

GCSE MATHS • GRADE 6

MATHS



MODEL ANSWERS

CIRCLE THEOREMS

SCORE

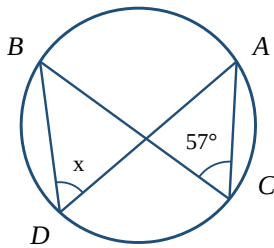
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PERCENTAGE

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STEM UP

- 1 A, B, C and D are points on the circumference of a circle. Angle $ACB = 57^\circ$.
Work out the size of angle ADB . Give a reason for your answer.

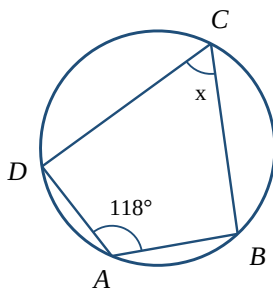


$$\text{angle } ADB = 57$$

57° (angles in the same segment are equal)

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- 2 A, B, C and D are points on the circumference of a circle. Angle $DAB = 118^\circ$.
Work out the size of angle BCD . Give a reason for your answer.

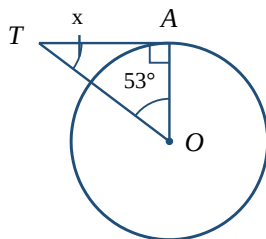


$$180 - 118 = 62$$

62° (opposite angles of a cyclic quadrilateral add up to 180°)

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- 3 TA is a tangent to the circle, centre O, at the point A. Angle $AOT = 53^\circ$.
Work out the size of angle ATO . Give reasons for your answer.

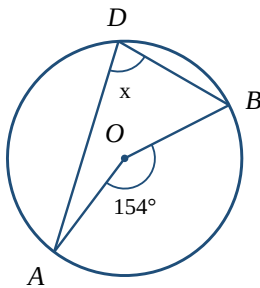


$$\begin{aligned} \text{Angle } OAT &= 90^\circ \\ 180 - 90 - 53 &= 37 \end{aligned}$$

37° (the angle between a tangent and a radius is 90° ; angles in a triangle add up to 180°)

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- 4 A, B and D are points on the circumference of a circle, centre O. Angle AOB = 154° .
Work out the size of angle ADB. Give a reason for your answer.

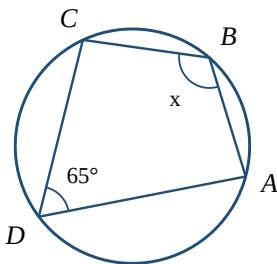


$$154 \div 2 = 77$$

77° (angle at the centre is twice the angle at the circumference)

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- 5 A, B, C and D are points on the circumference of a circle. Angle CDA = 65° .
Work out the size of angle ABC. Give a reason for your answer.

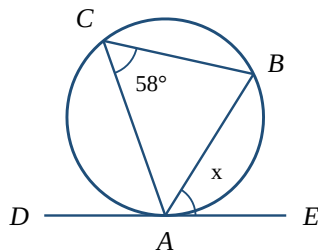


$$180 - 65 = 115$$

115° (opposite angles of a cyclic quadrilateral add up to 180°)

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- 6 DE is a tangent to the circle at A. B and C are points on the circumference. Angle ACB = 58° .
Work out the size of angle EAB. Give a reason for your answer.

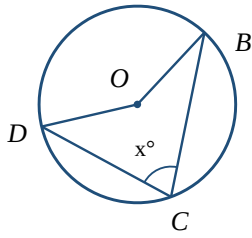


Alternate segment theorem: angle EAB = angle ACB = 58

58° (angle between tangent and chord equals the angle in the alternate segment)

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- 7 B, C and D are points on the circumference of a circle, centre O. Angle $BCD = x^\circ$. Find the size of angle BOD, in terms of x. Give a reason for your answer.

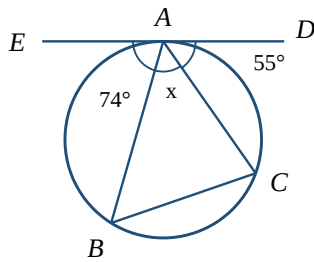


Angle $BOD = 2 \times \text{angle } BCD = 2x^\circ$ (angle at the centre is twice the angle at the circumference)

$2x^\circ$ (angle at the centre is twice the angle at the circumference)

/ 1

- 8 DE is a tangent to the circle at A. B and C are points on the circumference. Angle $EAB = 74^\circ$. Angle $DAC = 55^\circ$. Work out the size of angle BAC. Give a reason for your answer.

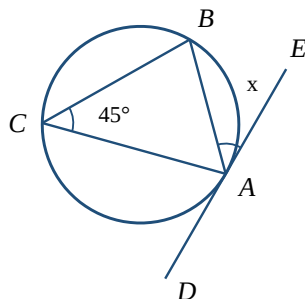


$$180 - 74 - 55 = 51$$

51° (angles on a straight line add up to 180°)

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- 9 DE is a tangent to the circle at A. B and C are points on the circumference. Angle $ACB = 45^\circ$. Work out the size of angle EAB. Give a reason for your answer.

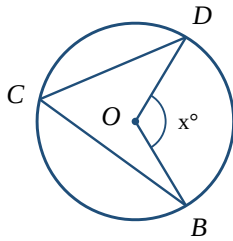


Alternate segment theorem: angle $EAB = \text{angle } ACB = 45$

45° (angle between tangent and chord equals the angle in the alternate segment)

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- 10** B, C and D are points on the circumference of a circle, centre O. Angle BOD = x° . Find the size of angle BCD, in terms of x. Give a reason for your answer.

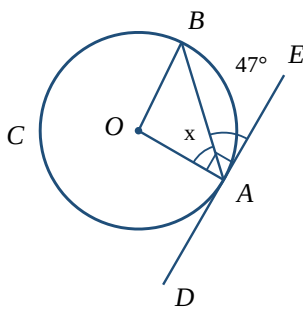


Angle BCD = angle BOD $\div$ 2 = $\frac{x}{2}^\circ$ (angle at the centre is twice the angle at the circumference)

$\frac{x}{2}^\circ$ (angle at the centre is twice the angle at the circumference)

1 / 1

- 11** DE is a tangent to the circle, centre O, at A. B is a point on the circle. Angle EAB = 47° . Work out the size of angle OAB. Give a reason for your answer.

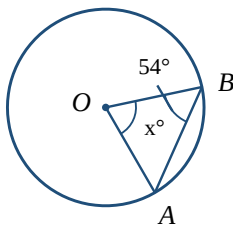


*Angle OAE = 90°
angle OAB = $90 - 47 = 43$*

43° (the angle between a tangent and a radius is 90°)

1 / 1

- 12** A and B are points on the circumference of a circle, centre O. Angle ABO = 54° . Find the size of angle AOB. Give a reason for your answer.

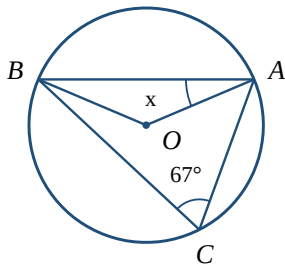


*Angle OAB = angle ABO = 54°
angle AOB = $180 - 2 \times 54 = 72$*

72° (OA = OB (radii) so the triangle is isosceles)

1 / 1

- 13 A, B and C are points on the circumference of a circle, centre O. Angle ACB = 67° . Work out the size of angle OAB. Give reasons for your answer.

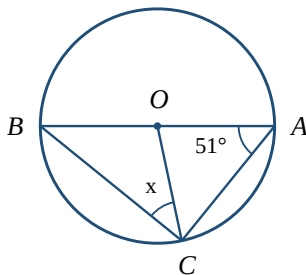


$$\begin{aligned}\text{Angle } AOB &= 2 \times 67 = 134 \\ \text{Angle } OAB &= (180 - 134) \div 2 \\ &= 23\end{aligned}$$

23° (angle at centre = $2 \times$ angle at circumference; triangle OAB is isosceles)

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- 14 AB is a diameter of a circle, centre O. C is a point on the circumference. Angle CAB = 51° . Work out the size of angle OCB. Give reasons for your answer.

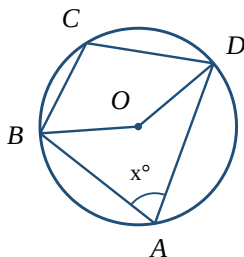


$$\begin{aligned}\text{Angle } ABC &= 180 - 90 - 51 = 39 \\ OB = OC \text{ so angle } OCB &= \text{angle } OBC = 39\end{aligned}$$

39° (the angle in a semicircle is 90° , so angle ABC = 39° ; triangle OBC is isosceles ($OB = OC$))

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- 15 A, B, C and D are points on the circumference of a circle, centre O. Angle BAD = x° . Find the size of angle BCD, in terms of x. Give a reason for your answer.

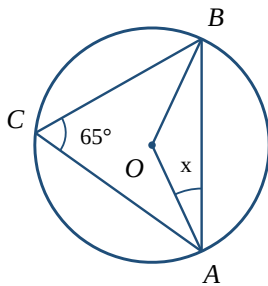


$$\text{Angle } BCD = 180 - x$$

$(180 - x)^\circ$ (opposite angles of a cyclic quadrilateral add up to 180°)

/ 1

- 16 A, B and C are points on the circumference of a circle, centre O. Angle ACB = 65° .
Work out the size of angle OAB. Give reasons for your answer.

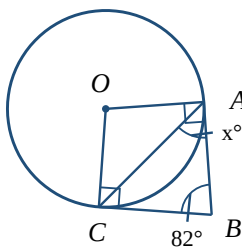


$$\begin{aligned}\text{Angle } AOB &= 2 \times 65 = 130 \\ \text{Angle } OAB &= (180 - 130) \div 2 \\ &= 25\end{aligned}$$

25° (angle at centre = $2 \times$ angle at circumference; triangle OAB is isosceles)

1 / 1

- 17 A and C are points on the circumference of a circle, centre O. AB and BC are tangents to the circle. Angle ABC = 82° .
Find the size of angle BAC. Give reasons for each stage of your working.

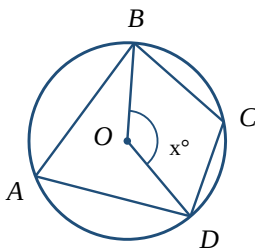


$$\begin{aligned}AB &= BC \text{ (tangents from a point are equal) so triangle } ABC \text{ is isosceles} \\ \text{angle } BAC &= (180 - 82) \div 2 = 49^\circ\end{aligned}$$

49°

1 / 1

- 18 A, B, C and D are points on the circumference of a circle, centre O. Angle BOD = x° .
Find the size of angle BAD, in terms of x. Give a reason for your answer.

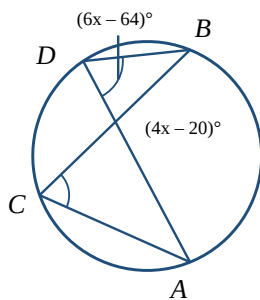


$$\text{Angle } BAD = x \div 2 = \frac{x}{2}$$

$\frac{x}{2}^\circ$ (angle at the centre is twice the angle at the circumference)

1 / 1

- 19 A, B, C and D are points on the circumference of a circle. Angle $ACB = (4x - 20)^\circ$. Angle $ADB = (6x - 64)^\circ$.
Work out the value of x .

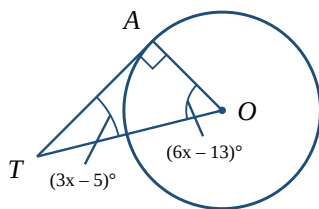


Angles in the same segment are equal
 $4x - 20 = 6x - 64$

$$x = 22$$

1 / 1

- 20 TA is a tangent to the circle, centre O, at the point A. Angle $AOT = (6x - 13)^\circ$. Angle $ATO = (3x - 5)^\circ$.
Work out the value of x .

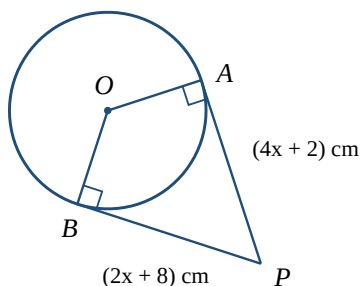


Angle $OAT = 90^\circ$
 $6x - 13 + 3x - 5 = 90$
 $9x - 18 = 90$

$$x = 12$$

1 / 1

- 21 PA and PB are tangents to a circle, centre O. $PA = (4x + 2)$ cm and $PB = (2x + 8)$ cm.
Work out the value of x and the length of PA. Give a reason for your answer.

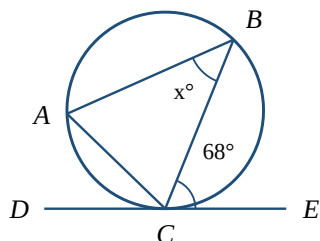


$PA = PB$
 $4x + 2 = 2x + 8$
 $2x = 6$
 $PA = 4 \times 3 + 2 = 14$

$x = 3$, $PA = 14$ cm (tangents from an external point are equal in length)

2 / 2

- 22 A, B and C are points on the circumference of a circle. DCE is a tangent to the circle. $AB = BC$. Angle $BCE = 68^\circ$.
Find the size of angle ABC. Give reasons for each stage of your working.

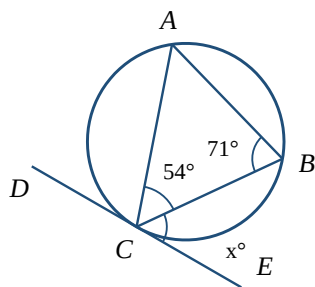


Angle $BAC = 68^\circ$ (alternate segment theorem)
 angle $BCA = 68^\circ$ ($AB = BC$, isosceles triangle)
 angle $ABC = 180 - 68 - 68 = 44^\circ$ (angles in a triangle add up to 180°)

44°

2 / 2

- 23 A, B and C are points on the circumference of a circle. DCE is a tangent to the circle. Angle $ABC = 71^\circ$. Angle $ACB = 54^\circ$. Angle $BCE = x^\circ$.
Find the value of x. Give reasons for each stage of your working.

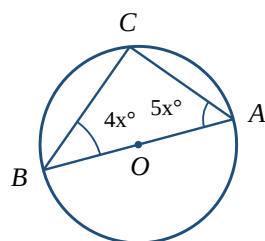


Angle $BAC = 180 - 71 - 54 = 55^\circ$ (angles in a triangle add up to 180°)
 $x = 55$ (angle between a tangent and a chord is equal to the angle in the alternate segment)

x = 55

2 / 2

- 24 A, B and C are points on the circumference of a circle, centre O. AB is a diameter. Angle $CAB = 5x^\circ$. Angle $ABC = 4x^\circ$.
Find the value of x. You must show all your working.



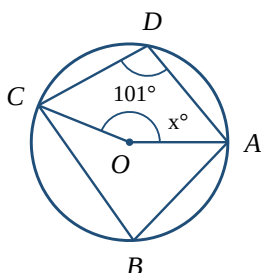
Angle $ACB = 90^\circ$ (angle in a semicircle)
 $5x + 4x + 90 = 180$
 $9x = 90$

x = 10

2 / 2

- 25 A, B, C and D are points on the circumference of a circle, centre O. Angle $ADC = 101^\circ$. Angle $AOC = x^\circ$.

Work out the value of x. You must show all your working.



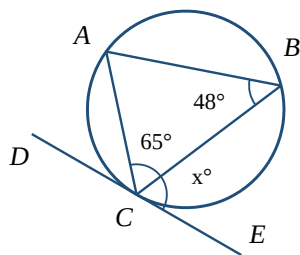
*Angle $ABC = 180 - 101 = 79$ (opposite angles of a cyclic quadrilateral)
 $x = 2 \times 79 = 158$ (angle at the centre is twice the angle at the circumference)*

$$x = 158$$

2 / 2

- 26 A, B and C are points on the circumference of a circle. DCE is a tangent to the circle. Angle $ABC = 48^\circ$. Angle $ACB = 65^\circ$. Angle $BCE = x^\circ$.

Find the value of x. Give reasons for each stage of your working.



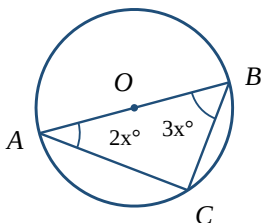
*Angle $BAC = 180 - 48 - 65 = 67^\circ$ (angles in a triangle add up to 180°)
 $x = 67$ (angle between a tangent and a chord is equal to the angle in the alternate segment)*

$$x = 67$$

2 / 2

- 27 A, B and C are points on the circumference of a circle, centre O. AB is a diameter. Angle $CAB = 2x^\circ$. Angle $ABC = 3x^\circ$.

Find the value of x. You must show all your working.

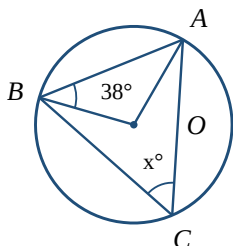


*Angle $ACB = 90^\circ$ (angle in a semicircle)
 $2x + 3x + 90 = 180$
 $5x = 90$*

$$x = 18$$

2 / 2

- 28 A and B are points on the circumference of a circle, centre O. C is a point on the circumference. Angle ABO = 38° .
Find the size of angle ACB. Give reasons for each stage of your working.

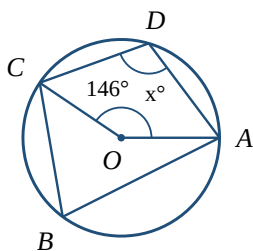


Angle OAB = 38° (OA = OB, isosceles triangle)
 angle AOB = $180 - 2 \times 38 = 104^\circ$
 angle ACB = $104 \div 2 = 52^\circ$

52° (angle at the centre is twice the angle at the circumference)

2 / 2

- 29 A, B, C and D are points on the circumference of a circle, centre O. Angle AOC = 146° . Angle ADC = x° .
Work out the value of x. Give reasons for each stage of your working.

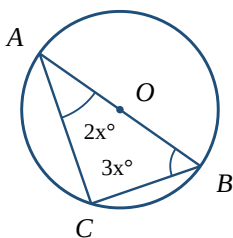


Angle ABC = $146 \div 2 = 73$ (angle at the centre is twice the angle at the circumference)
 $x = 180 - 73 = 107$ (opposite angles of a cyclic quadrilateral)

$x = 107$

2 / 2

- 30 A, B and C are points on the circumference of a circle, centre O. AB is a diameter. Angle CAB = $2x^\circ$. Angle ABC = $3x^\circ$.
Find the value of x. You must show all your working.

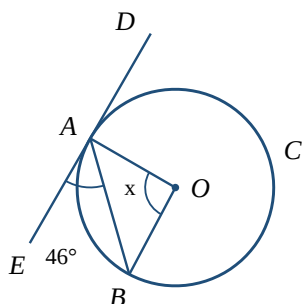


Angle ACB = 90° (angle in a semicircle)
 $2x + 3x + 90 = 180$
 $5x = 90$

$x = 18$

2 / 2

- 31 DE is a tangent to the circle, centre O, at A. B is a point on the circle. Angle EAB = 46° . Work out the size of angle AOB. Give reasons for your answer.

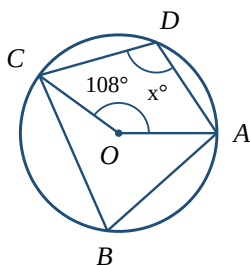


$$\begin{aligned}\text{Angle } OAB &= 90 - 46 = 44 \\ \text{angle } OBA &= 44 \\ \text{angle } AOB &= 180 - 2 \times 44 = 92\end{aligned}$$

92° (the angle between a tangent and a radius is 90° , so angle $OAB = 44^\circ$; triangle OAB is isosceles)

2 / 2

- 32 A, B, C and D are points on the circumference of a circle, centre O. Angle ADC = 108° . Angle AOC = x° . Work out the value of x. You must show all your working.

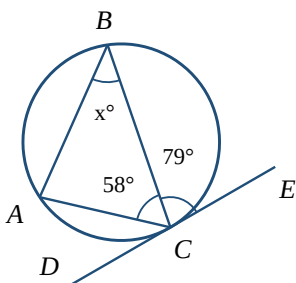


$$\begin{aligned}\text{Angle } ABC &= 180 - 108 = 72 \text{ (opposite angles of a cyclic quadrilateral)} \\ x &= 2 \times 72 = 144 \text{ (angle at the centre is twice the angle at the circumference)}\end{aligned}$$

$$x = 144$$

2 / 2

- 33 A, B and C are points on the circumference of a circle. DCE is a tangent to the circle. Angle BCE = 79° . Angle ACB = 58° . Find the size of angle ABC. Give reasons for each stage of your working.

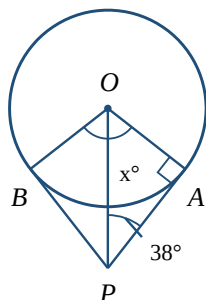


$$\begin{aligned}\text{Angle } BAC &= 79^\circ \text{ (angle between a tangent and a chord is equal to the angle in the alternate segment)} \\ \text{angle } ABC &= 180 - 79 - 58 = 43^\circ \text{ (angles in a triangle add up to } 180^\circ)\end{aligned}$$

$$43^\circ$$

2 / 2

- 34 PA and PB are tangents to a circle, centre O. Angle OPA = 38° .
Find the size of angle AOB. Give reasons for each stage of your working.

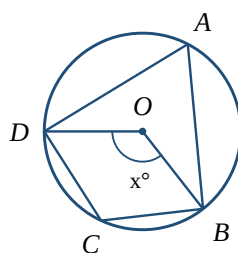


Angle OAP = 90° (tangent and radius)
 angle AOP = $180 - 90 - 38 = 52^\circ$
 angle AOB = $2 \times 52 = 104^\circ$ (triangles OAP and OBP are congruent)

104°

2 / 2

- 35 A, B, C and D are points on the circumference of a circle, centre O. Angle BOD = x° .
Find the size of angle BCD, in terms of x. Give reasons for each stage of your working.

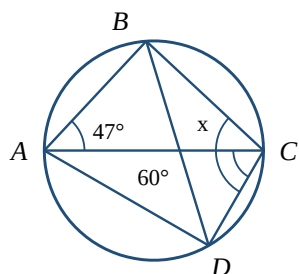


Angle BAD = $\frac{x}{2}^\circ$ (angle at the centre is twice the angle at the circumference)
 angle BCD = $(180 - \frac{x}{2})^\circ$ (opposite angles of a cyclic quadrilateral add up to 180°)

$(180 - \frac{x}{2})^\circ$

2 / 2

- 36 A, B, C and D are points on a circle. AC is a diameter. Angle CAB = 47° . Angle ACD = 60° .
Work out the size of angle BCD. Give reasons for your answer.

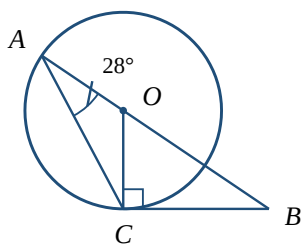


Angle ABC = 90°
 angle ACB = $180 - 90 - 47 = 43$
 angle BCD = $43 + 60 = 103$

103° (the angle in a semicircle is 90° , so angle ACB = 43° ; angles in a triangle add up to 180°)

2 / 2

- 37 A and C are points on the circumference of a circle, centre O. BC is a tangent to the circle. AOB is a straight line. Angle CAB = 28° . Find the size of angle ABC. You must show all your working.

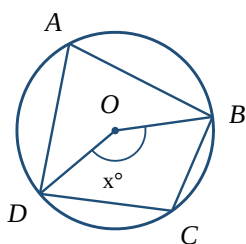


Angle COB = $2 \times 28 = 56^\circ$ (angle at the centre is twice the angle at the circumference)
 angle OCB = 90° (tangent and radius)
 angle ABC = $180 - 90 - 56 = 34^\circ$

34°

2 / 2

- 38 A, B, C and D are points on the circumference of a circle, centre O. Angle BOD = x° . Find the size of angle BCD, in terms of x. Give reasons for each stage of your working.

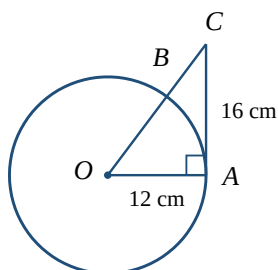


Angle BAD = $\frac{x}{2}^\circ$ (angle at the centre is twice the angle at the circumference)
 angle BCD = $(180 - \frac{x}{2})^\circ$ (opposite angles of a cyclic quadrilateral add up to 180°)

$(180 - \frac{x}{2})^\circ$

2 / 2

- 39 A and B are points on the circumference of a circle, centre O. AC is a tangent to the circle. OBC is a straight line. OA = 12 cm. AC = 16 cm. Find the length of BC. You must show all your working.

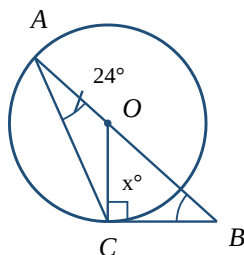


Angle OAC = 90° (tangent and radius)
 $OC^2 = 12^2 + 16^2 = 400$
 $OC = 20$
 $BC = 20 - 12 = 8$

BC = 8 cm

3 / 3

- 40 A and C are points on the circumference of a circle, centre O. BC is a tangent to the circle. AOB is a straight line. Angle CAB = 24° .
Find the size of angle ABC. You must show all your working.

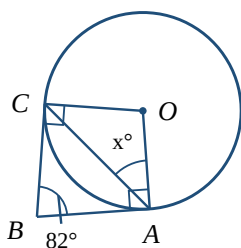


Angle COB = $2 \times 24 = 48^\circ$ (angle at the centre is twice the angle at the circumference)
angle OCB = 90° (tangent and radius)
angle ABC = $180 - 90 - 48 = 42^\circ$

42°

3 / 3

- 41 A and C are points on the circumference of a circle, centre O. AB and BC are tangents to the circle. Angle ABC = 82° .
Find the size of angle OAC. Give reasons for each stage of your working.

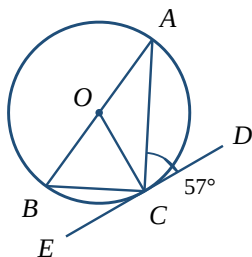


Angle OAB = angle OCB = 90° (tangent and radius)
angle AOC = $360 - 90 - 90 - 82 = 98^\circ$ (angles in a quadrilateral add up to 360°)
angle OAC = $(180 - 98) \div 2 = 41^\circ$ (OA = OC, isosceles triangle)

41°

3 / 3

- 42 A, B and C are points on the circumference of a circle, centre O. AOB is a diameter and DCE is a tangent to the circle. Angle ACD = 57° .
(a) Find the size of angle ACO. (b) Find the size of angle ABC.
Give reasons for each stage of your working.

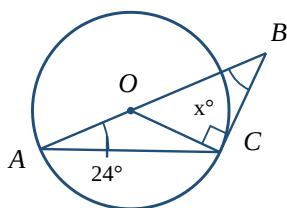


(a) Angle OCD = 90° (tangent and radius)
angle ACO = $90 - 57 = 33^\circ$
(b) angle ABC = 57° (angle between a tangent and a chord is equal to the angle in the alternate segment)

(a) 33° (b) 57°

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- 43 A and C are points on the circumference of a circle, centre O. BC is a tangent to the circle. AOB is a straight line. Angle CAB = 24° . Find the size of angle ABC. You must show all your working.

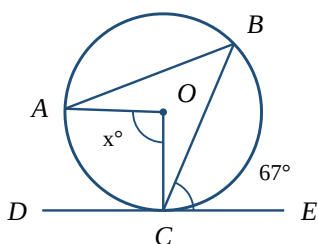


Angle COB = $2 \times 24 = 48^\circ$ (angle at the centre is twice the angle at the circumference)
 angle OCB = 90° (tangent and radius)
 angle ABC = $180 - 90 - 48 = 42^\circ$

42°

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- 44 A, B and C are points on the circumference of a circle, centre O. DCE is a tangent to the circle. AB = BC. Angle BCE = 67° . Find the size of angle AOC. You must show all your working.

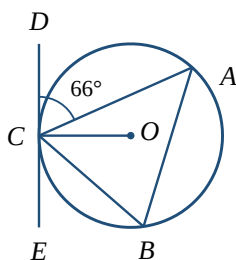


Angle BAC = 67° (alternate segment theorem)
 angle BCA = 67° (AB = BC, isosceles triangle)
 angle ABC = $180 - 2 \times 67 = 46^\circ$
 angle AOC = $2 \times 46 = 92^\circ$ (angle at the centre is twice the angle at the circumference)

92°

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- 45 A, B and C are points on the circumference of a circle, centre O. DCE is a tangent to the circle. Angle ACD = 66° .
 (a) Find the size of angle ACO. (b) Find the size of angle ABC.
 Give reasons for each stage of your working.

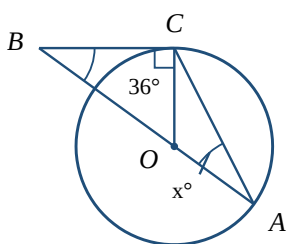


(a) Angle OCD = 90° (tangent and radius)
 angle ACO = $90 - 66 = 24^\circ$
 (b) angle ABC = 66° (angle between a tangent and a chord is equal to the angle in the alternate segment)

(a) 24° (b) 66°

3 / 3

- 46 A and C are points on the circumference of a circle, centre O. BC is a tangent to the circle. AOB is a straight line. Angle ABC = 36° . Find the size of angle CAB. You must show all your working.

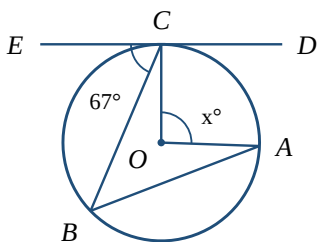


Angle $OCB = 90^\circ$ (tangent and radius)
 angle $COB = 180 - 90 - 36 = 54^\circ$
 angle $CAB = 54 \div 2 = 27^\circ$ (angle at the centre is twice the angle at the circumference)

27°

3 / 3

- 47 A, B and C are points on the circumference of a circle, centre O. DCE is a tangent to the circle. AB = BC. Angle BCE = 67° . Find the size of angle AOC. You must show all your working.

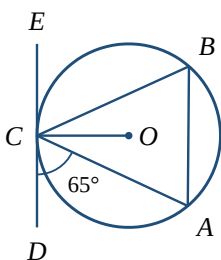


Angle $BAC = 67^\circ$ (alternate segment theorem)
 angle $BCA = 67^\circ$ (AB = BC, isosceles triangle)
 angle $ABC = 180 - 2 \times 67 = 46^\circ$
 angle $AOC = 2 \times 46 = 92^\circ$ (angle at the centre is twice the angle at the circumference)

92°

3 / 3

- 48 A, B and C are points on the circumference of a circle, centre O. DCE is a tangent to the circle. Angle ACD = 65° .
 (a) Find the size of angle ACO. (b) Find the size of angle ABC.
 Give reasons for each stage of your working.

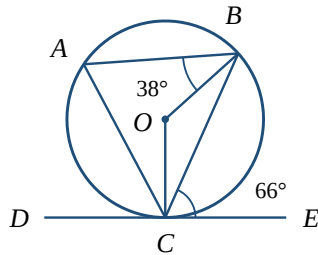


(a) Angle $OCD = 90^\circ$ (tangent and radius)
 angle $ACO = 90 - 65 = 25^\circ$
 (b) angle $ABC = 65^\circ$ (angle between a tangent and a chord is equal to the angle in the alternate segment)

(a) 25° (b) 65°

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- 49 A, B and C are points on the circumference of a circle, centre O. DCE is a tangent to the circle. Angle ABO = 38° . Angle BCE = 66° . Find the size of angle ACO. Give reasons for each stage of your working.

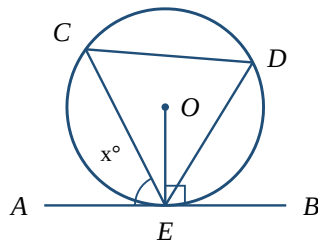


Angle OAB = 38° (OA = OB, isosceles triangle)
 angle BAC = 66° (alternate segment theorem)
 angle OAC = $66 - 38 = 28^\circ$
 angle ACO = 28° (OA = OC, isosceles triangle)

28°

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- 50 C, D and E are points on a circle, centre O. AEB is a tangent to the circle at E. CD = DE. Angle AEC = x° . Find the size of angle OED, in terms of x. Give reasons for each stage of your working.



Angle CDE = x° (alternate segment theorem)
 angle DEC = $(180 - x) \div 2 = (90 - \frac{x}{2})^\circ$ (CD = DE, isosceles triangle)
 angle OEC = $(90 - x)^\circ$ (tangent and radius)
 angle OED = $(90 - \frac{x}{2}) - (90 - x) = \frac{x}{2}$

$\frac{x}{2}^\circ$

3 / 3