Please check the examination details below before entering your candidate information

Candidate surname	StemUp Model Solution	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Wednesday 13 May 2020

Morning (Time: 1 hour 30 minutes)	Paper Reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. Given that

$$(3pq^2)^4 \times 2p\sqrt{q^8} \equiv ap^bq^c$$

find the values of the constants a, b and c.

(3)

$(3pq^2)^4 = 3^4p^4q^8 = 81p^4q^8$

$2p\sqrt{(q^8)} = 2p \times q^4 = 2pq^4$

$(3pq^2)^4 \times 2p\sqrt{(q^8)} = 81p^4q^8 \times 2pq^4 = 162p^5q^{12}$

$a = 162, \quad b = 5, \quad c = 12$

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Question 1 continued

Q1

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2. $f(x) = 3 + 12x - 2x^2$

(a) Express $f(x)$ in the form

$$a - b(x + c)^2$$

where a, b and c are integers to be found.

(3)

The curve with equation $y = f(x) - 7$ crosses the x-axis at the points P and Q and crosses the y-axis at the point R.

(b) Find the area of the triangle PQR, giving your answer in the form $m\sqrt{n}$ where m and n are integers to be found.

(4)

(a) $f(x) = 3 + 12x - 2x^2 = 3 - 2(x^2 - 6x)$

$= 3 - 2[(x - 3)^2 - 9]$

$= 3 - 2(x - 3)^2 + 18$

$f(x) = 21 - 2(x - 3)^2$ $\quad (a = 21,\ b = 2,\ c = -3)$

(b) $y = f(x) - 7 = -4 + 12x - 2x^2$

$x = 0$: $y = -4$, so R is $(0, -4)$

$y = 0$: $2x^2 - 12x + 4 = 0 \Rightarrow x^2 - 6x + 2 = 0$

$(x - 3)^2 = 7 \Rightarrow x = 3 \pm \sqrt{7}$

P and Q are $(3 - \sqrt{7}, 0)$ and $(3 + \sqrt{7}, 0)$, so $PQ = 2\sqrt{7}$

Area of $PQR = \frac{1}{2} \times PQ \times \text{height} = \frac{1}{2} \times ((3 + \sqrt{7}) - (3 - \sqrt{7})) \times 4$

$= \frac{1}{2} \times 2\sqrt{7} \times 4$

Area $= 4\sqrt{7}$ $\quad (m = 4,\ n = 7)$

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Question 2 continued

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Question 2 continued

Q2

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3.

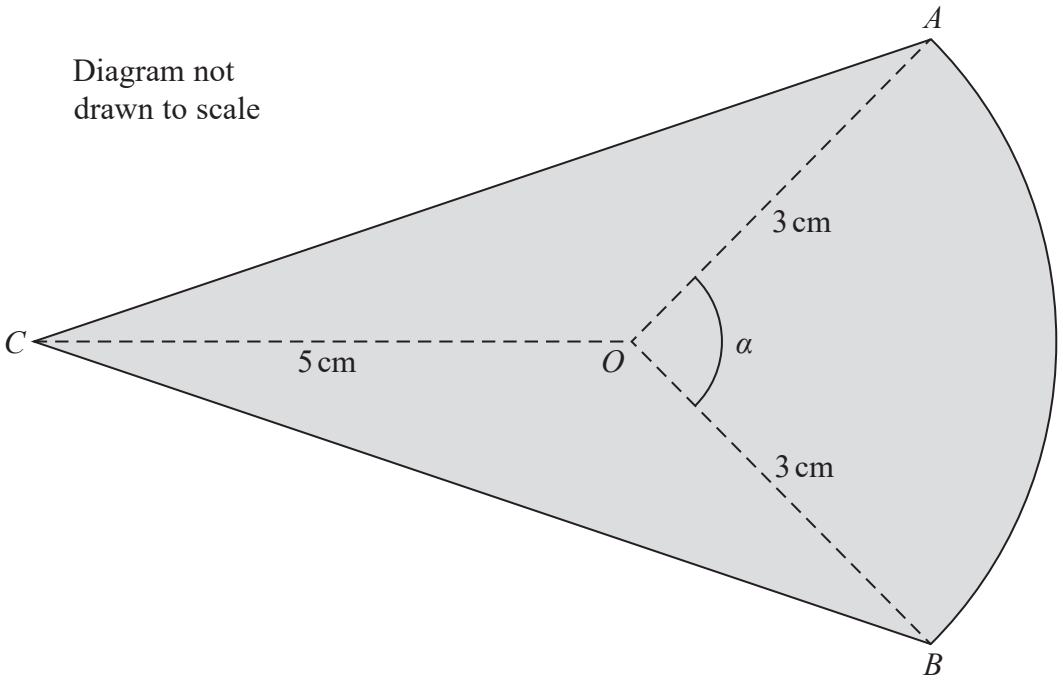


Figure 1

Figure 1 shows the design for a badge.

The design consists of two congruent triangles, AOC and BOC, joined to a sector AOB of a circle centre O.

- Angle $AOB = \alpha$
- $AO = OB = 3\,\text{cm}$
- $OC = 5\,\text{cm}$

Given that the area of sector AOB is $7.2\,\text{cm}^2$

(a) show that $\alpha = 1.6$ radians.

(2)

(b) Hence find

(i) the area of the badge, giving your answer in cm^2 to 2 significant figures,

(ii) the perimeter of the badge, giving your answer in cm to one decimal place.

(8)

(a) Sector area $= \frac{1}{2} r^2\alpha \Rightarrow \frac{1}{2} \times 3^2 \times \alpha = 7.2$

$4.5\alpha = 7.2 \Rightarrow \alpha = \frac{7.2}{4.5} = 1.6 \quad \therefore \alpha = 1.6$ radians

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Question 3 continued

(b) (i) Angle COA $= \frac{1}{2}(2\pi - 1.6) = 2.3415...$ rad

Area COA $= \frac{1}{2} \times 5 \times 3 \times \sin(2.3415...) = 5.3801...$ cm^2

Total area $= 2 \times \frac{1}{2} \times 5 \times 3 \times \sin(2.3415...) + 7.2$

$= 10.7603... + 7.2 = 17.9603...$

Area of badge = 18 cm^2 (2 s.f.)

(ii) Arc AB $= r\alpha = 3 \times 1.6 = 4.8$ cm

$AC^2 = 5^2 + 3^2 - 2 \times 5 \times 3 \times \cos(2.3415...)$

$AC^2 = 34 + 20.9012... = 54.9012... \Rightarrow AC = 7.4095...$ cm

Perimeter $= 2 \times AC +$ arc AB $= 2 \times 7.4095... + 4.8$

$= 19.6191...$

Perimeter of badge = 19.6 cm (1 d.p.)

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Question 3 continued

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Question 3 continued

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Q3

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4. Use algebra to solve the simultaneous equations

$$y - 3x = 4$$
$$x^2 + y^2 + 6x - 4y = 4$$

You must show all stages of your working.

(7)

$y - 3x = 4 \Rightarrow y = 3x + 4$

$x^2 + (3x + 4)^2 + 6x - 4(3x + 4) = 4$

$x^2 + 9x^2 + 24x + 16 + 6x - 12x - 16 = 4$

$10x^2 + 18x - 4 = 0$

$5x^2 + 9x - 2 = 0$

$(5x - 1)(x + 2) = 0$

$x = \frac{1}{5} = 0.2$ or $x = -2$

$x = 0.2$: $y = 3 \times \frac{1}{5} + 4 = 4.6$

$x = -2$: $y = 3 \times (-2) + 4 = -2$

$x = 0.2, y = 4.6$ and $x = -2, y = -2$

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Question 4 continued

Q4

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5. (i)

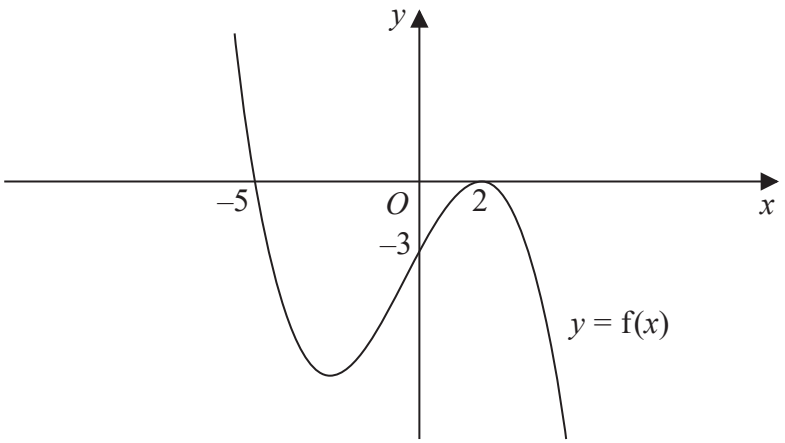


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-5, 0)$ and $(0, -3)$ and touches the x-axis at the point $(2, 0)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 2)$

(b) $y = f(-x)$

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

(6)

(ii)

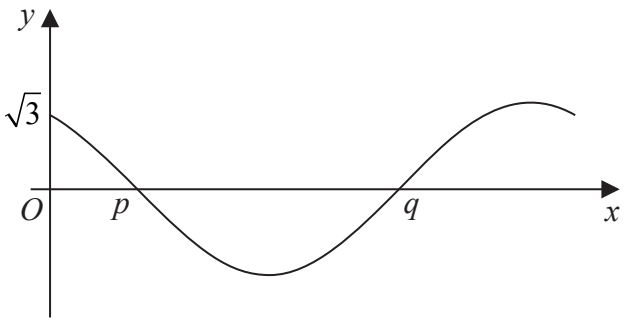


Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = k\cos\left(x + \frac{\pi}{6}\right) \qquad 0 \leqslant x \leqslant 2\pi$$

where k is a constant.

The curve meets the y-axis at the point $(0, \sqrt{3})$ and passes through the points $(p, 0)$ and $(q, 0)$.

Find

(a) the value of k,

(b) the exact value of p and the exact value of q.

(3)

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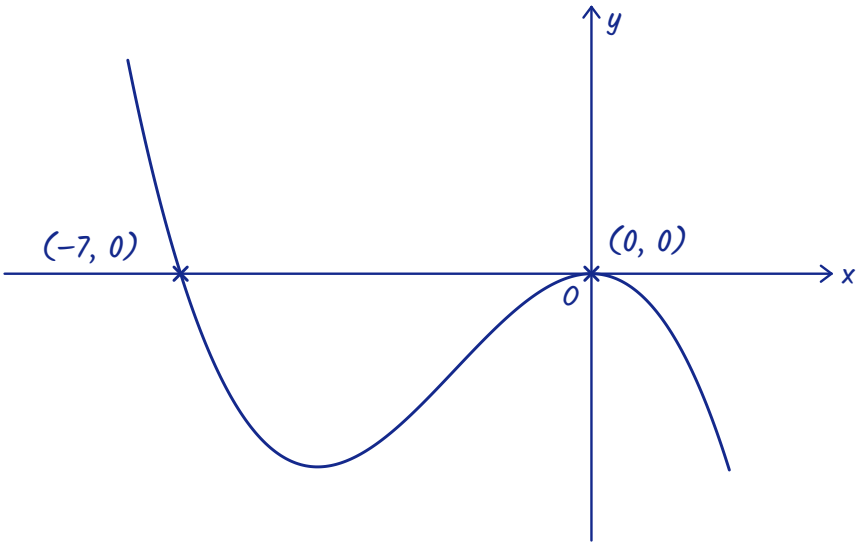
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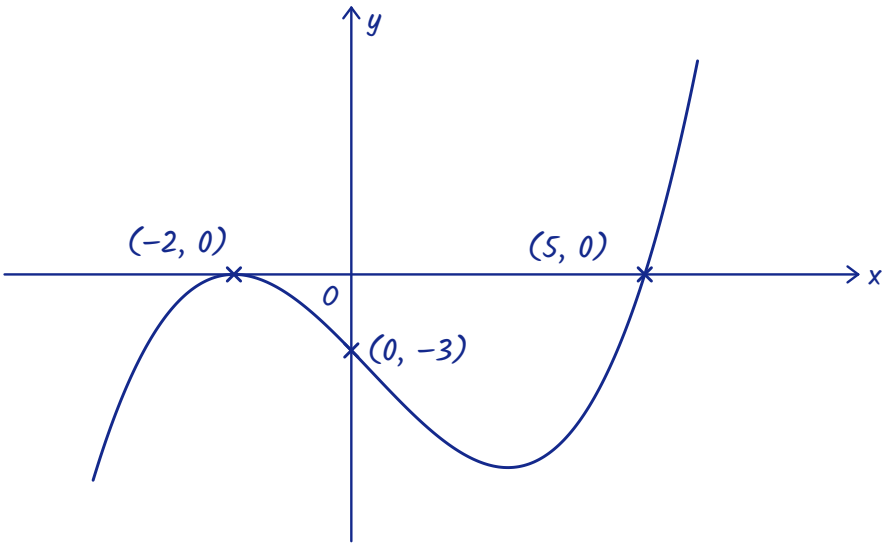
(i)(a)

$y = f(x + 2)$



(b)

$y = f(-x)$



Question 5 continued

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(ii)(a) $x = 0$: $y = k\cos\frac{\pi}{6} = \sqrt{3}$

$k \times \frac{\sqrt{3}}{2} = \sqrt{3} \Rightarrow k = 2$

(b) $2\cos\left(x + \frac{\pi}{6}\right) = 0 \Rightarrow x + \frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$ or $x = \frac{3\pi}{2} - \frac{\pi}{6} = \frac{4\pi}{3}$

$p = \frac{\pi}{3}$, $q = \frac{4\pi}{3}$

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Question 5 continued

Q5

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6. The point A has coordinates $(-4, 11)$ and the point B has coordinates $(8, 2)$.

(a) Find the gradient of the line AB, giving your answer as a fully simplified fraction.

(2)

The point M is the midpoint of AB. The line l passes through M and is perpendicular to AB.

(b) Find an equation for l, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

(4)

The point C lies on l such that the area of triangle ABC is 37.5 square units.

(c) Find the two possible pairs of coordinates of point C.

(5)

(a) Gradient $AB = \frac{2 - 11}{8 - (-4)} = \frac{-9}{12} = -\frac{3}{4}$

(b) $M = \left(\frac{-4 + 8}{2}, \frac{11 + 2}{2}\right) = \left(2, \frac{13}{2}\right)$

Gradient of $l = -1 \div \left(-\frac{3}{4}\right) = \frac{4}{3}$

$y - \frac{13}{2} = \frac{4}{3}(x - 2)$

$6y - 39 = 8x - 16$

$8x - 6y + 23 = 0$

(c) $AB = \sqrt{((8 - (-4))^2 + (2 - 11)^2)} = \sqrt{(144 + 81)} = \sqrt{225} = 15$

Area $ABC = \frac{1}{2} \times AB \times MC = 37.5$ $(MC \perp AB, C \text{ on } l)$

$\frac{1}{2} \times 15 \times MC = 37.5 \Rightarrow MC = 5$

Gradient $\frac{4}{3}$: 3 across, 4 up is a distance $\sqrt{(3^2 + 4^2)} = 5$

$C = \left(2 + 3, \frac{13}{2} + 4\right)$ or $C = \left(2 - 3, \frac{13}{2} - 4\right)$

$C = (5, 10.5)$ or $C = (-1, 2.5)$

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Question 6 continued

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7. The curve C has equation

$$y = \frac{1}{2 - x}$$

(a) Sketch the graph of C. On your sketch you should show the coordinates of any points of intersection with the coordinate axes and state clearly the equations of any asymptotes.

(3)

The line l has equation $y = 4x + k$, where k is a constant.

Given that l meets C at two distinct points,

(b) show that

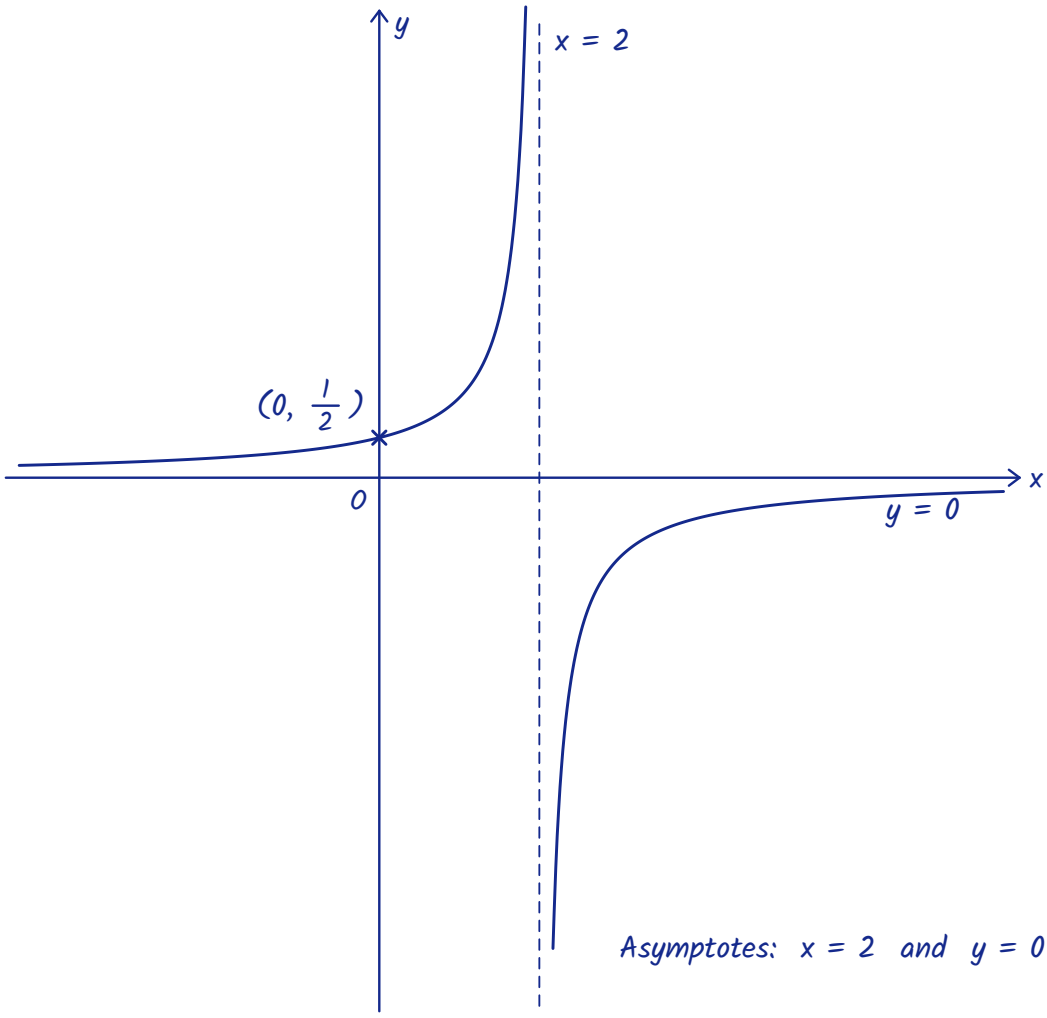
$$k^2 + 16k + 48 > 0$$

(4)

(c) Hence find the range of possible values for k.

(4)

(a)



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Question 7 continued

(b) $4x + k = \frac{1}{2 - x} \Rightarrow (4x + k)(2 - x) = 1$

$8x - 4x^2 + 2k - kx = 1$

$4x^2 + (k - 8)x + 1 - 2k = 0$

Two distinct points $\Rightarrow b^2 - 4ac > 0, \quad a = 4, \quad b = k - 8, \quad c = 1 - 2k$

$(k - 8)^2 - 4 \times 4 \times (1 - 2k) > 0$

$k^2 - 16k + 64 - 16 + 32k > 0$

$k^2 + 16k + 48 > 0$ as required

(c) $k^2 + 16k + 48 = 0 \Rightarrow (k + 12)(k + 4) = 0$

$k = -12$ or $k = -4$

Positive quadratic, so > 0 outside the roots:

$k < -12$ or $k > -4$

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Question 7 continued

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Question 7 continued

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8. The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{dy}{dx} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

(a) $y = (x - 2)(x - 4)^2 = (x - 2)(x^2 - 8x + 16)$

$y = x^3 - 8x^2 + 16x - 2x^2 + 16x - 32$

$y = x^3 - 10x^2 + 32x - 32$

$\frac{dy}{dx} = 3x^2 - 20x + 32$ as required

(b) $x = 6$: $y = (6 - 2)(6 - 4)^2 = 4 \times 4 = 16$

$\frac{dy}{dx} = 3(6)^2 - 20(6) + 32 = 108 - 120 + 32 = 20$

$y - 16 = 20(x - 6)$

$y = 20x - 104$

(c) Parallel $\Rightarrow$ same gradient: $3x^2 - 20x + 32 = 20$

$3x^2 - 20x + 12 = 0$

$(3x - 2)(x - 6) = 0 \Rightarrow x = 6$ (this is l_1) or $x = 2/3$

$\alpha = \frac{2}{3}$

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Question 8 continued

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9. A curve with equation $y = f(x)$ passes through the point (9, 10).

Given that

$$f'(x) = 27x^2 - \frac{21x^3 - 5x}{2\sqrt{x}} \qquad x > 0$$

find $f(x)$, fully simplifying each term.

(6)

$\frac{21x^3 - 5x}{2\sqrt{x}} = \frac{21x^3}{2x^{1/2}} - \frac{5x}{2x^{1/2}} = \frac{21}{2}x^{5/2} - \frac{5}{2}x^{1/2}$

$f'(x) = 27x^2 - \frac{21}{2}x^{5/2} + \frac{5}{2}x^{1/2}$

$f(x) = \frac{27x^3}{3} - \frac{21}{2} \times \frac{x^{7/2}}{7/2} + \frac{5}{2} \times \frac{x^{3/2}}{3/2} + c$

$f(x) = 9x^3 - 3x^{7/2} + \frac{5}{3}x^{3/2} + c$

$f(9) = 10: \quad 9(9)^3 - 3(9)^{7/2} + \frac{5}{3}(9)^{3/2} + c = 10$

$6561 - 6561 + 45 + c = 10 \quad \Rightarrow \quad c = -35$

$f(x) = 9x^3 - 3x^{7/2} + \frac{5}{3}x^{3/2} - 35$

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Question 9 continued

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