

MATHS



MODEL ANSWERS

SEQUENCES HIGHER

SCORE

/92

PERCENTAGE

%

STEM UP

- 1 An arithmetic series has 32 terms. The first term is 11 and the last term is 290.
Work out the sum of all the terms.

$$\begin{aligned} S &= \frac{n}{2}(a + l) \\ &= \frac{32}{2}(11 + 290) \\ &= 32 \times 301 \div 2 \\ &= 4816 \end{aligned}$$

4816

/ 1

- 2 The first term of an arithmetic series is -2 and the common difference is 7.
Work out the sum of the first 35 terms of the series.

$$\begin{aligned} S(n) &= \frac{n}{2}(2a + (n - 1)d) \\ S(35) &= \frac{35}{2}(2 \times (-2) + 34 \times 7) \\ &= \frac{35}{2} \times 234 \\ &= 4095 \end{aligned}$$

4095

/ 1

- 3 Work out $1 + 2 + 3 + \dots + 100$.

$$\begin{aligned} a &= 1 \\ l &= 100 \\ n &= 100 \\ S &= \frac{100}{2}(1 + 100) \\ &= 5050 \end{aligned}$$

5050

/ 1

- 4 An arithmetic series has 14 terms. The first term is 12 and the last term is 90.
Work out the sum of all the terms.

$$\begin{aligned} S &= \frac{n}{2}(a + l) \\ &= \frac{14}{2}(12 + 90) \\ &= 14 \times 102 \div 2 \\ &= 714 \end{aligned}$$

714

/ 1

- 5 The first term of an arithmetic series is 7 and the common difference is 5.
Work out the sum of the first 35 terms of the series.

$$\begin{aligned} S(n) &= \frac{n}{2}(2a + (n - 1)d) \\ S(35) &= \frac{35}{2}(2 \times 7 + 34 \times 5) \\ &= \frac{35}{2} \times 184 \\ &= 3220 \end{aligned}$$

3220

/ 1

- 6 Here are the first three patterns in a sequence made from matchsticks. Each square uses 4 matchsticks.

- (a) Find an expression, in terms of n , for the number of matchsticks in Pattern n .
(b) Pattern n uses 151 matchsticks. Find the value of n .



Pattern 1



Pattern 2



Pattern 3

- (a) 4, 7, 10: the difference is 3
(b) $3n + 1 = 151$
 $3n = 150$

(a) $3n + 1$ (b) $n = 50$

/ 1

- 7 An arithmetic series has 30 terms. The first term is 3 and the last term is 206.
Work out the sum of all the terms.

$$\begin{aligned}
 S &= \frac{n}{2}(a + l) \\
 &= \frac{30}{2}(3 + 206) \\
 &= 30 \times 209 \div 2 \\
 &= 3135
 \end{aligned}$$

3135

1 / 1

- 8 The first term of an arithmetic series is 4 and the common difference is 5.
Work out the sum of the first 31 terms of the series.

$$\begin{aligned}
 S(n) &= \frac{n}{2}(2a + (n - 1)d) \\
 S(31) &= \frac{31}{2}(2 \times 4 + 30 \times 5) \\
 &= \frac{31}{2} \times 158 \\
 &= 2449
 \end{aligned}$$

2449

1 / 1

- 9 Here are the first three patterns in a sequence made from matchsticks. Each square uses 4 matchsticks.

- (a) Find an expression, in terms of n , for the number of matchsticks in Pattern n .
(b) Pattern n uses 61 matchsticks. Find the value of n .



Pattern 1



Pattern 2



Pattern 3

- (a) 4, 7, 10: the difference is 3
(b) $3n + 1 = 61$
 $3n = 60$

(a) $3n + 1$ (b) $n = 20$

1 / 1

- 10 Work out the sum of the first 20 odd numbers.

$$1 + 3 + 5 + \dots + 39$$

$$a = 1$$

$$d = 2$$

$$S = \frac{20}{2}(2 \times 1 + 19 \times 2)$$

$$= 10 \times 40$$

$$= 400$$

400

1 / 1

- 11 The first term of an arithmetic series is 4 and the common difference is 6.

Work out the sum of the first 31 terms of the series.

$$S(n) = \frac{n}{2}(2a + (n - 1)d)$$

$$S(31) = \frac{31}{2}(2 \times 4 + 30 \times 6)$$

$$= \frac{31}{2} \times 188$$

$$= 2914$$

2914

1 / 1

- 12 Here is an arithmetic series.

$$15 + 23 + 31 + \dots + 263$$

Work out the sum of the series.

$$d = 8$$

$$l = a + (n - 1)d: 263 = 15 + (n - 1) \times 8$$

$$n = 32$$

$$S = \frac{32}{2}(15 + 263)$$

$$= 4448$$

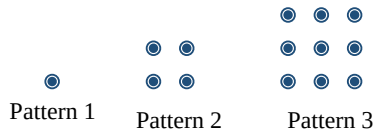
4448

1 / 1

13 Here are the first three patterns in a sequence of dot patterns.

(a) Find an expression, in terms of n , for the number of dots in Pattern n .

(b) Work out the number of dots in Pattern 40.



(a) rows $\times$ columns = $n \times n = n^2$

(b) $40^2 = 1600$

(a) n^2 (b) 1600

1 / 1

14 The first term of an arithmetic series is 6 and the common difference is -3 .

Work out the sum of the first 26 terms of the series.

$$S(n) = \frac{n}{2}(2a + (n-1)d)$$

$$S(26) = \frac{26}{2}(2 \times 6 + 25 \times (-3))$$

$$= \frac{26}{2} \times -63$$

$$= -819$$

-819

1 / 1

15 The n th term of a sequence is $9n + 1$.

Is 271 a term of this sequence?

$$9n + 1 = 271$$

$$9n = 270$$

$$n = 30$$

n is a whole number so 271 is term 30

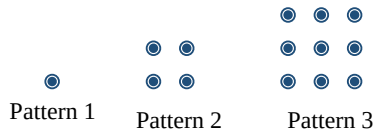
Yes, it is term number 30

1 / 1

16 Here are the first three patterns in a sequence of dot patterns.

(a) Find an expression, in terms of n , for the number of dots in Pattern n .

(b) Work out the number of dots in Pattern 25.



(a) rows $\times$ columns = $n \times n = n^2$

(b) $25^2 = 625$

(a) n^2 (b) 625

1 / 1

17 Here is an arithmetic series.

$$10 + 13 + 16 + \dots + 58$$

Work out the sum of the series.

$$d = 3$$

$$l = a + (n - 1)d: 58 = 10 + (n - 1) \times 3$$

$$n = 17$$

$$S = \frac{17}{2}(10 + 58)$$

$$= 578$$

578

1 / 1

18 The n th term of a sequence is $7n + 7$.

Is 245 a term of this sequence?

$$7n + 7 = 245$$

$$7n = 238$$

$$n = 34$$

n is a whole number so 245 is term 34

Yes, it is term number 34

1 / 1

- 19 Work out the sum of all the multiples of 3 from 3 to 99 inclusive.

multiples: 3, 6, ..., 99

number of terms = $99 \div 3 = 33$

$$S = \frac{33}{2}(3 + 99)$$

$$= 1683$$

1683

1 / 1

- 20 Here is an arithmetic series.

$$10 + 17 + 24 + \dots + 185$$

Work out the sum of the series.

$$d = 7$$

$$l = a + (n - 1)d: 185 = 10 + (n - 1) \times 7$$

$$n = 26$$

$$S = \frac{26}{2}(10 + 185)$$

$$= 2535$$

2535

1 / 1

- 21 The n th term of a sequence is $6n - 4$.

Is 106 a term of this sequence?

You must show your working.

$$6n - 4 = 106$$

$$6n = 110$$

$$n = \frac{110}{6}$$

n is not a whole number so 106 is not a term

No, n is not an integer

2 / 2

- 22 Logs are stacked in rows. The top row has 5 logs. Each row below has 1 more log than the row above. There are 14 rows.

- (a) How many logs are in the bottom row?
 (b) Work out the total number of logs.

$$(a) 5 + 13 \times 1 = 18$$

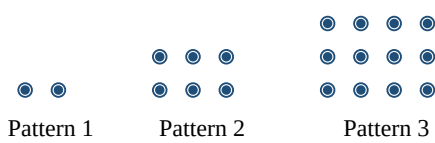
$$(b) S = \frac{14}{2}(5 + 18) = 161$$

(a) 18 (b) 161

2 / 2

- 23 Here are the first three patterns in a sequence of dot patterns.

- (a) Find an expression, in terms of n , for the number of dots in Pattern n .
 (b) Pattern n has 240 dots. Find the value of n .



$$(a) n \text{ rows of } (n + 1) \text{ dots} = n(n + 1) = n^2 + n$$

$$(b) n(n + 1) = 240$$

$$15 \times 16 = 240$$

(a) $n(n + 1)$ (b) $n = 15$

2 / 2

- 24 The sum of the first 8 terms of an arithmetic series is 200.

The common difference is 4.

Find the first term.

$$200 = \frac{8}{2}(2a + 7 \times 4)$$

$$200 = 4(2a + 28)$$

$$2a + 28 = 50$$

$$2a = 22$$

$$a = 11$$

11

2 / 2

- 25 Sam saves money each week. In week 1 Sam saves £5. In each following week Sam saves £5 more than in the week before.

- (a) How much does Sam save in week 52?
 (b) Work out the total Sam has saved after 52 weeks.

$$(a) 5 + 51 \times 5 = 260$$

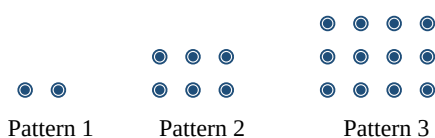
$$(b) S = \frac{52}{2}(5 + 260) = 6890$$

$$(a) \text{ £260 } (b) \text{ £6890}$$

2 / 2

- 26 Here are the first three patterns in a sequence of dot patterns.

- (a) Find an expression, in terms of n , for the number of dots in Pattern n .
 (b) Pattern n has 650 dots. Find the value of n .



$$(a) n \text{ rows of } (n + 1) \text{ dots} = n(n + 1) = n^2 + n$$

$$(b) n(n + 1) = 650$$

$$25 \times 26 = 650$$

$$(a) n(n + 1) (b) n = 25$$

2 / 2

- 27 The sum of the first 6 terms of an arithmetic series is 126.

The first term is 11.

Find the common difference.

$$126 = \frac{6}{2}(2 \times 11 + 5d)$$

$$126 = 3(22 + 5d)$$

$$22 + 5d = 42$$

$$5d = 20$$

$$d = 4$$

4

2 / 2

- 28 Sam saves money each week. In week 1 Sam saves £6. In each following week Sam saves £5 more than in the week before.

- (a) How much does Sam save in week 41?
 (b) Work out the total Sam has saved after 41 weeks.

$$(a) 6 + 40 \times 5 = 206$$

$$(b) S = \frac{41}{2}(6 + 206) = 4346$$

$$(a) \text{ £}206 \quad (b) \text{ £}4346$$

2 / 2

- 29 The 2nd term of an arithmetic sequence is 3 and the 6th term is 35.

- (a) Find the common difference and the first term.
 (b) Work out the 32nd term.

$$(a) 4d = 35 - 3 = 32$$

$$a = 3 - 1 \times 8 = -5$$

$$(b) u(32) = -5 + 31 \times 8 = 243$$

$$(a) d = 8, a = -5 \quad (b) 243$$

2 / 2

- 30 The n th term of a sequence is $2n^2 - 3n - 1$.

134 is a term in the sequence. Which term is it?

$$2n^2 - 3n - 1 = 134$$

$$\text{try } n = 9: 2 \times 81 + (-3) \times 9 + (-1) = 134$$

The 9th term

2 / 2

- 31 A theatre has 27 rows of seats. There are 14 seats in the first row. Each row has 2 more seats than the row in front.

- (a) How many seats are there in the last row?
 (b) Work out the total number of seats in the theatre.

$$(a) 14 + 26 \times 2 = 66$$

$$(b) S = \frac{27}{2}(14 + 66) = 1080$$

(a) 66 (b) 1080

2 / 2

- 32 The n th term of a sequence is $7n + 10$.

Work out the sum of the terms from the 12th term to the 32nd term inclusive.

$$12\text{th term} = 94$$

$$32\text{nd term} = 234$$

$$\text{number of terms} = 32 - 12 + 1 = 21$$

$$S = \frac{21}{2}(94 + 234)$$

$$= 3444$$

3444

2 / 2

- 33 The 4th term of an arithmetic sequence is 28 and the 6th term is 38.

- (a) Find the common difference and the first term.
 (b) Work out the 29th term.

$$(a) 2d = 38 - 28 = 10$$

$$a = 28 - 3 \times 5 = 13$$

$$(b) u(29) = 13 + 28 \times 5 = 153$$

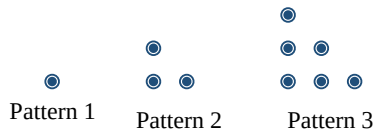
(a) $d = 5$, $a = 13$ (b) 153

2 / 2

34 Here are the first three patterns in a sequence of dot patterns.

(a) Find an expression, in terms of n , for the number of dots in Pattern n .

(b) Work out the number of dots in Pattern 40.



$$(a) 1, 3, 6: \text{ add } 1 + 2 + \dots + n = \frac{n}{2}(1 + n)$$

$$(b) \frac{40}{2} \times 41 = 820$$

$$(a) \frac{n(n+1)}{2} \quad (b) 820$$

2 / 2

35 The n th term of a sequence is $2n - 3$.

Work out the sum of the terms from the 8th term to the 16th term inclusive.

$$8\text{th term} = 13$$

$$16\text{th term} = 29$$

$$\text{number of terms} = 16 - 8 + 1 = 9$$

$$S = \frac{9}{2}(13 + 29)$$

$$= 189$$

$$189$$

2 / 2

36 The n th term of sequence A is $4n + 2$.

The n th term of sequence B is $3n + 9$.

There is one value of n for which the n th terms of A and B are equal.

Find this value of n and the term.

$$4n + 2 = 3n + 9$$

$$4n - 3n = 9 - 2$$

$$n = 7, \text{ term} = 30$$

2 / 2

- 37 The first term of an arithmetic series is 2 and the common difference is 3.

Find the smallest value of n for which the sum of the first n terms is greater than 1031.

$$S(n) = \frac{n}{2}(4 + (n - 1) \times 3)$$

$$\text{try } n = 26: S = 1027 \text{ (not greater than 1031)}$$

$$\text{try } n = 27: S = 1107 \text{ (greater than 1031)}$$

27

2 / 2

- 38 The n th term of a sequence is $5n + 8$.

Work out the sum of the terms from the 6th term to the 24th term inclusive.

$$6\text{th term} = 38$$

$$24\text{th term} = 128$$

$$\text{number of terms} = 24 - 6 + 1 = 19$$

$$S = \frac{19}{2}(38 + 128)$$

$$= 1577$$

1577

2 / 2

- 39 The n th term of sequence A is $7n + 6$.

The n th term of sequence B is $5n + 14$.

There is one value of n for which the n th terms of A and B are equal.

Find this value of n and the term.

$$7n + 6 = 5n + 14$$

$$7n - 5n = 14 - 6$$

$$2n = 8$$

$n = 4$, term = 34

3 / 3

- 40 The first term of an arithmetic series is 3 and the common difference is 4.

Find the smallest value of n for which the sum of the first n terms is greater than 535.

$$S(n) = \frac{n}{2}(6 + (n - 1) \times 4)$$

$$\text{try } n = 16: S = 528 \text{ (not greater than 535)}$$

$$\text{try } n = 17: S = 595 \text{ (greater than 535)}$$

17

3 / 3

- 41 The n th term of a sequence is $8n + 4$.

Work out the sum of the terms from the 10th term to the 21st term inclusive.

$$10\text{th term} = 84$$

$$21\text{st term} = 172$$

$$\text{number of terms} = 21 - 10 + 1 = 12$$

$$S = \frac{12}{2}(84 + 172)$$

$$= 1536$$

1536

3 / 3

- 42 The sum of the first n terms of an arithmetic series is 90.

The first term is 8 and the common difference is 5.

Find the value of n .

$$90 = \frac{n}{2}(2 \times 8 + (n - 1) \times 5)$$

$$90 = \frac{n(5n + 11)}{2}$$

$$5n^2 + 11n - 180 = 0$$

$$n = 5 \text{ or } n = \text{negative}$$

$$n \text{ must be positive so } n = 5$$

$n = 5$

3 / 3

- 43 The n th term of a sequence is $2n^2 + 2n - 2$.

Show that 311 is not a term of the sequence.

$$n = 12 \text{ gives } 310$$

$$n = 13 \text{ gives } 362$$

the sequence is increasing so 311 lies between consecutive terms: it is not a term

Not a term

3 / 3

- 44 The n th term of a sequence is $4n + 3$.

Show that the sum of the first n terms of the sequence is $\frac{n(4n + 10)}{2}$.

$$a = 7$$

$$d = 4$$

$$S(n) = \frac{n}{2}(2 \times 7 + (n - 1) \times 4)$$

$$= \frac{n}{2}(14 + 4n - 4)$$

$$= \frac{n}{2}(4n + 10)$$

Shown

3 / 3

- 45 The sum of the first n terms of an arithmetic series is 35.

The first term is 5 and the common difference is 1.

Find the value of n .

$$35 = \frac{n}{2}(2 \times 5 + (n - 1) \times 1)$$

$$35 = \frac{n(n + 9)}{2}$$

$$n^2 + 9n - 70 = 0$$

$$n = 5 \text{ or } n = -14$$

n must be positive so $n = 5$

$n = 5$

3 / 3

- 46 Here are the first five terms of a quadratic sequence.

7, 11, 19, 31, 47

Find an expression, in terms of n , for the n th term.

first differences: 4, 8, 12, 16

second differences: 4, 4, so $a = 2$

subtract $2n^2$: 5, 3, 1, -1

this is linear: $-2n + 7$

n th term = $2n^2 - 2n + 7$

$$2n^2 - 2n + 7$$

3 / 3

- 47 The n th term of a sequence is $2n + 6$.

Show that the sum of the first n terms of the sequence is $\frac{n(2n + 14)}{2}$.

$$a = 8$$

$$d = 2$$

$$S(n) = \frac{n}{2}(2 \times 8 + (n - 1) \times 2)$$

$$= \frac{n}{2}(16 + 2n - 2)$$

$$= \frac{n}{2}(2n + 14)$$

Shown

3 / 3

- 48 The sum of the first n terms of an arithmetic series is 1260.

The first term is 6 and the common difference is 6.

Find the value of n .

$$1260 = \frac{n}{2}(2 \times 6 + (n - 1) \times 6)$$

$$1260 = \frac{n(6n + 6)}{2}$$

$$6n^2 + 6n - 2520 = 0$$

$$n = 20 \text{ or } n = -21$$

n must be positive so $n = 20$

$$n = 20$$

3 / 3

- 49 Here are the first five terms of a quadratic sequence.

5, 2, -5, -16, -31

Find an expression, in terms of n , for the n th term.

first differences: -3, -7, -11, -15

second differences: -4, -4, so $a = -2$

subtract $-2n^2$: 7, 10, 13, 16

this is linear: $3n + 4$

n th term = $-2n^2 + 3n + 4$

$$-2n^2 + 3n + 4$$

3 / 3

- 50 Here are the first five terms of a quadratic sequence.

3, 11, 23, 39, 59

Find an expression, in terms of n , for the n th term.

first differences: 8, 12, 16, 20

second differences: 4, 4, so $a = 2$

subtract $2n^2$: 1, 3, 5, 7

this is linear: $2n - 1$

n th term = $2n^2 + 2n - 1$

$$2n^2 + 2n - 1$$

3 / 3
