how many counters were in the bag before any were removed.

let blue counters = x and all counters = $2x$

$$P(\text{both blue}) = \frac{x}{2x} \times \frac{x-1}{2x-1}$$

$$= \frac{x-1}{2} (2x-1)$$

$$\frac{x-1}{2} (2x-1) = \frac{5}{22}$$

$$22(x-1) = 10(2x-1)$$

$$22x - 22 = 20x - 10$$

$$2x = 12$$

$$x = 6$$

$$\text{all counters} = 2 \times 6 = 12$$

$$12$$

2 / 2

- 37 There are 7 white balls and y black balls in a box.
Priya takes a ball at random, replaces it, then takes another ball at random.

The probability that the first ball is white and the second is black is $\frac{84}{361}$.

Work out the value of y .

$$\begin{aligned} P(\text{white then black}) &= \frac{7}{7+y} \times \frac{y}{7+y} \\ &= \frac{7y}{(7+y)^2} \\ \frac{7y}{(7+y)^2} &= \frac{84}{361} \\ 361 \times 7y &= 84(7+y)^2 \\ 2527y &= 84y^2 + 1176y + 4116 \\ 84y^2 - 1351y + 4116 &= 0 \\ 12y^2 - 193y + 588 &= 0 \\ (12y - 49)(y - 12) &= 0 \\ y &= \frac{49}{12} \text{ or } y = 12 \\ y &= 12 \left(\frac{49}{12} \text{ is not a whole number} \right) \end{aligned}$$

$$y = 12$$

2 / 2

- 38 There are n counters in a bag. 3 of the counters are green and the rest are yellow. Zara takes a counter from the bag at random and does not replace it. She then takes another counter at random from the bag.

The probability that Zara takes two green counters is $\frac{3}{136}$.

(a) Show that $n^2 - n - 272 = 0$

(b) Find the value of n .

$$\begin{aligned} \frac{6}{n(n-1)} &= \frac{3}{136} \\ 136 \times 6 &= 3 \times n(n-1) \\ 816 &= 3n^2 - 3n \\ 3n^2 - 3n - 816 &= 0 \\ n^2 - n - 272 &= 0 \\ (n+16)(n-17) &= 0 \\ n &= -16 \text{ or } n = 17 \\ n &= 17 \text{ (} n \text{ cannot be negative)} \end{aligned}$$

$$(b) n = 17$$

2 / 2

- 39 There are some black buttons and some grey buttons in a tin.
The ratio of black buttons to grey buttons is 1:1.
Two buttons are removed at random.

The probability that both the buttons taken are grey is $\frac{3}{13}$.

Work out how many grey buttons were in the tin before any were removed.

$$\begin{aligned} \text{let grey buttons} &= x \text{ and all buttons} = 2x \\ P(\text{both grey}) &= \frac{x}{2} \times \frac{x-1}{2x-1} \\ &= \frac{x-1}{2} (2x-1) \\ \frac{x-1}{2} (2x-1) &= \frac{3}{13} \\ 13(x-1) &= 6(2x-1) \\ 13x - 13 &= 12x - 6 \\ 1x &= 7 \\ x &= 7 \\ \text{grey buttons} &= 1 \times 7 = 7 \end{aligned}$$

$$7$$

3 / 3

- 40 There are 2 black buttons and y grey buttons in a tin.
Samir takes a button at random, replaces it, then takes another button at random.

The probability that both buttons are the same colour is $\frac{37}{49}$.

Work out the value of y .

$$\begin{aligned} P(\text{same}) &= \left(\frac{2}{2+y}\right)^2 + \left(\frac{y}{2+y}\right)^2 \\ &= \frac{4+y^2}{(2+y)^2} \\ \frac{4+y^2}{(2+y)^2} &= \frac{37}{49} \\ 49 \times (4+y^2) &= 37(2+y)^2 \\ 49y^2 + 196 &= 37y^2 + 148y + 148 \\ 12y^2 - 148y + 48 &= 0 \\ 3y^2 - 37y + 12 &= 0 \\ (3y-1)(y-12) &= 0 \\ y = \frac{1}{3} \text{ or } y = 12 \\ y = 12 \left(\frac{1}{3} \text{ is not a whole number}\right) \end{aligned}$$

$$y = 12$$

3 / 3

- 41 There are n counters in a bag. 3 of the counters are red and the rest are blue. Leo takes a counter from the bag at random and does not replace it. He then takes another counter at random from the bag.

The probability that Leo takes two red counters is $\frac{3}{10}$.

(a) Show that $n^2 - n - 20 = 0$

(b) Find the value of n .

$$\begin{aligned} \frac{6}{n(n-1)} &= \frac{3}{10} \\ 10 \times 6 &= 3 \times n(n-1) \\ 60 &= 3n^2 - 3n \\ 3n^2 - 3n - 60 &= 0 \\ n^2 - n - 20 &= 0 \\ (n+4)(n-5) &= 0 \\ n = -4 \text{ or } n = 5 \\ n = 5 \text{ (} n \text{ cannot be negative)} \end{aligned}$$

$$(b) n = 5$$

3 / 3

- 42 There are 3 mint sweets and x toffee sweets in a bag. Two sweets are taken at random without replacement. The probability that both are toffee is $\frac{6}{11}$.

(a) Work out the value of x .

(b) Work out the probability that the two sweets are different colours.

$$\begin{aligned} (a) \frac{x(x-1)}{(x+3)(x+2)} &= \frac{6}{11} \\ 11x(x-1) &= 6(x+3)(x+2) \\ 5x^2 - 41x - 36 &= 0 \\ (5x+4)(x-9) &= 0 \\ (b) \text{ total} &= 3 + 9 = 12 \\ P(\text{different}) &= 2 \times \frac{3}{12} \times \frac{9}{11} = \frac{9}{22} \end{aligned}$$

$$(a) x = 9 \quad (b) \frac{9}{22}$$

3 / 3

- 43 There are 3 orange cubes and y purple cubes in a box.
Alice takes a cube at random, replaces it, then takes another cube at random.

The probability that both cubes are the same colour is $\frac{205}{289}$.

Work out the value of y .

$$\begin{aligned} P(\text{same}) &= \left(\frac{3}{3+y}\right)^2 + \left(\frac{y}{3+y}\right)^2 \\ &= \frac{9+y^2}{(3+y)^2} \\ \frac{9+y^2}{(3+y)^2} &= \frac{205}{289} \\ 289 \times (9+y^2) &= 205(3+y)^2 \\ 289y^2 + 2601 &= 205y^2 + 1230y + 1845 \\ 84y^2 - 1230y + 756 &= 0 \\ 14y^2 - 205y + 126 &= 0 \\ (14y-9)(y-14) &= 0 \\ y = \frac{9}{14} \text{ or } y = 14 \\ y = 14 \left(\frac{9}{14} \text{ is not a whole number} \right) \end{aligned}$$

$$y = 14$$

3 / 3

- 44 There are n balls in a box. 9 of the balls are white and the rest are black. Samir takes a ball from the box at random and does not replace it. He then takes another ball at random from the box.

The probability that Samir takes two white balls is $\frac{9}{34}$.

(a) Show that $n^2 - n - 272 = 0$

(b) Find the value of n .

$$\begin{aligned} \frac{72}{n(n-1)} &= \frac{9}{34} \\ 34 \times 72 &= 9 \times n(n-1) \\ 2448 &= 9n^2 - 9n \\ 9n^2 - 9n - 2448 &= 0 \\ n^2 - n - 272 &= 0 \\ (n+16)(n-17) &= 0 \\ n &= -16 \text{ or } n = 17 \\ n &= 17 \text{ (} n \text{ cannot be negative)} \end{aligned}$$

$$(b) n = 17$$

3 / 3

- 45 There are 5 orange cubes and x purple cubes in a box. Two cubes are taken at random without replacement. The probability that both are purple is $\frac{11}{24}$.

(a) Work out the value of x .

(b) Work out the probability that the two cubes are different colours.

$$\begin{aligned} (a) \frac{x(x-1)}{(x+5)(x+4)} &= \frac{11}{24} \\ 24 \times x(x-1) &= 11(x+5)(x+4) \\ 13x^2 - 123x - 220 &= 0 \\ (13x+20)(x-11) &= 0 \\ (b) \text{ total} &= 5 + 11 = 16 \\ P(\text{different}) &= 2 \times \frac{5}{16} \times \frac{11}{15} = \frac{11}{24} \end{aligned}$$

$$(a) x = 11 \quad (b) \frac{11}{24}$$

3 / 3

- 46 There are n cubes in a box. 6 of the cubes are orange and the rest are purple. Sofia takes a cube from the box at random and does not replace it. She then takes another cube at random from the box.

The probability that Sofia takes two orange cubes is $\frac{5}{12}$.

(a) Show that $n^2 - n - 72 = 0$

(b) Find the value of n .

$$\begin{aligned}\frac{30}{n(n-1)} &= \frac{5}{12} \\ 12 \times 30 &= 5 \times n(n-1) \\ 360 &= 5n^2 - 5n \\ 5n^2 - 5n - 360 &= 0 \\ n^2 - n - 72 &= 0 \\ (b) (n+8)(n-9) &= 0 \\ n &= -8 \text{ or } n = 9 \\ n &= 9 \text{ (} n \text{ cannot be negative)}\end{aligned}$$

$$(b) n = 9$$

3 / 3

- 47 There are n cubes in a box. 3 of the cubes are orange and the rest are purple. Matt takes a cube from the box at random and does not replace it. He then takes another cube at random from the box.

The probability that Matt takes two orange cubes is $\frac{1}{5}$.

(a) Show that $n^2 - n - 30 = 0$

(b) Find the value of n .

$$\begin{aligned}P(\text{two orange}) &= \frac{3}{n} \times \frac{2}{n-1} \\ \frac{6}{n(n-1)} &= \frac{1}{5} \\ 5 \times 6 &= 1 \times n(n-1) \\ 30 &= n^2 - n \\ n^2 - n - 30 &= 0 \\ (b) (n+5)(n-6) &= 0 \\ n &= -5 \text{ or } n = 6 \\ n &= 6 \text{ (} n \text{ cannot be negative)}\end{aligned}$$

$$(b) n = 6$$

3 / 3

- 48 There are 5 mint sweets and x toffee sweets in a bag. Two sweets are taken at random without replacement. The probability that both are toffee is $\frac{33}{68}$.

(a) Work out the value of x .

(b) Work out the probability that the two sweets are different colours.

$$\begin{aligned}(a) \frac{x(x-1)}{(x+5)(x+4)} &= \frac{33}{68} \\ 68x(x-1) &= 33(x+5)(x+4) \\ 35x^2 - 365x - 660 &= 0 \\ (7x+11)(x-12) &= 0 \\ (b) \text{ total} &= 5 + 12 = 17 \\ P(\text{different}) &= 2 \times \frac{5}{17} \times \frac{12}{16} = \frac{15}{34}\end{aligned}$$

$$(a) x = 12 \quad (b) \frac{15}{34}$$

3 / 3

- 49 There are 8 red beads and x green beads in a jar. Two beads are taken at random without replacement. The probability that both are green is $\frac{1}{11}$.

(a) Work out the value of x .

(b) Work out the probability that the two beads are different colours.

$$(a) \frac{x(x-1)}{(x+8)(x+7)} = \frac{1}{11}$$

$$11x(x-1) = 1(x+8)(x+7)$$

$$10x^2 - 26x - 56 = 0$$

$$(5x+7)(x-4) = 0$$

$$(b) \text{ total} = 8 + 4 = 12$$

$$P(\text{different}) = 2 \times \frac{8}{12} \times \frac{4}{11} = \frac{16}{33}$$

$$(a) x = 4 \quad (b) \frac{16}{33}$$

3 / 3

- 50 There are 6 mint sweets and x toffee sweets in a bag. Two sweets are taken at random without replacement. The probability that both are toffee is $\frac{2}{15}$.

(a) Work out the value of x .

(b) Work out the probability that the two sweets are different colours.

$$(a) \frac{x(x-1)}{(x+6)(x+5)} = \frac{2}{15}$$

$$15x(x-1) = 2(x+6)(x+5)$$

$$13x^2 - 37x - 60 = 0$$

$$(13x+15)(x-4) = 0$$

$$(b) \text{ total} = 6 + 4 = 10$$

$$P(\text{different}) = 2 \times \frac{6}{10} \times \frac{4}{9} = \frac{8}{15}$$

$$(a) x = 4 \quad (b) \frac{8}{15}$$

3 / 3