Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Wednesday 6 November 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. A curve C has equation $y = 2x^2(x - 5)$

(a) Find, using calculus, the x coordinates of the stationary points of C.

(4)

(b) Hence find the values of x for which y is increasing.

(2)

(a) $y = 2x^2(x - 5) = 2x^3 - 10x^2$

$\frac{dy}{dx} = 6x^2 - 20x$

Stationary points: $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 20x = 0$

$2x(3x - 10) = 0$

$x = 0$ or $x = \frac{10}{3}$

(b) y increasing when $\frac{dy}{dx} > 0$: $2x(3x - 10) > 0$

Positive quadratic, so outside the roots:

$x \leq 0$ or $x \geq \frac{10}{3}$

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Question 1 continued

Q1

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2. The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032

(3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000

(3)

(a) Increase of 2% each year $\Rightarrow$ $r = 1.02$, $a = 25\,000$

2032 is 13 years after 2019, so it is the 14th term:

$u_{14} = ar^{13} = 25\,000 \times 1.02^{13}$

$= 32\,340.16...$

Adult population at start of 2032 $\approx$ 32 340

(b) Each year the money given = 5 × population, so $a = 5 \times 25\,000 = 125\,000$

From 2019 to 2032 inclusive is 14 years: $n = 14$, $r = 1.02$

$$S_{14} = \frac{a(r^n - 1)}{r - 1} = \frac{125\,000(1.02^{14} - 1)}{1.02 - 1}$$

$= 1\,996\,742.27...$

Total given $\approx$ £1 997 000

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Question 2 continued

Q2

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3. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$\left(1+\frac{x}{4}\right)^{12}$$

giving each coefficient in its simplest form.

(3)

(b) Find the term independent of x in the expansion of

$$\left(\frac{x^2+8}{x^5}\right)\left(1+\frac{x}{4}\right)^{12}$$

(3)

(a) $\left(1+\frac{x}{4}\right)^{12} = 1 + 12\left(\frac{x}{4}\right) + \frac{12 \times 11}{2!}\left(\frac{x}{4}\right)^2 + \frac{12 \times 11 \times 10}{3!}\left(\frac{x}{4}\right)^3 + \ldots$

$= 1 + 12 \times \frac{x}{4} + 66 \times \frac{x^2}{16} + 220 \times \frac{x^3}{64} + \ldots$

$= 1 + 3x + \frac{33}{8}x^2 + \frac{55}{16}x^3 + \ldots$

(b) Need the x^3 term and the x^5 term of the expansion:

x^5 term: ${}^{12}C_5\left(\frac{x}{4}\right)^5 = 792 \times \frac{x^5}{1024} = \frac{99}{128}x^5$

$\left(\frac{x^2+8}{x^5}\right)(\ldots)$: term independent of $x = \frac{x^2}{x^5} \times \frac{55}{16}x^3 + \frac{8}{x^5} \times \frac{99}{128}x^5$

$= \frac{55}{16} + \frac{99}{16} = \frac{154}{16}$

$= \frac{77}{8}$

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Question 3 continued

Q3

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4. $f(x) = (x - 3)(3x^2 + x + a) - 35$ where a is a constant

(a) State the remainder when $f(x)$ is divided by $(x - 3)$.

(1)

Given $(3x - 2)$ is a factor of $f(x)$,

(b) show that $a = -17$

(2)

(c) Using algebra and showing each step of your working, fully factorise $f(x)$.

(5)

(a) $f(3) = (3 - 3)(27 + 3 + a) - 35 = 0 - 35$

Remainder $= -35$

(b) $(3x - 2)$ is a factor $\Rightarrow f\left(\frac{2}{3}\right) = 0$

$\left(\frac{2}{3} - 3\right)\left(3\left(\frac{2}{3}\right)^2 + \frac{2}{3} + a\right) - 35 = 0$

$-\frac{7}{3}\left(\frac{4}{3} + \frac{2}{3} + a\right) = 35$

$-\frac{7}{3}(2 + a) = 35 \Rightarrow 2 + a = -15$

$a = -17$ as required

(c) $f(x) = (x - 3)(3x^2 + x - 17) - 35$

$= 3x^3 + x^2 - 17x - 9x^2 - 3x + 51 - 35$

$= 3x^3 - 8x^2 - 20x + 16$

$(3x - 2)$ is a factor: $3x^3 - 8x^2 - 20x + 16 = (3x - 2)(x^2 + px - 8)$

Compare x^2 terms: $-8 = 3p - 2 \Rightarrow p = -2$

Check x terms: $-24 - 2p = -24 + 4 = -20$ ✓

$f(x) = (3x - 2)(x^2 - 2x - 8)$

$f(x) = (3x - 2)(x + 2)(x - 4)$

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Question 4 continued

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Question 4 continued

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5. (a) Given $0 < a < 1$, sketch the curve with equation

$$y = a^x$$

showing the coordinates of the point at which the curve crosses the y-axis.

(2)

x	2	2.5	3	3.5	4
y	4.25	6.427	9.125	12.34	16.06

The table above shows corresponding values of x and y for $y = x^2 + \left(\frac{1}{2}\right)^x$

The values of y are given to 4 significant figures as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^4 \left(x^2 + \left(\frac{1}{2}\right)^x\right) \mathrm{d}x$

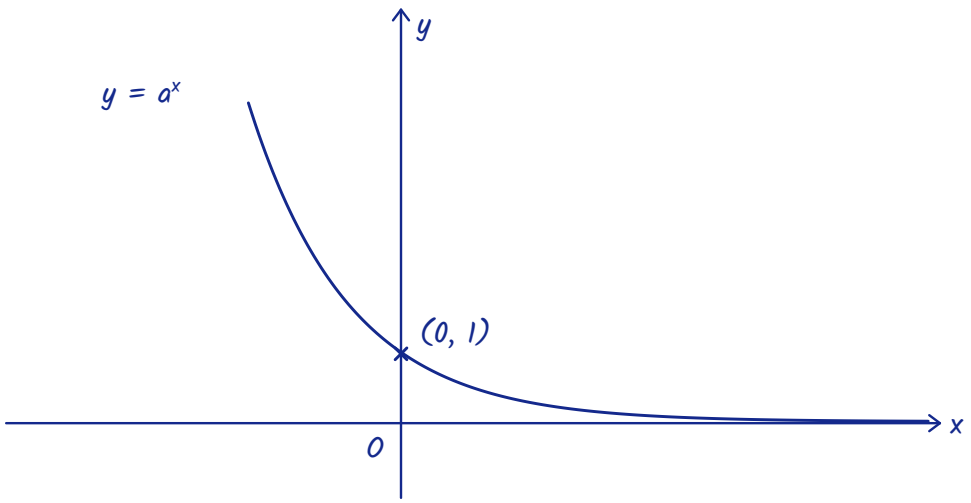
(3)

Using your answer to part (b) and making your method clear, estimate

(c) $\int_2^4 \left(x(x-3) + \left(\frac{1}{2}\right)^x\right) \mathrm{d}x$

(2)

(a)



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Question 5 continued

(b) $h = 0.5$

$$\int_2^4 \left(x^2 + \left(\frac{1}{2}\right)^x\right) dx \approx \frac{0.5}{2}\{4.25 + 16.06 + 2(6.427 + 9.125 + 12.34)\}$$

$$= 0.25 \times \{20.31 + 55.784\} = 0.25 \times 76.094$$

$$= 19.0235 \approx 19.0$$

(c) $x(x-3) + \left(\frac{1}{2}\right)^x = x^2 + \left(\frac{1}{2}\right)^x - 3x$

$$\int_2^4 \left(x(x-3) + \left(\frac{1}{2}\right)^x\right) dx = \int_2^4 \left(x^2 + \left(\frac{1}{2}\right)^x\right) dx - \int_2^4 3x\, dx$$

$$\int_2^4 3x\, dx = \left[\frac{3x^2}{2}\right]_2^4 = 24 - 6 = 18$$

So $\int_2^4 \left(x(x-3) + \left(\frac{1}{2}\right)^x\right) dx \approx 19.0235 - 18 = 1.0235 \approx 1.02$



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Question 5 continued

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Question 5 continued

Q5

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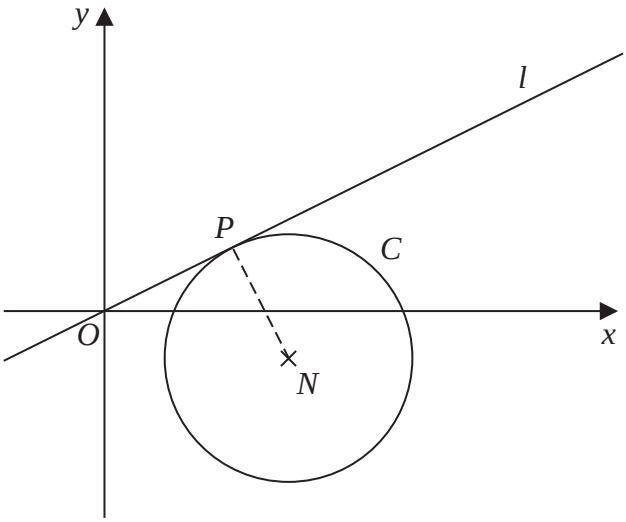


Figure 1

Figure 1 shows a sketch of a circle C with centre $N(4, -1)$.

The line l with equation $y = \frac{1}{2}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) the equation of C.

(5)

(a) Gradient of $l = \frac{1}{2}$; PN is perpendicular to l, so gradient of PN = -2

Through $N(4, -1)$: $y + 1 = -2(x - 4)$

$y + 1 = -2x + 8$

$y = -2x + 7$

(b) P is where l meets PN: $\frac{1}{2}x = -2x + 7 \Rightarrow \frac{5}{2}x = 7$

$x = \frac{14}{5}$, $y = \frac{1}{2} \times \frac{14}{5} = \frac{7}{5}$ $P\left(\frac{14}{5}, \frac{7}{5}\right)$

$r^2 = PN^2 = \left(4 - \frac{14}{5}\right)^2 + \left(-1 - \frac{7}{5}\right)^2$

$= \left(\frac{6}{5}\right)^2 + \left(-\frac{12}{5}\right)^2 = \frac{36}{25} + \frac{144}{25} = \frac{180}{25} = \frac{36}{5}$

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Question 6 continued

C: $(x - 4)^2 + (y + 1)^2 = \frac{36}{5}$

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Question 6 continued

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Question 6 continued

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7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$ **(2)**

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$ **(2)**

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$ **(3)**

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right) = \log_a a^{1/2} - \log_a b = \frac{1}{2}\log_a a - \log_a b$

$= \frac{1}{2} - k$

(ii) $\frac{\log_a a^2 b}{\log_a b^3} = \frac{\log_a a^2 + \log_a b}{3\log_a b} = \frac{2\log_a a + \log_a b}{3\log_a b}$

$= \frac{2 + k}{3k}$

(iii) $\log_a b^n = n\log_a b = nk$, so $k + \log_a b^n = k + nk = (n + 1)k$

$\sum_{n=1}^{50}(k + nk) = 2k + 3k + 4k + \ldots + 51k$

Arithmetic series with $a = 2k$, $d = k$, $n = 50$

$S_{50} = \frac{50}{2}(2 \times 2k + 49k) = 25 \times 53k$

$= 1325k$

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Question 7 continued

Q7

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8. Solutions relying on calculator technology are not acceptable in this question.

(i)

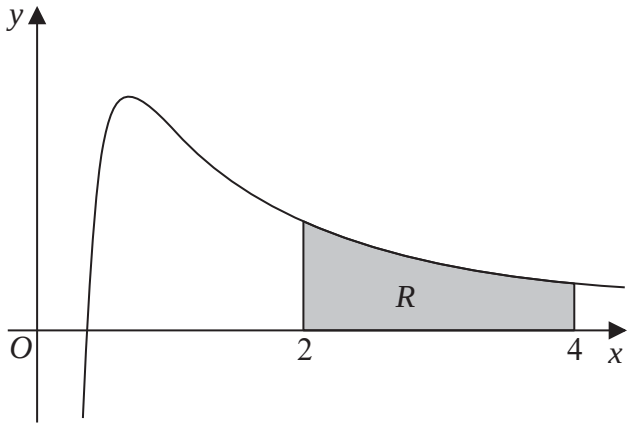


Figure 2

Figure 2 shows a sketch of part of a curve with equation

$$y = \frac{8\sqrt{x} - 5}{2x^2} \qquad x > 0$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

Find the exact area of R.

(5)

(ii) Find the value of the constant k such that

$$\int_{-3}^{6} \left(\frac{1}{2}x^2 + k \right) dx = 55$$

(4)

(i) Area $= \int_2^4 \frac{8\sqrt{x} - 5}{2x^2}\,dx = \int_2^4 \left(4x^{-3/2} - \frac{5}{2}x^{-2} \right) dx$

$= \left[\frac{4x^{-1/2}}{-1/2} - \frac{5x^{-1}}{2 \times (-1)} \right]_2^4 = \left[-8x^{-1/2} + \frac{5}{2}x^{-1} \right]_2^4$

$= \left(-\frac{8}{\sqrt{4}} + \frac{5}{8} \right) - \left(-\frac{8}{\sqrt{2}} + \frac{5}{4} \right)$

$= -4 + \frac{5}{8} + 4\sqrt{2} - \frac{5}{4}$

Area $= 4\sqrt{2} - \frac{37}{8}$

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Question 8 continued

(ii) $\int_{-3}^{6}\left(\frac{1}{2}x^2 + k\right)dx = \left[\frac{1}{6}x^3 + kx\right]_{-3}^{6}$

$= \left(\frac{216}{6} + 6k\right) - \left(\frac{-27}{6} - 3k\right) = 36 + 6k + \frac{9}{2} + 3k$

$= \frac{81}{2} + 9k$

$\frac{81}{2} + 9k = 55 \Rightarrow 9k = \frac{29}{2}$

$k = \frac{29}{18}$

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Question 8 continued

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Question 8 continued

Q8

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9. Solutions based entirely on graphical or numerical methods are not acceptable in this question.

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$3\sin(2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2\cos\alpha = 3\sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$,

(b) find, showing all working, the value of α to 3 decimal places.

(5)

(i) $\sin(2\theta - 10°) = \frac{1}{3}$, $\quad -10° \leqslant 2\theta - 10° < 350°$

$2\theta - 10° = \sin^{-1}\left(\frac{1}{3}\right) = 19.471...°$ or $180° - 19.471...° = 160.528...°$

$\theta = \frac{19.471... + 10}{2}, \frac{160.528... + 10}{2} = 14.735...°, 85.264...°$

$\theta = 14.7°, \ 85.3°$

(ii)(a) Common difference: $\frac{1}{\tan\alpha} - \sin\alpha = 2\sin\alpha - \frac{1}{\tan\alpha}$

$\frac{2}{\tan\alpha} = 3\sin\alpha$

Using $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$: $\frac{2\cos\alpha}{\sin\alpha} = 3\sin\alpha$

$2\cos\alpha = 3\sin^2\alpha$ as required

(b) Using $\sin^2\alpha = 1 - \cos^2\alpha$: $2\cos\alpha = 3(1 - \cos^2\alpha)$

$3\cos^2\alpha + 2\cos\alpha - 3 = 0$

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Question 9 continued

$\cos\alpha = \dfrac{-2 \pm \sqrt{(2^2 - 4 \times 3 \times (-3))}}{2 \times 3} = \dfrac{-2 \pm \sqrt{40}}{6}$

$\cos\alpha = 0.72075\ldots$ or $\cos\alpha = -1.387\ldots$ (no solution, since $\cos\alpha \geqslant -1$)

$\cos^{-1}(0.72075\ldots) = 0.76589\ldots$ and $\pi < \alpha < 2\pi$, so

$\alpha = 2\pi - 0.76589\ldots = 5.51728\ldots$

$\alpha = 5.517$

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Question 9 continued

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Question 9 continued

Q9

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10. The curve C has equation

$$y = ax^3 - 3x^2 + 3x + b$$

where a and b are constants.

Given that

- the point $(2, 5)$ lies on C
- the gradient of the curve at $(2, 5)$ is 7

(a) find the value of a and the value of b.

(4)

(b) Prove that C has no turning points.

(3)

(a) $(2, 5)$ on C: $5 = 8a - 12 + 6 + b \Rightarrow 8a + b = 11$

$\frac{dy}{dx} = 3ax^2 - 6x + 3$

At $x = 2$, $\frac{dy}{dx} = 7$: $12a - 12 + 3 = 7 \Rightarrow 12a = 16 \Rightarrow a = \frac{4}{3}$

$b = 11 - 8 \times \frac{4}{3} = 11 - \frac{32}{3} = \frac{1}{3}$

$a = \frac{4}{3}$, $b = \frac{1}{3}$

(b) $\frac{dy}{dx} = 3 \times \frac{4}{3} x^2 - 6x + 3 = 4x^2 - 6x + 3$

At a turning point $\frac{dy}{dx} = 0$: $4x^2 - 6x + 3 = 0$

Discriminant $b^2 - 4ac = (-6)^2 - 4 \times 4 \times 3 = 36 - 48 = -12$

$-12 < 0$, so $\frac{dy}{dx} = 0$ has no real roots,

hence C has no turning points.

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Question 10 continued

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Question 10 continued

Q10

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