Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Wednesday 20 May 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2-\frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

(b) Hence find the constant term in the series expansion of

$$\left(3-\frac{1}{x}\right)^{2}\left(2-\frac{x}{4}\right)^{10}$$

(3)

(a) $\left(2-\frac{x}{4}\right)^{10} = 2^{10} + \binom{10}{1}2^{9}\left(-\frac{x}{4}\right) + \binom{10}{2}2^{8}\left(-\frac{x}{4}\right)^{2} + \ldots$

$\ldots + \binom{10}{3}2^{7}\left(-\frac{x}{4}\right)^{3} + \ldots$

$= 1024 + 10 \times 512 \times \left(-\frac{x}{4}\right) + 45 \times 256 \times \frac{x^2}{16} + 120 \times 128 \times \left(-\frac{x^3}{64}\right)$

$= 1024 - 1280x + 720x^2 - 240x^3 + \ldots$

(b) $\left(3-\frac{1}{x}\right)^{2} = 9 - \frac{6}{x} + \frac{1}{x^2}$

$\left(9 - \frac{6}{x} + \frac{1}{x^2}\right)(1024 - 1280x + 720x^2 - 240x^3 + \ldots)$

Constant term $= 9 \times 1024 + \left(-\frac{6}{x}\right) \times (-1280x) + \frac{1}{x^2} \times 720x^2$

$= 9216 + 7680 + 720$

Constant term $= 17616$

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Question 1 continued

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Question 1 continued

Q1

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2.

$$y = \frac{2^x}{\sqrt{(5x^2 + 3)}}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	−0.25	0	0.25	0.5	0.75
y	0.462	0.577	0.653	0.686	0.698

(1)

(b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_{-0.25}^{0.75} \frac{2^x}{\sqrt{(5x^2 + 3)}}\,\mathrm{d}x$$

(3)

(a) $x = 0$: $y = 0.57735... = 0.577$, $x = 0.5$: $y = 0.68599... = 0.686$

(b) $h = 0.25$

$$\int \approx \frac{1}{2} \times 0.25 \times [0.462 + 0.698 + 2(0.577 + 0.653 + 0.686)]$$

$$= 0.125 \times [1.160 + 2(1.916)]$$

$$= 0.125 \times 4.992$$

$$= 0.624$$

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Question 2 continued

Q2

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3. $f(x) = ax^3 - x^2 + bx + 4$

where a and b are constants.

When $f(x)$ is divided by $(x + 4)$, the remainder is -108

(a) Use the remainder theorem to show that

$$16a + b = 24$$

(2)

Given also that $(2x - 1)$ is a factor of $f(x)$,

(b) find the value of a and the value of b.

(3)

(c) Find $f'(x)$.

(1)

(d) Hence find the exact coordinates of the stationary points of the curve with equation $y = f(x)$.

(4)

(a) $f(-4) = a(-4)^3 - (-4)^2 + b(-4) + 4 = -108$

$-64a - 16 - 4b + 4 = -108$

$-64a - 4b = -96 \Rightarrow 16a + b = 24$ (as required)

(b) $f\left(\frac{1}{2}\right) = a\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 4 = 0$

$\frac{a}{8} - \frac{1}{4} + \frac{b}{2} + 4 = 0 \Rightarrow a - 2 + 4b + 32 = 0 \Rightarrow a + 4b = -30$

$16a + b = 24 \Rightarrow b = 24 - 16a$

$a + 4(24 - 16a) = -30 \Rightarrow -63a = -126 \Rightarrow a = 2$

$b = 24 - 32 = -8$

$a = 2, \quad b = -8$

(c) $f(x) = 2x^3 - x^2 - 8x + 4$

$f'(x) = 6x^2 - 2x - 8$

(d) $f'(x) = 0$: $6x^2 - 2x - 8 = 0 \Rightarrow 3x^2 - x - 4 = 0$

$(3x - 4)(x + 1) = 0 \Rightarrow x = \frac{4}{3}, \quad x = -1$

$x = \frac{4}{3}$: $y = 2\left(\frac{4}{3}\right)^3 - \left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 4$

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Question 3 continued

$= \frac{128}{27} - \frac{48}{27} - \frac{288}{27} + \frac{108}{27} = -\frac{100}{27}$

$x = -1$: $y = 2(-1)^3 - (-1)^2 - 8(-1) + 4 = -2 - 1 + 8 + 4 = 9$

Stationary points: $\left(\frac{4}{3}, -\frac{100}{27}\right)$ and $(-1, 9)$

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Question 3 continued

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Question 3 continued

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Q3

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4. The points P and Q have coordinates $(-11, 6)$ and $(-3, 12)$ respectively.

Given that PQ is a diameter of the circle C,

(a) (i) find the coordinates of the centre of C,

(ii) find the radius of C.

(4)

(b) Hence find an equation of C.

(2)

(c) Find an equation of the tangent to C at the point Q giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(3)

(a) (i) Centre $= \left(\frac{-11 + (-3)}{2}, \frac{6 + 12}{2}\right) = (-7, 9)$

(ii) $r = \sqrt{((-3 - (-7))^2 + (12 - 9)^2)}$

$r = \sqrt{(16 + 9)} = \sqrt{25} = 5$

(b) $(x + 7)^2 + (y - 9)^2 = 25$

(c) Gradient of radius $CQ = \frac{12 - 9}{-3 - (-7)} = \frac{3}{4}$

Gradient of tangent $= -\frac{4}{3}$

$y - 12 = -\frac{4}{3}(x + 3)$

$3y - 36 = -4x - 12$

$4x + 3y - 24 = 0$

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Question 4 continued

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Question 4 continued

Q4

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5. Ben is saving for the deposit for a house over a period of 60 months.

Ben saves £100 in the first month and in each subsequent month, he saves £5 more than the previous month, so that he saves £105 in the second month, £110 in the third month, and so on, forming an arithmetic sequence.

(a) Find the amount Ben saves in the 40th month.

(2)

(b) Find the total amount Ben saves over the 60-month period.

(3)

Lina is also saving for a deposit for a house.

Lina saves £600 in the first month and in each subsequent month, she saves £10 less than the previous month, so that she saves £590 in the second month, £580 in the third month, and so on, forming an arithmetic sequence.

Given that, after n months, Lina will have saved exactly £18 200 for her deposit,

(c) form an equation in n and show that it can be written as

$$n^2 - 121n + 3640 = 0$$

(3)

(d) Solve the equation in part (c).

(2)

(e) State, with a reason, which of the solutions to the equation in part (c) is **not** a sensible value for n.

(1)

(a) $u_{40} = a + (n-1)d = 100 + (40-1) \times 5 = 100 + 195$

Amount saved in the 40th month = £295

(b) $S_{60} = \frac{n}{2}[2a + (n-1)d] = \frac{60}{2}[2 \times 100 + (60-1) \times 5]$

$= 30 \times [200 + 295] = 30 \times 495$

Total = £14 850

(c) $S_n = \frac{n}{2}[2 \times 600 + (n-1) \times (-10)] = 18\,200$

$\frac{n}{2}[1200 - 10n + 10] = 18\,200$

$n(1210 - 10n) = 36\,400$

$1210n - 10n^2 = 36\,400$

$10n^2 - 1210n + 36\,400 = 0$

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Question 5 continued

$n^2 - 121n + 3640 = 0$ (as required)

(d) $(n - 56)(n - 65) = 0$

$n = 56, \quad n = 65$

(e) $n = 65$ is not sensible: in month 65 Lina would save

$600 - 10 \times 64 = -£40$, a negative amount, which is not possible.

(She has already saved £18 200 after 56 months.)

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Question 5 continued

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Question 5 continued

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Q5

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6.

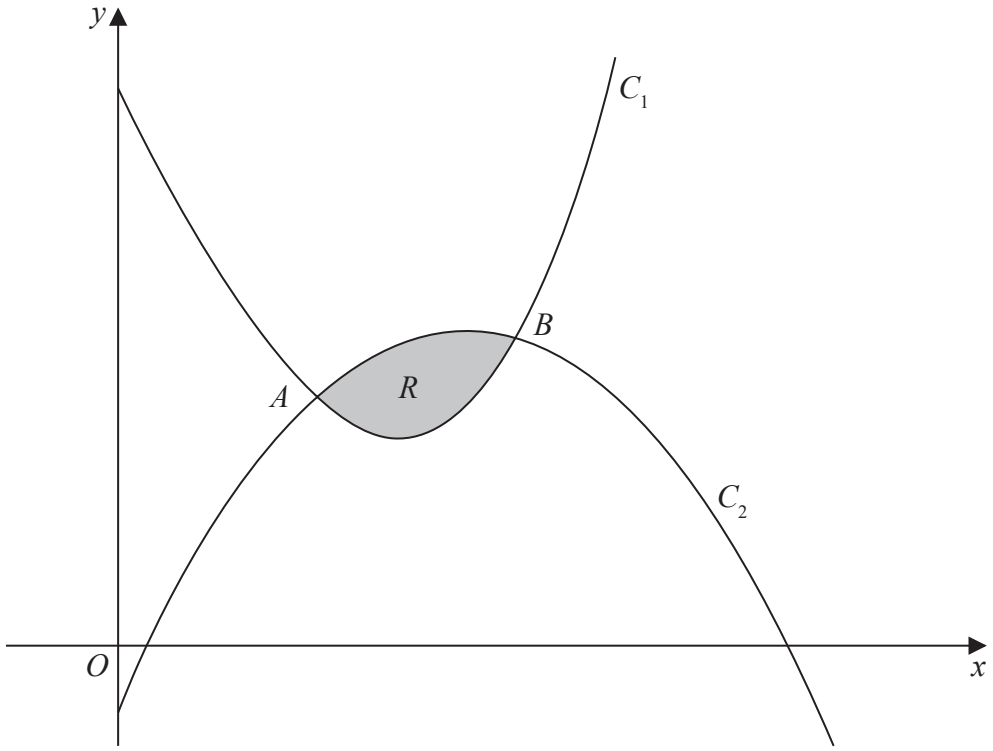


Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2 with equations

$$C_1 : y = x^3 - 6x + 9 \qquad x \geqslant 0$$
$$C_2 : y = -2x^2 + 7x - 1 \qquad x \geqslant 0$$

The curves C_1 and C_2 intersect at the points A and B as shown in Figure 1.

The point A has coordinates (1, 4).

Using algebra and showing all steps of your working,

(a) find the coordinates of the point B.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(5)

(a) $x^3 - 6x + 9 = -2x^2 + 7x - 1$

$x^3 + 2x^2 - 13x + 10 = 0$

$x = 1$ is a root (point A), so $(x - 1)$ is a factor:

$x^3 + 2x^2 - 13x + 10 = (x - 1)(x^2 + 3x - 10)$

$= (x - 1)(x - 2)(x + 5) = 0 \quad \Rightarrow \quad x = 1, 2, -5$

$x \geq 0$ and $B \neq A$, so $x = 2$: $\quad y = 2^3 - 6(2) + 9 = 5$

$B = (2, 5)$

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Question 6 continued

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(b) Area $R = \int_1^2 [(-2x^2 + 7x - 1) - (x^3 - 6x + 9)]\,dx$

$= \int_1^2 (-x^3 - 2x^2 + 13x - 10)\,dx$

$= \left[-\frac{x^4}{4} - \frac{2x^3}{3} + \frac{13x^2}{2} - 10x \right]_1^2$

$= \left(-\frac{16}{4} - \frac{16}{3} + \frac{52}{2} - 20 \right) - \left(-\frac{1}{4} - \frac{2}{3} + \frac{13}{2} - 10 \right)$

$= -\frac{10}{3} - \left(-\frac{53}{12} \right) = -\frac{40}{12} + \frac{53}{12}$

Area of $R = \frac{13}{12}$

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Question 6 continued

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Question 6 continued

Q6

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7. (i) Show that

$$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta} \qquad \theta \neq \frac{n\pi}{2} \qquad n \in \mathbb{Z}$$

(3)

(ii) Solve, for $0 \leqslant x < 90°$, the equation

$$3\cos^2(2x + 10°) = 1$$

giving your answers in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(i) $LHS = \tan\theta + \frac{1}{\tan\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta} = RHS$ (since $\sin^2\theta + \cos^2\theta \equiv 1$)

(ii) $3\cos^2(2x + 10°) = 1 \Rightarrow \cos^2(2x + 10°) = \frac{1}{3}$

$\cos(2x + 10°) = \pm\frac{1}{\sqrt{3}}$ $\quad (10° \leqslant 2x + 10° < 190°)$

$2x + 10° = 54.7356...°$ or $180° - 54.7356...° = 125.2643...°$

$2x = 44.7356...°$ or $115.2643...°$

$x = 22.3678...°$ or $57.6321...°$

$x = 22.4°, \quad x = 57.6°$

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Question 7 continued

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8. A geometric series has first term a and common ratio r.

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(3)

The second term of a geometric series is –320 and the fifth term is $\frac{512}{25}$

(b) Find the value of the common ratio.

(2)

(c) Hence find the sum of the first 13 terms of the series, giving your answer to 2 decimal places.

(3)

(a) $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \ldots + ar^n$

Subtracting: $S_n - rS_n = a - ar^n$

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ (as required)

(b) $ar = -320, \quad ar^4 = \frac{512}{25}$

$r^3 = \frac{512 \div 25}{-320} = -0.064$

$r = \sqrt[3]{(-0.064)} = -0.4$

(c) $a = \frac{-320}{r} = \frac{-320}{-0.4} = 800$

$S_{13} = \frac{800(1 - (-0.4)^{13})}{1 - (-0.4)} = \frac{800(1 + 0.4^{13})}{1.4}$

$= 571.43240\ldots$

$S_{13} = 571.43$ (2 d.p.)

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Question 8 continued

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Question 8 continued

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9. (i) Find the exact value of x for which

$$\log_3(x + 5) - 4 = \log_3(2x - 1)$$

(4)

(ii) Given that

$$3^{y+3} \times 2^{1-2y} = 108$$

(a) show that

$$0.75^y = 2$$

(4)

(b) Hence find the value of y, giving your answer to 3 decimal places.

(2)

(i) $\log_3(x + 5) - \log_3(2x - 1) = 4$

$\log_3\left(\frac{x+5}{2x-1}\right) = 4 \Rightarrow \frac{x+5}{2x-1} = 3^4 = 81$

$x + 5 = 81(2x - 1) = 162x - 81$

$161x = 86 \Rightarrow x = \frac{86}{161}$

(ii) (a) $3^{y+3} \times 2^{1-2y} = 108$

$3^y \times 3^3 \times 2 \times 2^{-2y} = 108 \Rightarrow 27 \times 2 \times \frac{3^y}{2^{2y}} = 108$

$\frac{3^y}{4^y} = \frac{108}{54} = 2$

$\left(\frac{3}{4}\right)^y = 2 \Rightarrow 0.75^y = 2$ (as required)

(b) $y = \log_{0.75} 2 = \frac{\ln 2}{\ln 0.75} = -2.40942...$

$y = -2.409$ (3 d.p.)

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Question 9 continued

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Question 9 continued

Q9

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