Pearson Edexcel International Advanced Level

Tuesday 19 May 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Write your answers in the D1 answer book for this paper.

1. The table below shows the distances, in metres, between six vertices, A, B, C, D, E and F, in a network.

	A	**B**	**C**	**D**	**E**	**F**
A	–	18	23	17	28	19
B	18	–	20	11	–	24
C	23	20	–	–	25	13
D	17	11	–	–	–	22
E	28	–	25	–	–	–
F	19	24	13	22	–	–

(a) Draw the weighted network using the vertices given in Diagram 1 in the answer book.

(2)

(b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the edges in the order that you consider them and state whether you are adding them to your minimum spanning tree.

(3)

(c) Draw the minimum spanning tree on Diagram 2 in the answer book and state its total weight.

(2)

(Total 7 marks)

2. (a) (i) Describe how to carry out the first pass of a bubble sort when it is used to sort a list of n numbers into ascending order.

(ii) Write down the circumstances under which a bubble sort stops.

(4)

A bubble sort, starting at the left-hand end of the list, is used to sort a list of ten numbers into ascending order. After a number of passes the list reads

0.9 1.2 1.5 0.5 1.4 1.1 0.7 1.7 2.2 3.2

(b) Determine the maximum number of passes that could have taken place on this list. You must give a reason for your answer.

(2)

(c) Complete the bubble sort to produce a list of the numbers in ascending order. You only need to give the state of the list after each complete pass.

(4)

(d) Use the first-fit decreasing bin packing algorithm to determine how the ten numbers listed above can be packed into bins of size 4

(3)

(Total 13 marks)

3. The table below shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	**B**	**C**	**D**	**E**	**F**
A	–	57	76	59	72	65
B	57	–	67	80	66	76
C	76	67	–	71	83	80
D	59	80	71	–	77	78
E	72	66	83	77	–	69
F	65	76	80	78	69	–

Mei must visit each town at least once. She will start and finish at A and wishes her route to minimise the total distance she will travel.

(a) Starting with the minimum spanning tree in the answer book, use the shortcut method to find an upper bound below 520 km for Mei's route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Mei's route.

(2)

(c) Starting by deleting E, and all of its arcs, find a lower bound for the length of Mei's route. Make your method clear.

(3)

(Total 7 marks)

4. (a) Draw the activity network described by the precedence table below, using activity on arc. Use dummies only where necessary.

(5)

Activity	Immediately preceding activities
A	–
B	–
C	A
D	A, B
E	C, D
F	D
G	C
H	G
I	G
J	E, F, I
K	F

Given that K is a critical activity,

(b) state which other activities must also be critical.

(1)

Given instead that all activities shown in the precedence table have the same duration and K is not necessarily critical,

(c) state the critical path for the network.

(1)

(Total 7 marks)

5.

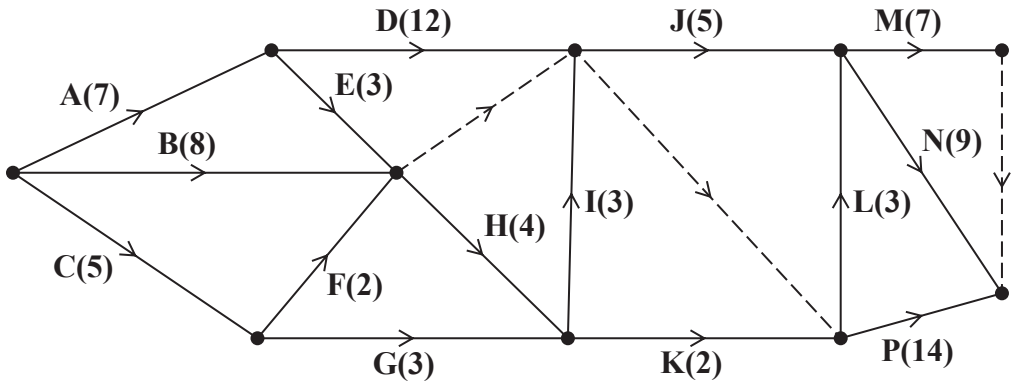


Figure 1

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the corresponding activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(b) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(2)

(c) Schedule the activities on Grid 1 in the answer book using the minimum number of workers so that the project is completed in the minimum time.

(4)

Additional resources become available, which can shorten the duration of one of activities D, G or P by one day.

(d) Determine which of these three activities should be shortened to allow the project to be completed in the minimum time. You must give reasons for your answer.

(2)

(Total 12 marks)

6.

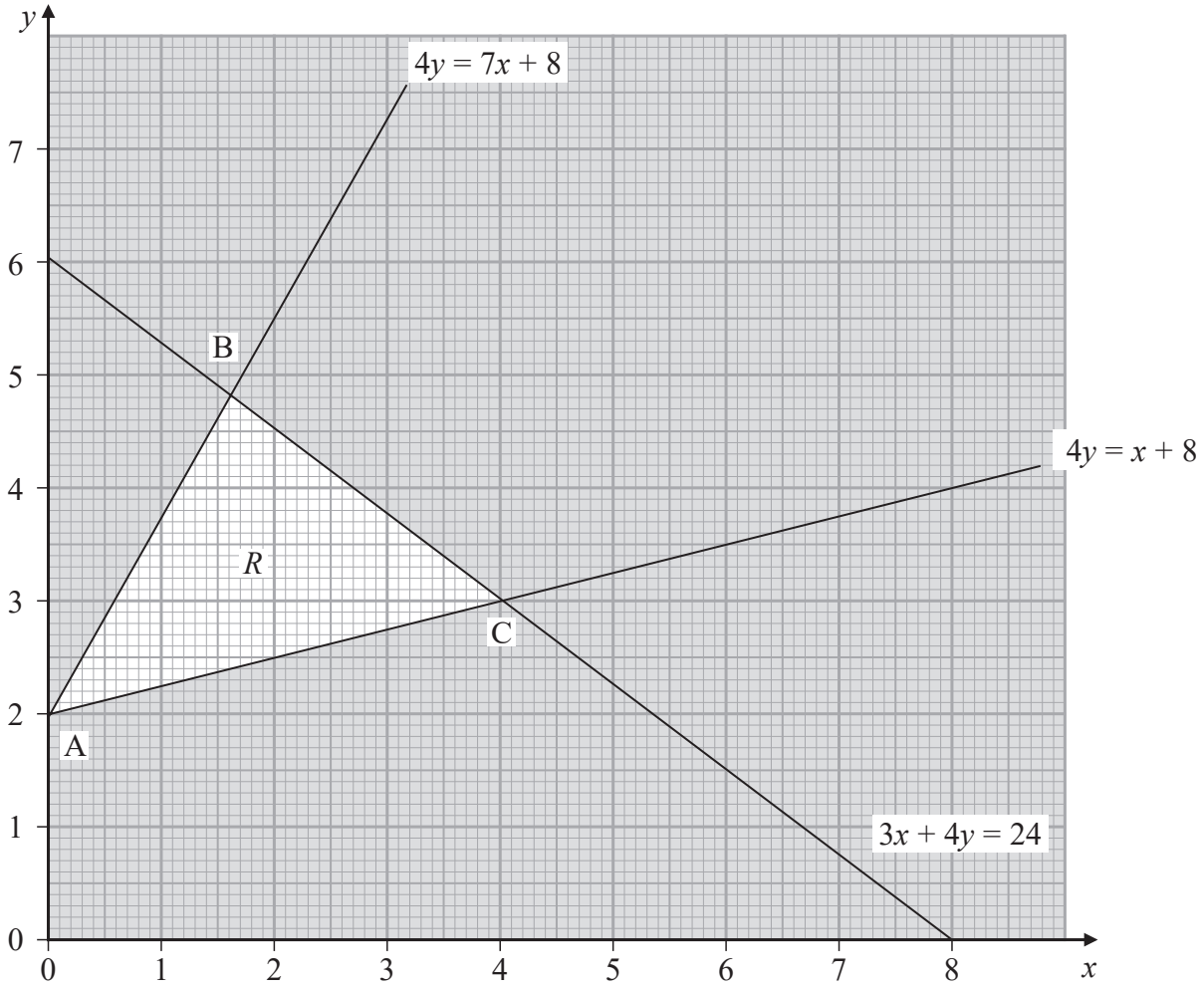


Figure 2

The graph in Figure 2 is being used to solve a linear programming problem in x and y. The three constraints have been drawn on the graph and the rejected regions have been shaded out. The three vertices of the feasible region R are labelled A, B and C.

(a) Determine the inequalities that define R.

(2)

The objective function, P, is given by

$$P = ax + by$$

where a and b are positive constants.

The minimum value of P is 8 and the maximum value of P occurs at C.

(b) Find the range of possible values of a. You must make your method clear.

(5)

(Total 7 marks)

7.

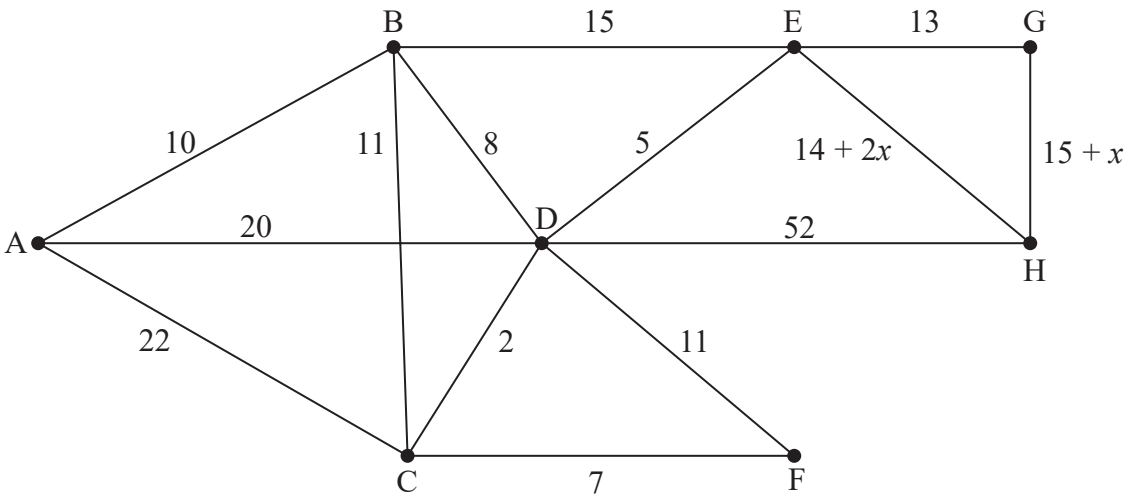


Figure 3

[*The total weight of the network is* $205 + 3x$]

Figure 3 represents a network of roads. The number on each arc represents the time taken, in minutes, to drive along the corresponding road.

Malcolm wishes to minimise the time spent driving from his home at A to his office at H. The delays from roadworks on two of the roads leading in to H vary daily, and so the time taken to drive along these roads is expressed in terms of x, where x is fixed for any given day and $x > 0$

(a) Use Dijkstra's algorithm to find the possible routes that minimise the driving time from A to H. State the length of each route, leaving your answer in terms of x where necessary.

(7)

On Monday, Malcolm needs to check each road. He must travel along each road at least once. He must start and finish at H and minimise the total time taken for his inspection route.

Malcolm finds that his minimum duration inspection route requires him to traverse exactly four roads twice and the total time it takes to complete his inspection route is 307 minutes.

(b) Calculate the minimum time taken for Malcolm to travel from A to H on Monday. You must make your method and working clear.

(4)

(Total 11 marks)

8. A bakery makes three types of doughnut. These are ring, jam and custard. The bakery has the following constraints on the number of doughnuts it must make each day.

- The total number of doughnuts made must be at least 200
- They must make at least three times as many ring doughnuts as jam doughnuts
- At most 70% of the doughnuts the bakery makes must be ring doughnuts
- At least a fifth of the doughnuts the bakery makes must be jam doughnuts

It costs 8 pence to make each ring doughnut, 10 pence to make each jam doughnut and 14 pence to make each custard doughnut. The bakery wants to minimise the total daily costs of making the required doughnuts.

Let x represent the number of ring doughnuts, let y represent the number of jam doughnuts and let z represent the number of custard doughnuts the bakery makes each day.

(a) Formulate this as a linear programming problem stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(6)

On a given day, instead of making **at least** 200 doughnuts, the bakery requires that **exactly** 200 doughnuts are made. Furthermore, the bakery decides to make the minimum number of jam doughnuts which satisfy all the remaining constraints.

Given that the bakery still wants to minimise the total cost of making the required doughnuts, use algebra to

(b) (i) calculate the number of each type of doughnut the bakery will make on that day,

(ii) calculate the corresponding total cost of making all the doughnuts.

(5)

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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Mathematics

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Decision Mathematics D1

Answer Book
Do not return the question paper with the answer book.

Total Marks

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1.

	A	B	C	D	E	F
A	–	18	23	17	28	19
B	18	–	20	11	–	24
C	23	20	–	–	25	13
D	17	11	–	–	–	22
E	28	–	25	–	–	–
F	19	24	13	22	–	–

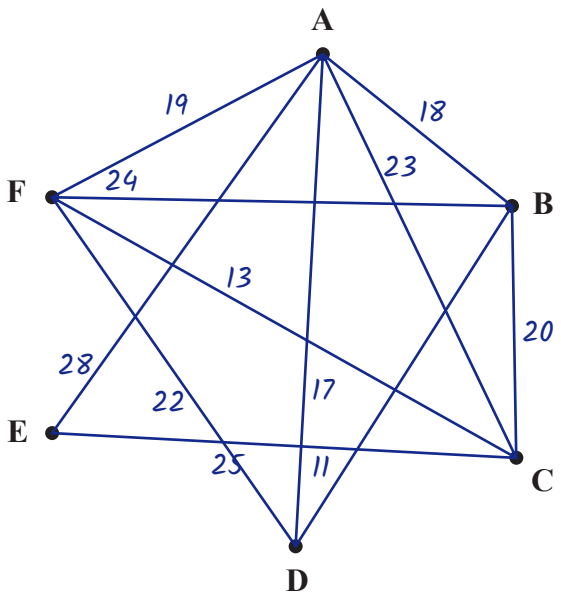


Diagram 1

(b) Kruskal's algorithm, arcs in order of weight:

BD (11) add

CF (13) add

AD (17) add

AB (18) reject (would make cycle A – B – D)

AF (19) add

BC (20) reject (would make cycle)

DF (22) reject (would make cycle)

AC (23) reject (would make cycle)

BF (24) reject (would make cycle)

CE (25) add

All six vertices are now connected (5 arcs), so stop.

Arcs in MST: BD, CF, AD, AF, CE

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Question 1 continued

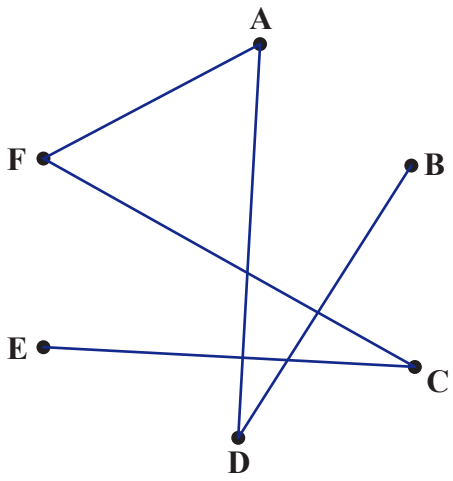


Diagram 2

Weight of minimum spanning tree 11 + 13 + 17 + 19 + 25 = 85 metres

Q1

(Total 7 marks)

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2.

(a)(i) Compare the 1st number with the 2nd number and swap them if the 1st is larger than the 2nd. Then compare the 2nd number with the 3rd and swap if the 2nd is larger than the 3rd. Continue comparing each adjacent pair in turn in this way until the last two numbers (the (n – 1)th and nth) have been compared.

(ii) The bubble sort stops when a complete pass is made with no swaps, or when there is only one number left to consider.

(b) Maximum number of passes = 3

Only the three largest values (1.7, 2.2 and 3.2) are in their correct final positions. After each pass at least one more of the largest values is fixed, so 4 passes would have put 1.5 in place.

(c) Given:	0.9	1.2	1.5	0.5	1.4	1.1	0.7	1.7	2.2	3.2
Pass 1:	0.9	1.2	0.5	1.4	1.1	0.7	1.5	1.7	2.2	3.2
Pass 2:	0.9	0.5	1.2	1.1	0.7	1.4	1.5	1.7	2.2	3.2
Pass 3:	0.5	0.9	1.1	0.7	1.2	1.4	1.5	1.7	2.2	3.2
Pass 4:	0.5	0.9	0.7	1.1	1.2	1.4	1.5	1.7	2.2	3.2
Pass 5:	0.5	0.7	0.9	1.1	1.2	1.4	1.5	1.7	2.2	3.2
Pass 6:	0.5	0.7	0.9	1.1	1.2	1.4	1.5	1.7	2.2	3.2

No swaps in pass 6, so the list is sorted:

0.5 0.7 0.9 1.1 1.2 1.4 1.5 1.7 2.2 3.2

(d) Sorted into descending order:

3.2 2.2 1.7 1.5 1.4 1.2 1.1 0.9 0.7 0.5

First-fit decreasing, bins of size 4:

Bin 1: 3.2 0.7

Bin 2: 2.2 1.7

Bin 3: 1.5 1.4 1.1

Bin 4: 1.2 0.9 0.5

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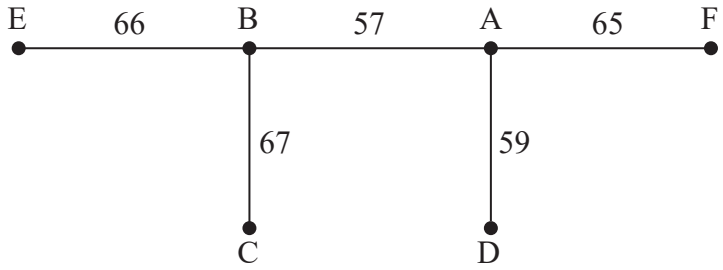
Q2

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3.

	A	B	C	D	E	F
A	–	57	76	59	72	65
B	57	–	67	80	66	76
C	76	67	–	71	83	80
D	59	80	71	–	77	78
E	72	66	83	77	–	69
F	65	76	80	78	69	–



The total weight of the minimum spanning tree is 314 km

(a) Initial upper bound = 2 × 314 = 628 km

Shortcut: use EF (69) instead of E – B – A – F (66 + 57 + 65)

Route: A – D – A – F – E – B – C – B – A

Upper bound = 628 – 66 – 57 – 65 + 69 = 509 km

(b) Nearest neighbour starting at A:

A – B (57), B – E (66), E – F (69), F – D (78),

D – C (71), C – A (76)

Route: A – B – E – F – D – C – A

Upper bound = 57 + 66 + 69 + 78 + 71 + 76 = 417 km

(c) Delete E. RMST on A, B, C, D, F (Prim's from A):

AB (57), AD (59), AF (65), BC (67)

Length of RMST = 57 + 59 + 65 + 67 = 248

Two least arcs from E: EB (66) and EF (69)

Lower bound = 248 + 66 + 69 = 383 km

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Question 3 continued

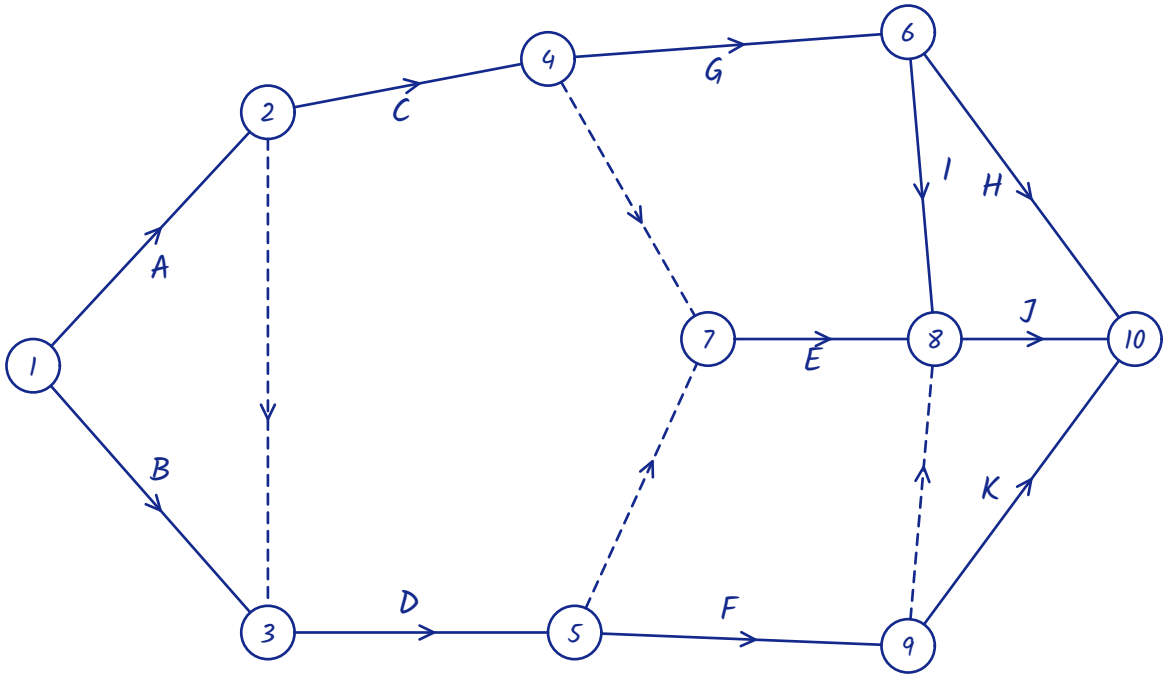
	A	B	C	D	E	F
A	–	57	76	59	72	65
B	57	–	67	80	66	76
C	76	67	–	71	83	80
D	59	80	71	–	77	78
E	72	66	83	77	–	69
F	65	76	80	78	69	–

Q3

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4.



Dashed arcs are dummies

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Question 4 continued

(b) D and F must also be critical

(K can only start after F, and F can only start after D)

(c) All durations equal, so the critical path is the path with the most activities:

Critical path: A – C – G – I – J

Q4

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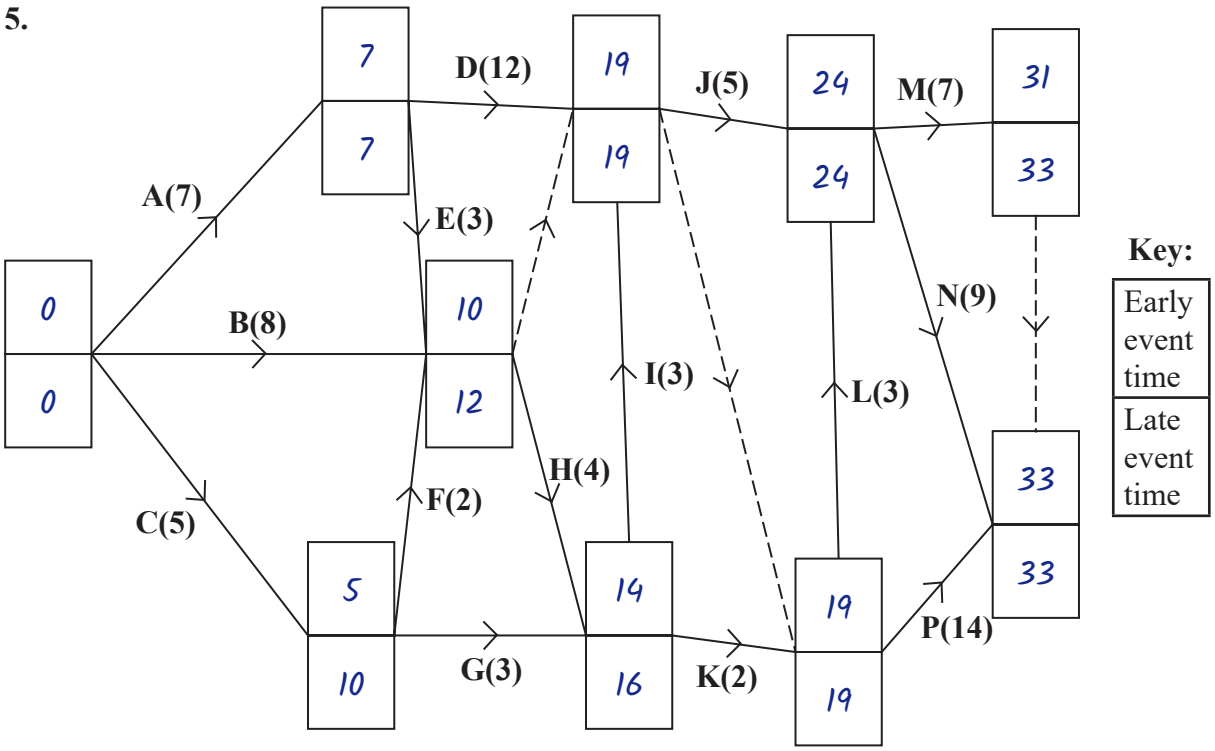


Diagram 1

(b) Sum of durations = 7 + 8 + 5 + 12 + 3 + 2 + 3 + 4 + 3 + 5
+ 2 + 3 + 7 + 9 + 14 = 87

Lower bound = 87 ÷ 33 = 2.6363... ⇒ 3 workers

(c) See Grid 1 (3 workers, finishing at 33)

(d) G is not critical: total float = 16 − 5 − 3 = 8 days, so shortening G would not reduce the completion time.

D and P are both critical. Critical paths (length 33):

A − D − P and A − D − J − N

D lies on both critical paths, but P is only on one, so shortening P leaves A − D − J − N still taking 33 days.

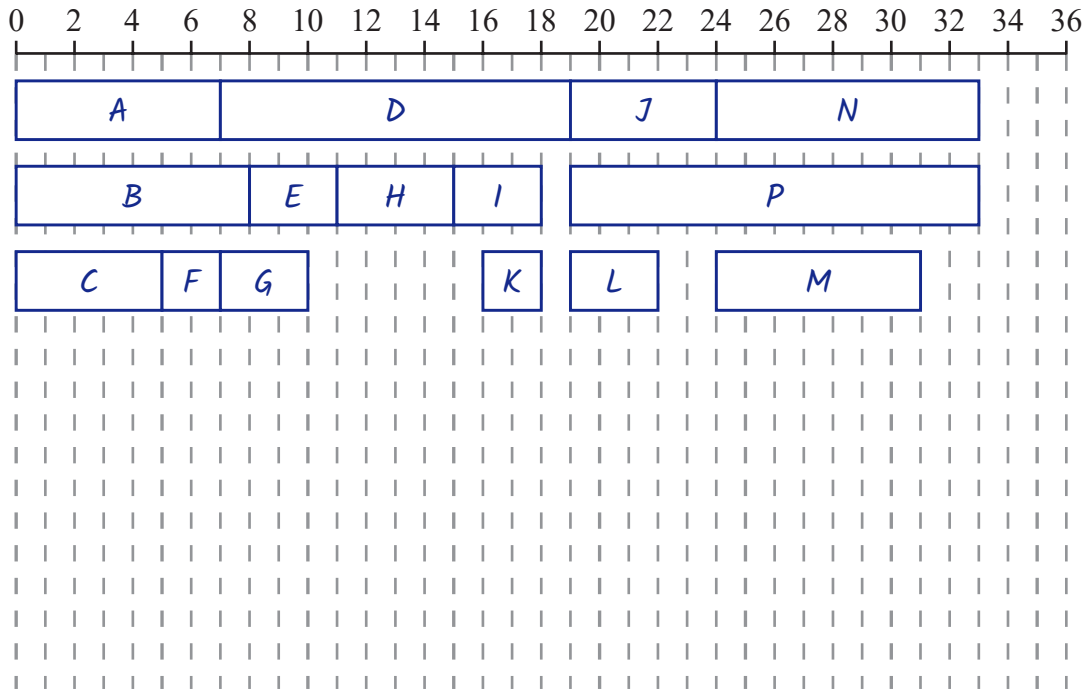
∴ Activity D should be shortened (project then takes 32 days)

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Question 5 continued



Grid 1

Q5

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6.

(a) $4y \leqslant 7x + 8$, $4y \geqslant x + 8$, $3x + 4y \leqslant 24$

(b) $a, b > 0$, so the minimum value of P is at A(0, 2):

$P = 0a + 2b = 8 \Rightarrow b = 4$

B: $4y = 7x + 8$ and $3x + 4y = 24$

$3x + 7x + 8 = 24 \Rightarrow 10x = 16 \Rightarrow x = 8/5,\ y = 24/5$

C: $4y = x + 8$ and $3x + 4y = 24$

$3x + x + 8 = 24 \Rightarrow 4x = 16 \Rightarrow x = 4,\ y = 3$

At C(4, 3): $P = 4a + 4(3) = 4a + 12$

At B(8/5, 24/5): $P = (8/5)a + 4(24/5) = (8/5)a + 96/5$

Maximum at C $\Rightarrow 4a + 12 > (8/5)a + 96/5$

$20a + 60 > 8a + 96$

$12a > 36$

$a > 3$

(At A, $P = 8 < 4a + 12$ for all $a > 0$, so A is never the maximum.)

$\therefore\ a > 3$

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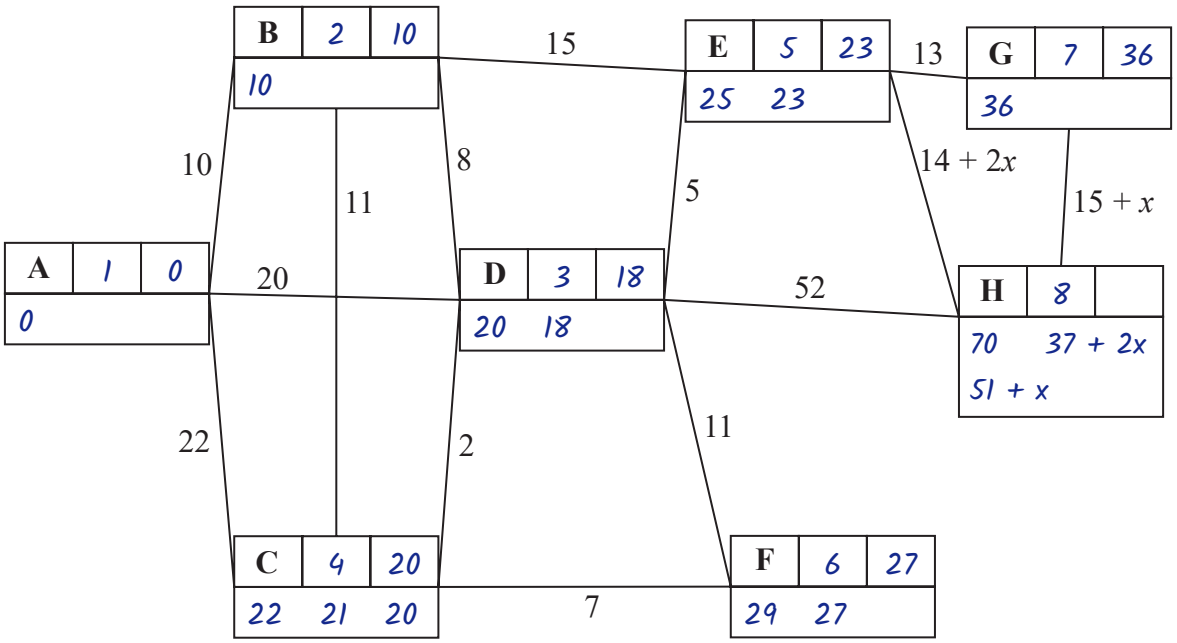
Question 6 continued

Q6

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7.



Key:

Vertex	Order of labelling	Final value
Working values		

(a) Working values at H: 70 (via DH), 37 + 2x (via EH),
51 + x (via GH). Possible shortest routes from A to H:

A – B – D – H length 70

A – B – D – E – H length 37 + 2x

A – B – D – E – G – H length 51 + x

(ABDEH is shortest when x < 14, ABDEGH when 14 < x < 19,
ABDH when x > 19)

(b) Orders of vertices: A 3, B 4, C 4, D 6, E 4, F 2, G 2, H 3
The only odd vertices are A and H, so the inspection route must
repeat the shortest path from A to H. Exactly four roads are
repeated, so this path is A – B – D – E – H (37 + 2x).

(205 + 3x) + (37 + 2x) = 307

5x + 242 = 307 ⇒ 5x = 65 ⇒ x = 13

Monday: ABDEH = 37 + 2(13) = 63, ABDEGH = 51 + 13 = 64,
ABDH = 70

Minimum time from A to H on Monday = 63 minutes

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Question 7 continued

Q7

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8.

(a) Minimise $P = 8x + 10y + 14z$

subject to:

$x + y + z \geqslant 200$

$x \geqslant 3y$

$x \leqslant 0.7(x + y + z) \Rightarrow 10x \leqslant 7x + 7y + 7z \Rightarrow 3x \leqslant 7y + 7z$

$y \geqslant (1/5)(x + y + z) \Rightarrow 5y \geqslant x + y + z \Rightarrow x + z \leqslant 4y$

$x, y, z \geqslant 0$

(b)(i) $x + y + z = 200 \Rightarrow z = 200 - x - y$

$3x \leqslant 7y + 7z$: $3x \leqslant 7y + 7(200 - x - y) = 1400 - 7x$

$10x \leqslant 1400 \Rightarrow x \leqslant 140$

$x + z \leqslant 4y$: $x + (200 - x - y) \leqslant 4y$

$200 \leqslant 5y \Rightarrow y \geqslant 40$

Minimum number of jam doughnuts: $y = 40$

Then $x \geqslant 3y = 120$ and $x \leqslant 140$, $z = 200 - x - 40 = 160 - x$

Cost $= 8x + 10(40) + 14(160 - x) = 2640 - 6x$

so cost is least when x is greatest: $x = 140$

$z = 200 - 140 - 40 = 20$

140 ring doughnuts, 40 jam doughnuts, 20 custard doughnuts

(ii) Cost $= 8(140) + 10(40) + 14(20) = 1120 + 400 + 280$

$= 1800$ pence $= £18$

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Question 8 continued

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Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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