

MATHS



MODEL ANSWERS

DIFFERENTIATION

SCORE

/92

PERCENTAGE

%

STEM UP

- 1 Given that $y = 5x - 9$, find $\frac{dy}{dx}$.

$5x$ gives 5

-9 differentiates to 0

$$\frac{dy}{dx} = 5$$

5

1 / 1

- 2 $y = (x - 5)(x + 4)$

Find $\frac{dy}{dx}$.

expand: $(x - 5)(x + 4) = x^2 - x - 20$

differentiate each term: $\frac{dy}{dx} = 2x - 1$

$2x - 1$

1 / 1

- 3 $y = 6x^3 + 3x^2 + 6x$.

Work out the gradient of the curve at the point where $x = 3$.

$$\frac{dy}{dx} = 18x^2 + 6x + 6$$

when $x = 3$: gradient = 186

186

1 / 1

- 4 Given that $y = 7x^3 - 3x^2 + 8x + 9$, find $\frac{dy}{dx}$.

$7x^3$ gives $21x^2$

$-3x^2$ gives $-6x$

$8x$ gives 8

9 differentiates to 0

$$\frac{dy}{dx} = 21x^2 - 6x + 8$$

$$21x^2 - 6x + 8$$

/ 1

- 5 $y = 3x^2(4x - 1)$

Find $\frac{dy}{dx}$.

expand: $3x^2(4x - 1) = 12x^3 - 3x^2$

$$\frac{dy}{dx} = 36x^2 - 6x$$

$$36x^2 - 6x$$

/ 1

- 6 $y = -9 + \frac{9}{x}$.

Find $\frac{dy}{dx}$.

write each term as a power of x

-9 differentiates to 0

$9x^{-1}$ gives $-9x^{-2}$

$$\frac{dy}{dx} = -9x^{-2}$$

$$-9x^{-2}$$

/ 1

7 $y = -8x^3 + 5x^2 + 7x.$

Work out the gradient of the curve at the point where $x = 3.$

$$\frac{dy}{dx} = -24x^2 + 10x + 7$$

when $x = 3$: gradient = -179

-179

1 / 1

8 $y = \frac{6x^3 + x^2 - x}{x}$

Find $\frac{dy}{dx}.$

divide each term by x : $y = 6x^2 + x - 1$

$$\frac{dy}{dx} = 12x + 1$$

12x + 1

1 / 1

9 $y = 6 + \frac{8}{x}.$

Find $\frac{dy}{dx}.$

write each term as a power of x

6 differentiates to 0

$8x^{-1}$ gives $-8x^{-2}$

$$\frac{dy}{dx} = -8x^{-2}$$

$-8x^{-2}$

1 / 1

10 $f(x) = 3x^4 - 4x^2 + 3x - 5$

(a) Find $f'(x)$.

(b) Work out the value of $f'(-3)$.

(a) $3x^4$ gives $12x^3$

$-4x^2$ gives $-8x$

$3x$ gives 3

-5 differentiates to 0

$f'(x) = 12x^3 - 8x + 3$

(b) $f'(-3) = -297$

(a) $12x^3 - 8x + 3$ (b) -297

/ 1

11 $y = \frac{5x^3 + 8x^2 + 6x}{x}$

Find $\frac{dy}{dx}$.

divide each term by x : $y = 5x^2 + 8x + 6$

$\frac{dy}{dx} = 10x + 8$

$10x + 8$

/ 1

12 The temperature, T °C, of a liquid t minutes after it is heated is $T = 2t^3 - 6t^2 + 30t + 10$.

Work out the rate of change of the temperature when $t = 4$.

$\frac{dT}{dt} = 6t^2 - 12t + 30$

when $t = 4$: rate = 78

78 °C per minute

/ 1

13 $f(x) = -6x^4 - 4x^3 - 8x^2 - 3$

(a) Find $f'(x)$.

(b) Work out the value of $f'(3)$.

(a) $-6x^4$ gives $-24x^3$

$-4x^3$ gives $-12x^2$

$-8x^2$ gives $-16x$

-3 differentiates to 0

$f'(x) = -24x^3 - 12x^2 - 16x$

(b) $f'(3) = -804$

(a) $-24x^3 - 12x^2 - 16x$ (b)
 -804

/ 1

14 $y = x^3 + 6x^2 + 2x$.

(a) Find $\frac{dy}{dx}$.

(b) Find $\frac{d^2y}{dx^2}$.

(c) Work out the value of $\frac{d^2y}{dx^2}$ when $x = 3$.

(b) differentiate again: $6x + 12$

(c) substitute $x = 3$: 30

(a) $3x^2 + 12x + 2$ (b) $6x + 12$
 (c) 30

/ 1

15 The temperature, T °C, of a liquid t minutes after it is heated is $T = 3t^3 - 12t^2 + 30t + 16$.

Work out the rate of change of the temperature when $t = 1$.

$\frac{dT}{dt} = 9t^2 - 24t + 30$

when $t = 1$: rate = 15

15 °C per minute

/ 1

16 $y = 3x^2 + 24x + 57$.

Use differentiation to find the coordinates of the turning point of the curve.

State whether this point is a maximum or a minimum.

$$\frac{dy}{dx} = 6x + 24$$

$$6x + 24 = 0$$

$$x = -4$$

$$y = 9$$

$$\frac{d^2y}{dx^2} = 6 \text{ so minimum}$$

$(-4, 9)$, minimum

/ 1

17 $y = -9x^4 + 3x^2 - 2x$.

(a) Find $\frac{dy}{dx}$.

(b) Find $\frac{d^2y}{dx^2}$.

(c) Work out the value of $\frac{d^2y}{dx^2}$ when $x = -1$.

(b) differentiate again: $-108x^2 + 6$

(c) substitute $x = -1$: -102

(a) $-36x^3 + 6x - 2$ (b) $-108x^2 + 6$ (c) -102

/ 1

18 $y = -2x^3 - 18x^2 - 48x + 1$.

Show that the curve has a stationary point at $x = -4$.

$$\frac{dy}{dx} = -6x^2 - 36x - 48$$

when $x = -4$: $\frac{dy}{dx} = 0$

so $\frac{dy}{dx} = 0$ and the curve is stationary

Shown

/ 1

19 $y = -2x^2 + 8x - 14$.

Use differentiation to find the coordinates of the turning point of the curve.

State whether this point is a maximum or a minimum.

$$\frac{dy}{dx} = -4x + 8$$

$$-4x + 8 = 0$$

$$x = 2$$

$$y = -6$$

$$\frac{d^2y}{dx^2} = -4 \text{ so maximum}$$

(2, -6), maximum

1 / 1

20 $y = -4x^2 + \frac{1}{x} + \frac{9}{x^2}$.

Find an equation of the tangent to the curve at the point where $x = 1$.

Give your answer in the form $y = mx + c$.

$$y = 6 \text{ when } x = 1$$

$$\frac{dy}{dx} = -8x - x^{-2} - 18x^{-3}$$

$$m = -27$$

$$y - 6 = (-27)(x - 1)$$

$$y = -27x + 33$$

1 / 1

21 $y = x^3 + 6x^2 + 1$.

Show that the curve has a stationary point at $x = -4$.

$$\frac{dy}{dx} = 3x^2 + 12x$$

$$\text{when } x = -4: \frac{dy}{dx} = 0$$

$$\text{so } \frac{dy}{dx} = 0 \text{ and the curve is stationary}$$

Shown

2 / 2

22 $y = 2x^3 + 3x^2 - 9x - 3.$

Find the coordinates of the points on the curve where the gradient is 27.

$$\frac{dy}{dx} = 6x^2 + 6x - 9$$

$$6x^2 + 6x - 9 = 27$$

$$6x^2 + 6x - 36 = 0$$

$$x = -3 \text{ or } x = 2$$

$$y \text{ values: } -3 \text{ and } 7$$

$$(-3, -3) \text{ and } (2, 7)$$

/ 2

23 $y = -x^3 + 3x^2 + 8.$

Find an equation of the tangent to the curve at the point where $x = 3$.

Give your answer in the form $y = mx + c$.

$$y = 8 \text{ when } x = 3$$

$$\frac{dy}{dx} = -3x^2 + 6x$$

$$m = -9$$

$$y - 8 = (-9)(x - 3)$$

$$y = -9x + 35$$

/ 2

24 $y = x^3 + kx^2 + 6x + 9$

The gradient of the curve at the point where $x = -2$ is 14.

Find the value of k .

$$\frac{dy}{dx} = 3x^2 + 2kx + 6$$

$$x = -2: 3(4) + 2k(-2) + 6 = 14$$

$$-4k + 18 = 14$$

$$k = 1$$

/ 2

25 $y = 2x^3 - 9x + 3.$

Find the coordinates of the points on the curve where the gradient is 15.

$$\frac{dy}{dx} = 6x^2 - 9$$

$$6x^2 - 9 = 15$$

$$6x^2 - 24 = 0$$

$$x = -2 \text{ or } x = 2$$

$$y \text{ values: } 5 \text{ and } 1$$

$$(-2, 5) \text{ and } (2, 1)$$

$$\boxed{2} / 2$$

26 $y = -5x^3 + 7x^2 + 5.$

Find an equation of the tangent to the curve at the point where $x = 2$.

Give your answer in the form $y = mx + c$.

$$y = -7 \text{ when } x = 2$$

$$\frac{dy}{dx} = -15x^2 + 14x$$

$$m = -32$$

$$y - (-7) = (-32)(x - 2)$$

$$y = -32x + 57$$

$$\boxed{2} / 2$$

27 $y = x^3 + kx^2 + 6x + 5$

The gradient of the curve at the point where $x = -1$ is -3 .

Find the value of k .

$$\frac{dy}{dx} = 3x^2 + 2kx + 6$$

$$x = -1: 3(1) + 2k(-1) + 6 = -3$$

$$-2k + 9 = -3$$

$$k = 6$$

$$\boxed{2} / 2$$

28 $y = 2x^3 - 15x^2 + 36x - 7$.

The curve has stationary points at $x = 2$ and $x = 3$.

By considering the gradient on either side of each stationary point, determine whether each is a maximum or a minimum.

$$\frac{dy}{dx} = 6x^2 - 30x + 36$$

$x = 2$: gradient is positive just before and negative just after, so maximum

$x = 3$: gradient is negative just before and positive just after, so minimum

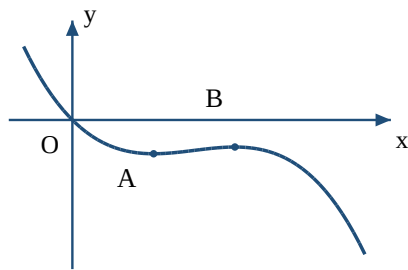
$x = 2$: maximum; $x = 3$:
minimum

2 / 2

29 The diagram shows a sketch of the curve with equation $y = -2x^3 + 9x^2 - 12x$.

The curve has two turning points, A and B.

Find the coordinates of A and B. Diagram NOT accurately drawn.



$$\frac{dy}{dx} = -6x^2 + 18x - 12$$

$$\frac{dy}{dx} = 0: -6(x - 1)(x - 2) = 0$$

$$x = 1 \text{ or } x = 2$$

$$y = -5 \text{ and } -4$$

A (1, -5), B (2, -4)

2 / 2

30 $y = -5x^2 + 8x - 1$.

The point P on the curve has x coordinate 2.

Find an equation of the normal to the curve at P.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

$$y = -5$$

$$\text{gradient of curve} = -12$$

$$\text{gradient of normal} = \frac{1}{12}$$

$$y - (-5) = \frac{1}{12}(x - 2)$$

$$x - 12y - 62 = 0$$

2 / 2

31 $y = 2x^3 + 9x^2 - 24x - 2$.

The curve has stationary points at $x = -4$ and $x = 1$.

By considering the gradient on either side of each stationary point, determine whether each is a maximum or a minimum.

$$\frac{dy}{dx} = 6x^2 + 18x - 24$$

$x = -4$: gradient is positive just before and negative just after, so maximum

$x = 1$: gradient is negative just before and positive just after, so minimum

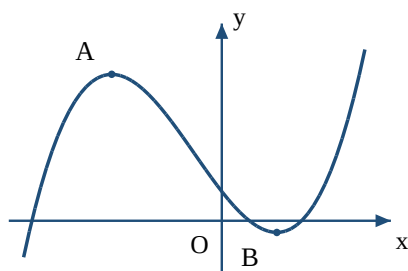
$x = -4$: maximum; $x = 1$:
minimum

2 / 2

32 The diagram shows a sketch of the curve with equation $y = 2x^3 + 3x^2 - 12x + 5$.

The curve has two turning points, A and B.

Find the coordinates of A and B. Diagram NOT accurately drawn.



$$\frac{dy}{dx} = 6x^2 + 6x - 12$$

$$\frac{dy}{dx} = 0: 6(x + 2)(x - 1) = 0$$

$$x = -2 \text{ or } x = 1$$

$$y = 25 \text{ and } -2$$

A (-2, 25), B (1, -2)

2 / 2

33 A particle moves in a straight line. At time t seconds its distance from a fixed point O is s metres, where $s = t^3 - 6t^2 + 9t$.

(a) Find an expression for the velocity, v m/s, of the particle at time t .

(b) Work out the velocity when $t = 5$.

(c) Find the two times when the particle is stationary.

(d) Work out the acceleration when $t = 5$.

$$(a) v = \frac{ds}{dt} = 3t^2 - 12t + 9$$

$$(b) v(5) = 24$$

$$(c) v = 0: 3(t - 1)(t - 3) = 0$$

$$t = 1 \text{ or } t = 3$$

$$(d) a = \frac{dv}{dt} = 6t - 12$$

$$a(5) = 18$$

(a) $3t^2 - 12t + 9$ (b) 24 m/s
(c) $t = 1$ and $t = 3$ (d) 18 m/s²

2 / 2

34 $y = -2x^3 - x^2 + 6$.

The point P on the curve has x coordinate -1 .

Find an equation of the normal to the curve at P.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

$$y = 7$$

$$\text{gradient of curve} = -4$$

$$\text{gradient of normal} = \frac{1}{4}$$

$$y - 7 = \frac{1}{4}(x - (-1))$$

$$x - 4y + 29 = 0$$

2 / 2

- 35 A company makes x thousand items each week. Its weekly profit is £P thousand, where $P = -5x^2 + 80x - 5$.

(a) Find $\frac{dP}{dx}$.

(b) Find the number of items that gives the maximum profit.

(c) Work out the maximum weekly profit.

(d) Show that this value of P is a maximum.

$$(a) \frac{dP}{dx} = -10x + 80$$

$$(b) -10x + 80 = 0$$

$$x = 8 \text{ so } 8000 \text{ items}$$

$$(c) P = 315$$

$$(d) \frac{d^2P}{dx^2} = -10 < 0 \text{ so maximum}$$

$$(a) -10x + 80 \quad (b) 8,000 \quad (c) \text{£}315 \text{ thousand} \quad (d) \text{negative second derivative}$$

2 / 2

- 36 A particle moves in a straight line. At time t seconds its distance from a fixed point O is s metres, where $s = t^3 - 9t^2 + 15t$.

(a) Find an expression for the velocity, v m/s, of the particle at time t .

(b) Work out the velocity when $t = 3$.

(c) Find the two times when the particle is stationary.

(d) Work out the acceleration when $t = 3$.

$$(a) v = \frac{ds}{dt} = 3t^2 - 18t + 15$$

$$(b) v(3) = -12$$

$$(c) v = 0: 3(t - 1)(t - 5) = 0$$

$$t = 1 \text{ or } t = 5$$

$$(d) a = \frac{dv}{dt} = 6t - 18$$

$$a(3) = 0$$

$$(a) 3t^2 - 18t + 15 \quad (b) -12 \text{ m/s} \quad (c) t = 1 \text{ and } t = 5 \quad (d) 0 \text{ m/s}^2$$

2 / 2

37 $y = 8x^2 + 9x + 3$.

The point P on the curve has x coordinate -1 .

Find an equation of the normal to the curve at P.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

$$y = 2$$

$$\text{gradient of curve} = -7$$

$$\text{gradient of normal} = \frac{1}{7}$$

$$y - 2 = \frac{1}{7}(x - (-1))$$

$$x - 7y + 15 = 0$$

2 / 2

- 38 A company makes x thousand items each week. Its weekly profit is £P thousand, where $P = -3x^2 + 90x - 5$.

(a) Find $\frac{dP}{dx}$.

(b) Find the number of items that gives the maximum profit.

(c) Work out the maximum weekly profit.

(d) Show that this value of P is a maximum.

$$(a) \frac{dP}{dx} = -6x + 90$$

$$(b) -6x + 90 = 0$$

$$x = 15 \text{ so } 15000 \text{ items}$$

$$(c) P = 670$$

$$(d) \frac{d^2P}{dx^2} = -6 < 0 \text{ so maximum}$$

$$(a) -6x + 90 \quad (b) 15,000 \quad (c) \text{£}670 \text{ thousand} \quad (d) \text{negative second derivative}$$

2 / 2

39 $f(x) = x^3 - 12x - 1$

(a) Find $f'(x)$.

(b) Find the range of values of x for which $f(x)$ is decreasing.

$$(a) f'(x) = 3x^2 - 12$$

$$(b) f'(x) < 0: 3(x + 2)(x - 2) < 0$$

$$\text{critical values } x = -2 \text{ and } x = 2$$

sketch the U-shaped quadratic

$$(a) 3x^2 - 12 \quad (b) -2 < x < 2$$

3 / 3

40 $y = x^3 - 9x^2 + 24x + 10.$

(a) Find $\frac{dy}{dx}$.

(b) Find the coordinates of the stationary points of the curve.

(c) Determine the nature of each stationary point.

(a) $\frac{dy}{dx} = 3x^2 - 18x + 24$

(b) $\frac{dy}{dx} = 0: 3(x - 2)(x - 4) = 0$

$x = 2$ or $x = 4$

$y = 30$ and 26

(c) $\frac{d^2y}{dx^2} = 6x - 18$

at $x = 2$: -6 so maximum

at $x = 4$: 6 so minimum

(a) $3x^2 - 18x + 24$ (b) $(2, 30), (4, 26)$ (c) maximum at $x = 2$, minimum at $x = 4$

3 / 3

41 $y = x + \frac{4}{x}.$

(a) Find $\frac{dy}{dx}$.

(b) Find the coordinates of the stationary points of the curve.

(c) Work out the nature of the stationary point where x is positive.

(a) $y = x + 4x^{-1}$
 $\frac{dy}{dx} = 1 - 4x^{-2}$

(b) $1 - 4x^{-2} = 0$
 $1 = \frac{4}{x^2}$

$x^2 = 4$

$x = \pm 2$

$y = 4$ and -4

(c) $\frac{d^2y}{dx^2} = \frac{8}{x^3}$

at $x = 2$ the value is positive so a minimum

(a) $1 - 4x^{-2}$ (b) $(2, 4)$ and $(-2, -4)$ (c) minimum

3 / 3

42 $f(x) = 2x^3 - 9x^2 - 60x + 6$

(a) Find $f'(x)$.

(b) Find the range of values of x for which $f(x)$ is increasing.

(a) $f'(x) = 6x^2 - 18x - 60$

(b) $f'(x) > 0: 6(x + 2)(x - 5) > 0$

critical values $x = -2$ and $x = 5$

sketch the U-shaped quadratic

(a) $6x^2 - 18x - 60$ (b) $x < -2$
or $x > 5$

3 / 3

43 $y = 2x^3 + 6x^2 - 18x - 6$.

(a) Find $\frac{dy}{dx}$.

(b) Find the coordinates of the stationary points of the curve.

(c) Determine the nature of each stationary point.

(a) $\frac{dy}{dx} = 6x^2 + 12x - 18$

(b) $\frac{dy}{dx} = 0: 6(x + 3)(x - 1) = 0$

$x = -3$ or $x = 1$

$y = 48$ and -16

(c) $\frac{d^2y}{dx^2} = 12x + 12$

at $x = -3$: -24 so maximum

at $x = 1$: 24 so minimum

(a) $6x^2 + 12x - 18$ (b) $(-3, 48), (1, -16)$ (c) maximum at $x = -3$, minimum at $x = 1$

3 / 3

44 $y = x + \frac{25}{x}$.

(a) Find $\frac{dy}{dx}$.

(b) Find the coordinates of the stationary points of the curve.

(c) Work out the nature of the stationary point where x is positive.

(a) $y = x + 25x^{-1}$
 $\frac{dy}{dx} = 1 - 25x^{-2}$

(b) $1 - 25x^{-2} = 0$
 $1 = \frac{25}{x^2}$

$x^2 = 25$

$x = \pm 5$

$y = 10$ and -10

(c) $\frac{d^2y}{dx^2} = \frac{50}{x^3}$

at $x = 5$ the value is positive so a minimum

(a) $1 - 25x^{-2}$ (b) $(5, 10)$ and $(-5, -10)$ (c) minimum

3 / 3

45 A farmer has 80 m of fencing. She uses all of it to make three sides of a rectangular field. The fourth side is a wall.

The sides of the field perpendicular to the wall are each x metres long.

(a) Show that the area of the field, $A \text{ m}^2$, is $A = 80x - 2x^2$.

(b) Use calculus to find the value of x for which A is a maximum.

(c) Work out the maximum area.

(a) length parallel to wall $= 80 - 2x$

$A = x(80 - 2x) = 80x - 2x^2$

(b) $\frac{dA}{dx} = 80 - 4x = 0$

$\frac{d^2A}{dx^2} = -4 < 0$ so maximum

(c) $A = 80 \times 20 - 2 \times 20^2 = 800$

(a) Shown (b) $x = 20$ (c) 800 m^2

3 / 3

- 46 A square piece of card has side 24 cm. A square of side x cm is cut from each corner. The card is folded to make an open box.

The volume of the box is $V \text{ cm}^3$.

(a) Show that $V = 4x^3 - 96x^2 + 576x$.

(b) Find the value of x for which V is a maximum.

(c) Work out this maximum volume. Diagram NOT accurately drawn.

(a) $V = x(24 - 2x)^2 = x(576 - 96x + 4x^2) = 4x^3 - 96x^2 + 576x$

(b) $\frac{dV}{dx} = 12x^2 - 192x + 576 = 0$

$12(x - 4)(x - 12) = 0$

$x = 4 \text{ or } x = 12$

$x = 4 \text{ as } x < 12$

(c) $V = 4 \times (24 - 8)^2 = 1024$

(a) Shown (b) $x = 4$ (c) 1024 cm^3

3 / 3

- 47 An open box has a square base of side x cm and height h cm. The volume of the box is 108 cm^3 .

(a) Show that the outside surface area, $S \text{ cm}^2$, is $S = x^2 + \frac{432}{x}$.

(b) Find the value of x for which S is a minimum.

(c) Work out the minimum value of S . Diagram NOT accurately drawn.

(a) $h = \frac{108}{x^2}$

$S = x^2 + 4xh = x^2 + \frac{432}{x}$

(b) $\frac{dS}{dx} = 2x - \frac{432}{x^2} = 0$

$2x^3 = 432$

$x^3 = 216$

(c) $S = 36 + \frac{432}{6} = 108$

(a) Shown (b) $x = 6$ (c) 108 cm^2

3 / 3

- 48 A farmer has 60 m of fencing. She uses all of it to make three sides of a rectangular field. The fourth side is a wall.

The sides of the field perpendicular to the wall are each x metres long.

(a) Show that the area of the field, $A \text{ m}^2$, is $A = 60x - 2x^2$.

(b) Use calculus to find the value of x for which A is a maximum.

(c) Work out the maximum area.

(a) length parallel to wall = $60 - 2x$

$A = x(60 - 2x) = 60x - 2x^2$

(b) $\frac{dA}{dx} = 60 - 4x = 0$

$\frac{d^2A}{dx^2} = -4 < 0$ so maximum

(c) $A = 60 \times 15 - 2 \times 15^2 = 450$

(a) Shown (b) $x = 15$ (c) 450 m^2

3 / 3

- 49 A square piece of card has side 36 cm. A square of side x cm is cut from each corner. The card is folded to make an open box.

The volume of the box is $V \text{ cm}^3$.

(a) Show that $V = 4x^3 - 144x^2 + 1296x$.

(b) Find the value of x for which V is a maximum.

(c) Work out this maximum volume. Diagram NOT accurately drawn.

$$(a) V = x(36 - 2x)^2 = x(1296 - 144x + 4x^2) = 4x^3 - 144x^2 + 1296x$$

$$(b) \frac{dV}{dx} = 12x^2 - 288x + 1296 = 0$$

$$12(x - 6)(x - 18) = 0$$

$$x = 6 \text{ or } x = 18$$

$$x = 6 \text{ as } x < 18$$

$$(c) V = 6 \times (36 - 12)^2 = 3456$$

$$(a) \text{ Shown } (b) x = 6 (c) 3456 \text{ cm}^3$$

3 / 3

- 50 An open box has a square base of side x cm and height h cm. The volume of the box is 4 cm^3 .

(a) Show that the outside surface area, $S \text{ cm}^2$, is $S = x^2 + \frac{16}{x}$.

(b) Find the value of x for which S is a minimum.

(c) Work out the minimum value of S . Diagram NOT accurately drawn.

$$(a) h = \frac{4}{x^2}$$

$$S = x^2 + 4xh = x^2 + \frac{16}{x}$$

$$(b) \frac{dS}{dx} = 2x - \frac{16}{x^2} = 0$$

$$2x^3 = 16$$

$$x^3 = 8$$

$$(c) S = 4 + \frac{16}{2} = 12$$

$$(a) \text{ Shown } (b) x = 2 (c) 12 \text{ cm}^2$$

3 / 3