

GCSE MATHS • GRADE 9

MATHS



MODEL ANSWERS

PERPENDICULAR LINES

SCORE

/92

PERCENTAGE

%

STEMUP

- 1 A straight line has equation $y = x - 9$

Write down the gradient of a line that is perpendicular to this line.

gradient of the line is 1

perpendicular gradient is -1 divided by 1 = -1

-1

1 / 1

- 2 A straight line passes through the points $(-1, 2)$ and $(4, -13)$

Find the equation of the straight line.

gradient = $\frac{-13 - 2}{4 - (-1)} = -3$

$y = -3x + c$

put in $(-1, 2)$: $c = -1$

$y = -3x - 1$

1 / 1

- 3 Here are the equations of 5 straight lines.

$P : y = 3x - 4$ $Q : y = -\frac{1}{3}x + 1$ $R : y = \frac{2}{3}x + 4$

$S : y = -3x - 1$ $T : y = \frac{1}{3}x + 2$

(a) Write down the letter of the line that is parallel to $y = \frac{1}{3}x + 3$

(b) Write down the letter of the line that is perpendicular to $y = \frac{1}{3}x - 4$

(a) gradient of $y = \frac{1}{3}x + 3$ is $\frac{1}{3}$, so line T

(b) perpendicular gradient is -3, so line S

(a) T (b) S

1 / 1

- 4 A straight line has equation $y = -4x + 4$

Write down the gradient of a line that is perpendicular to this line.

gradient of the line is -4

perpendicular gradient is -1 divided by $-4 = \frac{1}{4}$

$$\frac{1}{4}$$

/ 1

- 5 A straight line passes through the points (4, 9) and (6, 21)

Find the equation of the straight line.

$$\text{gradient} = \frac{21-9}{6-4} = 6$$

$$y = 6x + c$$

$$\text{put in (4, 9): } c = -15$$

$$y = 6x - 15$$

/ 1

- 6 Here are the equations of 5 straight lines.

$$P : y = 4x + 8 \quad Q : y = -\frac{1}{2}x - 3 \quad R : y = 2x + 2$$

$$S : y = -2x - 8 \quad T : y = \frac{1}{2}x - 4$$

(a) Write down the letter of the line that is parallel to $y = 2x + 4$

(b) Write down the letter of the line that is perpendicular to $y = 2x + 1$

(a) gradient of $y = 2x + 4$ is 2, so line R

(b) perpendicular gradient is $-\frac{1}{2}$, so line Q

(a) R (b) Q

/ 1

- 7 A straight line has equation $y = -2x - 1$

Write down the gradient of a line that is perpendicular to this line.

gradient of the line is -2

perpendicular gradient is -1 divided by $-2 = \frac{1}{2}$

$$\frac{1}{2}$$

/ 1

- 8 A straight line passes through the points $(3, -24)$ and $(-1, 8)$

Find the equation of the straight line.

$$\text{gradient} = \frac{8 - (-24)}{-1 - 3} = -8$$

$$y = -8x + c$$

$$\text{put in } (3, -24): c = 0$$

$$y = -8x$$

/ 1

- 9 Here are the equations of 5 straight lines.

$$P: y = 3x - 5 \quad Q: y = -3x - 2 \quad R: y = \frac{1}{3}x - 7$$

$$S: y = \frac{3}{2}x - 1 \quad T: y = -\frac{1}{3}x + 5$$

(a) Write down the letter of the line that is parallel to $y = 3x + 3$

(b) Write down the letter of the line that is perpendicular to $y = 3x - 2$

(a) gradient of $y = 3x + 3$ is 3 , so line P

(b) perpendicular gradient is $-\frac{1}{3}$, so line T

(a) P (b) T

/ 1

- 10 A straight line has equation $y = 3x + 6$

Write down the gradient of a line that is perpendicular to this line.

gradient of the line is 3

perpendicular gradient is -1 divided by $3 = -\frac{1}{3}$

$-\frac{1}{3}$

/ 1

- 11 Find the gradient of a line that is perpendicular to the line $2x - 5y = 0$

make y the subject: $y = \frac{2}{5}x$

gradient is $\frac{2}{5}$

perpendicular gradient = $-\frac{5}{2}$

$-\frac{5}{2}$

/ 1

- 12 Here are the equations of 5 straight lines.

$P : y = \frac{3}{2}x + 3$ $Q : y = -\frac{1}{3}x + 7$ $R : y = 3x + 5$

$S : y = -3x - 8$ $T : y = \frac{1}{3}x + 4$

(a) Write down the letter of the line that is parallel to $y = 3x - 4$

(b) Write down the letter of the line that is perpendicular to $y = 3x - 3$

(a) gradient of $y = 3x - 4$ is 3, so line R

(b) perpendicular gradient is $-\frac{1}{3}$, so line Q

(a) R (b) Q

/ 1

- 13 A straight line passes through the points (3, 17) and (-2, 5)

Find the equation of the straight line.

$$\text{gradient} = \frac{5 - 17}{-2 - 3} = \frac{12}{5}$$

$$y = \frac{12}{5}x + c$$

$$\text{put in } (3, 17): c = \frac{49}{5}$$

$$y = \frac{12}{5}x + \frac{49}{5}$$

/ 1

- 14 Find the gradient of a line that is perpendicular to the line $5x - 4y = 6$

$$\text{make } y \text{ the subject: } y = \frac{5}{4}x - \frac{3}{2}$$

$$\text{gradient is } \frac{5}{4}$$

$$\text{perpendicular gradient} = -\frac{4}{5}$$

$$-\frac{4}{5}$$

/ 1

- 15 The line with equation $y = 6x + 7$ is perpendicular to the line with equation $y = kx - 3$

Find the value of k.

$$\text{gradient of the first line is } 6$$

$$\text{so } k = -\frac{1}{6}$$

$$-\frac{1}{6}$$

/ 1

- 16 A straight line, L, passes through the point with coordinates (8, -4) and is perpendicular to the line with equation $y = \frac{4}{3}x - 7$

Find an equation of the straight line L.

gradient of given line = $\frac{4}{3}$

perpendicular gradient = $-\frac{3}{4}$

$y = -\frac{3}{4}x + c$

put in (8, -4): $c = 2$

$y = -\frac{3}{4}x + 2$

/ 1

- 17 Find the gradient of a line that is perpendicular to the line $2x - 3y = 0$

make y the subject: $y = \frac{2}{3}x$

gradient is $\frac{2}{3}$

perpendicular gradient = $-\frac{3}{2}$

$-\frac{3}{2}$

/ 1

- 18 The line with equation $y = 4x - 8$ is perpendicular to the line with equation $y = kx + 7$
Find the value of k.

gradient of the first line is 4

so $k = -\frac{1}{4}$

$-\frac{1}{4}$

/ 1

- 19 Find the equation of the line that is perpendicular to $y = -2x + 2$ and passes through the point $(-4, -6)$.

gradient of given line = -2

perpendicular gradient = $\frac{1}{2}$

$$y = \frac{1}{2}x + c$$

put in $(-4, -6)$: $c = -4$

$$y = \frac{1}{2}x - 4$$

1 / 1

- 20 A is the point $(3, -2)$

B is the point $(9, 1)$

The equation of the straight line through A and B is $y = \frac{1}{2}x - \frac{7}{2}$

(a) Write down the gradient of any line parallel to AB.

(b) Find the equation of the line perpendicular to AB that passes through B.

(a) parallel lines have the same gradient: $\frac{1}{2}$

(b) perpendicular gradient = -2

$$y = -2x + c$$

put in $(9, 1)$: $c = 19$

$$(a) \frac{1}{2} \quad (b) y = -2x + 19$$

1 / 1

- 21 Show that the line $5y = 6x - 15$ is perpendicular to the line $6y = -5x - 16$

gradient of line 1 is $\frac{6}{5}$

gradient of line 2 is $-\frac{5}{6}$

$$\frac{6}{5} \times \left(-\frac{5}{6}\right) = -1$$

so perpendicular

Proved

2 / 2

- 22 Find the equation of the line that is perpendicular to $y = -\frac{3}{2}x - 2$ and passes through the point $(-9, 6)$.

$$\text{gradient of given line} = -\frac{3}{2}$$

$$\text{perpendicular gradient} = \frac{2}{3}$$

$$y = \frac{2}{3}x + c$$

$$\text{put in } (-9, 6): c = 12$$

$$y = \frac{2}{3}x + 12$$

2 / 2

- 23 A is the point $(3, -2)$

B is the point $(8, -8)$

The equation of the straight line through A and B is $y = -\frac{6}{5}x + \frac{8}{5}$

(a) Write down the gradient of any line parallel to AB.

(b) Find the equation of the line perpendicular to AB that passes through B.

$$(a) \text{ parallel lines have the same gradient: } -\frac{6}{5}$$

$$(b) \text{ perpendicular gradient} = \frac{5}{6}$$

$$y = \frac{5}{6}x + c$$

$$\text{put in } (8, -8): c = -\frac{44}{3}$$

$$(a) -\frac{6}{5} \quad (b) y = \frac{5}{6}x - \frac{44}{3}$$

2 / 2

- 24 Show that the line $5y = 6x - 2$ is perpendicular to the line $6y = -5x$

$$\text{gradient of line 1 is } \frac{6}{5}$$

$$\text{gradient of line 2 is } -\frac{5}{6}$$

$$\frac{6}{5} \times \left(-\frac{5}{6}\right) = -1$$

so perpendicular

Proved

2 / 2

- 25 Find the equation of the line that is perpendicular to $y = -6x + 3$ and passes through the point $(18, 8)$.

gradient of given line = -6

perpendicular gradient = $\frac{1}{6}$

$$y = \frac{1}{6}x + c$$

put in $(18, 8)$: $c = 5$

$$y = \frac{1}{6}x + 5$$

2 / 2

- 26 The line with equation $ky = 3x - 4$ is perpendicular to the line with equation $y = -3x - 4$. Find the value of k .

gradient of the line $ky = 3x - 4$ is $\frac{3}{k}$

gradient of the other line is -3

perpendicular so $(\frac{3}{k}) \times (-3) = -1$

$$k = 9$$

9

2 / 2

- 27 Show that the line $3y = 2x - 12$ is perpendicular to the line $2y = -3x + 1$

gradient of line 1 is $\frac{2}{3}$

gradient of line 2 is $-\frac{3}{2}$

$$\frac{2}{3} \times (-\frac{3}{2}) = -1$$

so perpendicular

Proved

2 / 2

- 28 A circle C has centre $(-3, -2)$

The point A $(-2, -3)$ lies on the circumference of the circle.

Find the equation of the tangent to the circle at A.

$$\text{gradient of radius} = \frac{-3 - (-2)}{-2 - (-3)} = -1$$

tangent is perpendicular to the radius: gradient
= 1

$$y = x + c$$

$$\text{put in } (-2, -3): c = -1$$

$$y = x - 1$$

2 / 2

- 29 A is the point $(0, -1)$

B is the point $(10, -5)$

The equation of the straight line through A and B is $y = -\frac{2}{5}x - 1$

(a) Write down the equation of another straight line parallel to $y = -\frac{2}{5}x - 1$

(b) Write down the equation of another straight line that passes through the point $(0, -1)$

(c) Find the equation of the line perpendicular to AB passing through B.

(a) any line with gradient $-\frac{2}{5}$, e.g. $y = -\frac{2}{5}x$

(b) any line through $(0, -1)$, e.g. $y = -1$

(c) perpendicular gradient = $\frac{5}{2}$

$$y = \frac{5}{2}x + c$$

$$\text{put in } (10, -5): c = -30$$

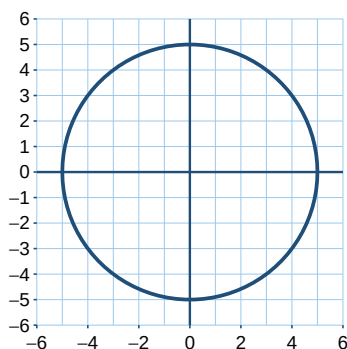
$$(a) \text{ e.g. } y = -\frac{2}{5}x \quad (b) \text{ e.g. } y = -1$$

$$(c) y = \frac{5}{2}x - 30$$

2 / 2

- 30 The diagram shows a circle of radius 5 cm, centre the origin.

Find the equation of the tangent to the circle at $(3, 4)$.



$$\text{gradient of radius} = \frac{4}{3} = \frac{4}{3}$$

$$\text{tangent gradient} = -\frac{3}{4}$$

$$y = -\frac{3}{4}x + c$$

$$\text{put in } (3, 4): c = \frac{25}{4}$$

$$y = -\frac{3}{4}x + \frac{25}{4}$$

2 / 2

- 31 A circle C has centre
- $(-4, 1)$

The point A $(2, -5)$ lies on the circumference of the circle.

Find the equation of the tangent to the circle at A.

$$\text{gradient of radius} = \frac{-5-1}{2-(-4)} = -1$$

tangent is perpendicular to the radius: gradient
= 1

$$y = x + c$$

$$\text{put in } (2, -5): c = -7$$

$$y = x - 7$$

2 / 2

- 32 A is the point
- $(0, -2)$

B is the point $(8, -5)$ The equation of the straight line through A and B is $y = -\frac{3}{8}x - 2$ (a) Write down the equation of another straight line parallel to $y = -\frac{3}{8}x - 2$ (b) Write down the equation of another straight line that passes through the point $(0, -2)$

(c) Find the equation of the line perpendicular to AB passing through B.

(a) any line with gradient $-\frac{3}{8}$, e.g. $y = -\frac{3}{8}x - 1$ (b) any line through $(0, -2)$, e.g. $y = -2$ (c) perpendicular gradient = $\frac{8}{3}$

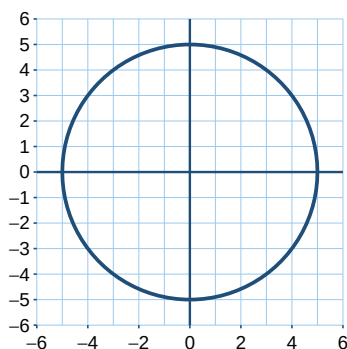
$$y = \frac{8}{3}x + c$$

$$\text{put in } (8, -5): c = -\frac{79}{3}$$

$$(a) \text{ e.g. } y = -\frac{3}{8}x - 1 \quad (b) \text{ e.g. } y = -2 \quad (c) y = \frac{8}{3}x - \frac{79}{3}$$

2 / 2

- 33 The diagram shows a circle of radius 5 cm, centre the origin.

Find the equation of the tangent to the circle at $(4, -3)$.

$$\text{gradient of radius} = -\frac{3}{4} = -\frac{3}{4}$$

$$\text{tangent gradient} = \frac{4}{3}$$

$$y = \frac{4}{3}x + c$$

$$\text{put in } (4, -3): c = -\frac{25}{3}$$

$$y = \frac{4}{3}x - \frac{25}{3}$$

2 / 2

34 A circle C has centre $(4, -1)$

The point A $(6, -5)$ lies on the circumference of the circle.

Find the equation of the tangent to the circle at A.

$$\text{gradient of radius} = \frac{-5 - (-1)}{6 - 4} = -2$$

$$\text{tangent is perpendicular to the radius: gradient} = \frac{1}{2}$$

$$y = \frac{1}{2}x + c$$

$$\text{put in } (6, -5): c = -8$$

$$y = \frac{1}{2}x - 8$$

2 / 2

35 The point A has the coordinates $(-4, 2)$

The point B has the coordinates $(-2, 0)$

(a) Find the mid-point of AB.

(b) Find the gradient of the line that passes through A and B.

(c) Find the equation of the perpendicular bisector of AB.

$$(a) \left(\frac{-4 + -2}{2}, \frac{2 + 0}{2} \right) = (-3, 1)$$

$$(b) \text{ gradient} = \frac{0 - 2}{-2 - (-4)} = -1$$

$$(c) \text{ perpendicular gradient} = 1$$

$$y = x + c$$

$$\text{put in } (-3, 1): c = 4$$

$$(a) (-3, 1) \quad (b) -1 \quad (c) y = x + 4$$

2 / 2

36 A is the point $(-3, 0)$

B is the point $(-2, -1)$

C is the point $(-3, -2)$

Show that angle $ABC = 90^\circ$

$$\text{gradient of AB} = -1$$

$$\text{gradient of BC} = 1$$

$$-1 \times 1 = -1$$

so AB is perpendicular to BC

$$\text{angle ABC} = 90^\circ$$

Proved

2 / 2

- 37 A circle C has centre $(-4, 3)$
 The point A $(-6, 12)$ lies on the circumference of the circle.
 Find the equation of the tangent to the circle at A.

$$\text{gradient of radius} = \frac{12-3}{-6-(-4)} = -\frac{9}{2}$$

$$\text{tangent is perpendicular to the radius: gradient} = \frac{2}{9}$$

$$y = \frac{2}{9}x + c$$

$$\text{put in } (-6, 12): c = \frac{40}{3}$$

$$y = \frac{2}{9}x + \frac{40}{3}$$

2

 / 2

- 38 The point A has the coordinates $(0, -3)$
 The point B has the coordinates $(4, 1)$
 (a) Find the mid-point of AB.
 (b) Find the gradient of the line that passes through A and B.
 (c) Find the equation of the perpendicular bisector of AB.

$$(a) \left(\frac{0+4}{2}, \frac{-3+1}{2}\right) = (2, -1)$$

$$(b) \text{ gradient} = \frac{1-(-3)}{4-0} = 1$$

$$(c) \text{ perpendicular gradient} = -1$$

$$y = -x + c$$

$$\text{put in } (2, -1): c = 1$$

$$(a) (2, -1) \quad (b) 1 \quad (c) y = -x + 1$$

2

 / 2

- 39 A is the point $(-6, 3)$
 B is the point $(-2, 1)$
 C is the point $(0, 5)$
 Show that angle $ABC = 90^\circ$

$$\text{gradient of AB} = -\frac{1}{2}$$

$$\text{gradient of BC} = 2$$

$$-\frac{1}{2} \times 2 = -1$$

$$\text{so AB is perpendicular to BC}$$

$$\text{angle } ABC = 90^\circ$$

 Proved

3

 / 3

- 40 A circle C has centre $(4, -3)$
 The point A $(-5, -1)$ lies on the circumference of the circle.
 Find the equation of the tangent to the circle at A.

$$\text{gradient of radius} = \frac{-1 - (-3)}{-5 - 4} = -\frac{2}{9}$$

$$\text{tangent is perpendicular to the radius: gradient} = \frac{9}{2}$$

$$y = \frac{9}{2}x + c$$

$$\text{put in } (-5, -1): c = \frac{43}{2}$$

$$y = \frac{9}{2}x + \frac{43}{2}$$

3 / 3

- 41 The point A has the coordinates $(2, 1)$
 The point B has the coordinates $(3, 0)$
 (a) Find the mid-point of AB.
 (b) Find the gradient of the line that passes through A and B.
 (c) Find the equation of the perpendicular bisector of AB.

$$(a) \left(\frac{2+3}{2}, \frac{1+0}{2}\right) = (2.5, 0.5)$$

$$(b) \text{gradient} = \frac{0-1}{3-2} = -1$$

$$(c) \text{perpendicular gradient} = 1$$

$$y = x + c$$

$$\text{put in } (2.5, 0.5): c = -2$$

$$(a) (2.5, 0.5) \quad (b) -1 \quad (c) y = x - 2$$

3 / 3

- 42 A circle has the equation $x^2 + y^2 = 50$
 (a) Write down the coordinates of the centre of the circle.
 (b) Write down the exact length of the radius of the circle.
 P is the point $(7, -1)$ on the circle.
 (c) Work out the equation of the tangent to the circle at P.

$$(b) \text{radius} = \sqrt{50} = 5\sqrt{2}$$

$$(c) \text{gradient of OP} = -\frac{1}{7} = -\frac{1}{7}$$

$$\text{tangent gradient} = 7$$

$$y = 7x + c$$

$$\text{put in } (7, -1): c = -50$$

$$(a) (0, 0) \quad (b) 5\sqrt{2} \quad (c) y = 7x - 50$$

3 / 3

- 43 The line L1 has equation $y = \frac{5}{2}x + 2$

The line L2 is perpendicular to L1 and passes through the point (10, -2).

Find the coordinates of the point where L1 and L2 intersect.

$$\text{gradient of L2} = -\frac{2}{5}$$

$$y = -\frac{2}{5}x + c$$

$$\text{put in } (10, -2): c = 2$$

$$\text{L2: } y = -\frac{2}{5}x + 2$$

$$\frac{5}{2}x + 2 = -\frac{2}{5}x + 2$$

solve for x

$$x = 0$$

$$y = 2$$

$$(0, 2)$$

3 / 3

- 44 A circle has the equation $x^2 + y^2 = 17$

(a) Write down the coordinates of the centre of the circle.

(b) Write down the exact length of the radius of the circle.

P is the point (4, -1) on the circle.

(c) Work out the equation of the tangent to the circle at P.

$$(b) \text{ radius} = \sqrt{17} = \sqrt{17}$$

$$(c) \text{ gradient of OP} = -\frac{1}{4} = -\frac{1}{4}$$

$$\text{tangent gradient} = 4$$

$$y = 4x + c$$

$$\text{put in } (4, -1): c = -17$$

$$(a) (0, 0) (b) \sqrt{17} (c) y = 4x - 17$$

3 / 3

- 45 A circle has the equation $x^2 + y^2 = 85$

(a) Write down the coordinates of the centre of the circle.

(b) Write down the exact length of the radius of the circle.

P is the point (7, 6) on the circle.

(c) Work out the equation of the tangent to the circle at P.

$$(b) \text{ radius} = \sqrt{85} = \sqrt{85}$$

$$(c) \text{ gradient of OP} = \frac{6}{7} = \frac{6}{7}$$

$$\text{tangent gradient} = -\frac{7}{6}$$

$$y = -\frac{7}{6}x + c$$

$$\text{put in } (7, 6): c = \frac{85}{6}$$

$$(a) (0, 0) (b) \sqrt{85} (c) y = -\frac{7}{6}x + \frac{85}{6}$$

3 / 3

- 46 The line L1 has equation $y = -5x + 21$

The line L2 is perpendicular to L1 and passes through the point $(-11, -2)$.

Find the coordinates of the point where L1 and L2 intersect.

$$\text{gradient of L2} = \frac{1}{5}$$

$$y = \frac{1}{5}x + c$$

$$\text{put in } (-11, -2): c = \frac{1}{5}$$

$$\text{L2: } y = \frac{1}{5}x + \frac{1}{5}$$

$$-5x + 21 = \frac{1}{5}x + \frac{1}{5}$$

solve for x

$$x = 4$$

$$y = 1$$

$(4, 1)$

3 / 3

- 47 The line L1 has equation $y = 3x + 9$

The line L2 is perpendicular to L1 and passes through the point $(0, -1)$.

Find the coordinates of the point where L1 and L2 intersect.

$$\text{gradient of L2} = -\frac{1}{3}$$

$$y = -\frac{1}{3}x + c$$

$$\text{put in } (0, -1): c = -1$$

$$\text{L2: } y = -\frac{1}{3}x - 1$$

$$3x + 9 = -\frac{1}{3}x - 1$$

solve for x

$$x = -3$$

$$y = 0$$

$(-3, 0)$

3 / 3

- 48 The line L1 has equation $y = -2x + 5$

The line L2 is perpendicular to L1 and passes through the point $(8, -1)$.

Find the coordinates of the point where L1 and L2 intersect.

$$\text{gradient of L2} = \frac{1}{2}$$

$$y = \frac{1}{2}x + c$$

$$\text{put in } (8, -1): c = -5$$

$$\text{L2: } y = \frac{1}{2}x - 5$$

$$-2x + 5 = \frac{1}{2}x - 5$$

solve for x

$$x = 4$$

$$y = -3$$

$(4, -3)$

3 / 3

49 The line L1 has equation $y = 4x + 7$

The line L2 is perpendicular to L1 and passes through the point $(-13, 6)$.

Find the coordinates of the point where L1 and L2 intersect.

$$\text{gradient of L2} = -\frac{1}{4}$$

$$y = -\frac{1}{4}x + c$$

$$\text{put in } (-13, 6): c = \frac{11}{4}$$

$$\text{L2: } y = -\frac{1}{4}x + \frac{11}{4}$$

$$4x + 7 = -\frac{1}{4}x + \frac{11}{4}$$

solve for x

$$x = -1$$

$$y = 3$$

$(-1, 3)$

3 / 3

50 The line L1 has equation $y = -4x + 5$

The line L2 is perpendicular to L1 and passes through the point $(-4, 4)$.

Find the coordinates of the point where L1 and L2 intersect.

$$\text{gradient of L2} = \frac{1}{4}$$

$$y = \frac{1}{4}x + c$$

$$\text{put in } (-4, 4): c = 5$$

$$\text{L2: } y = \frac{1}{4}x + 5$$

$$-4x + 5 = \frac{1}{4}x + 5$$

solve for x

$$x = 0$$

$$y = 5$$

$(0, 5)$

3 / 3